

## EXISTENCE OF FRACTAL BEHAVIOUR IN OZONE TIME SERIES

(Kewujudan Tingkah Laku Fraktal dalam Siri Masa Ozon)

NURYAZMIN AHMAT ZAINURI, ABDUL AZIZ JEMAIN & NORA MUDA

### ABSTRACT

Fractal has received a wide attention and has been used in many areas such as meteorology, stock market and also in medical field. It can be viewed as an object which has similar appearance when viewed at different scales which are known as self-similar. Thus, the scale invariance and scaling properties of the time series may be explored using fractal techniques. The aim of this study is to investigate the presence of fractal behaviour in the ozone time series. The presence of fractal behaviour can lead to the possibility of implementing the fractal approach in order to examine the properties of a time series. In this study, the daily average of ozone concentration from six selected air monitoring stations with different types of backgrounds in Peninsular Malaysia are used. From the autocorrelation function (ACF) plot obtained for the six stations, the slow decay in ACF values indicates that self-similarity are present where it shows that the series is having the property of fractal behaviour. In this paper, the existence of fractal behaviour is investigated by using the power spectrum method and the empirical probability distribution function. The result shows that the daily average of ozone concentration exhibits fractal behaviour for all the six monitoring stations considered.

*Keywords:* self-similar; power spectrum; empirical probability distribution function

### ABSTRAK

Fraktal telah mendapat perhatian yang meluas dan telah berkembang dalam pelbagai bidang seperti meteorologi, pasaran saham dan juga dalam bidang perubatan. Ia boleh dilihat sebagai suatu objek yang mempunyai penampilan yang sama apabila dilihat pada skala yang berbeza dan dikenali sebagai swasama. Oleh itu skala ketakvarianan dan sifat penskalaan bagi sesuatu siri masa boleh diterokai dengan menggunakan teknik fraktal. Tujuan kajian ini adalah untuk menyiasat kehadiran tingkah laku fraktal dalam siri masa ozon. Kehadiran tingkah laku fraktal ini membolehkan kemungkinan penggunaan pendekatan fraktal bagi mengkaji sifat-sifat siri masa. Dalam kajian ini, data purata kepekatan ozon harian bagi enam stesen pemantauan dengan latar belakang yang berbeza di Semenanjung Malaysia digunakan. Daripada plot fungsi autokorelasi (ACF) yang diperolehi untuk keenam-enam stesen, penyusutan yang sangat perlahan dalam nilai ACF menggambarkan kewujudan swasama yang mana ini memberikan petanda kehadiran tingkah laku fraktal dalam siri ini. Dalam makalah ini, kewujudan tingkah laku fraktal disiasat dengan menggunakan kaedah spektrum kuasa dan taburan kebarangkalian empirik. Keputusan menunjukkan bahawa purata kepekatan ozon harian mempamerkan kewujudan tingkah laku fraktal bagi semua stesen pemantauan yang digunakan.

*Kata kunci:* swasama; spektrum kuasa; taburan kebarangkalian empirik

## 1. Introduction

Air pollution is an important and serious problem which can cause adverse environmental and ecological effects. One of the major pollutants that is harmful and may results in serious effect to humans and environment is ozone (Afroz *et al.* 2003; Gurjar *et al.* 2008; Mills *et al.* 2011; Neidell & Kinney 2010; Pudasainee *et al.* 2010). Ozone is an odourless gas created by a

chemical reaction in the presence of sunlight between nitrogen oxides (NO<sub>x</sub>) and volatile organic compounds.

The increase in ozone concentrations has received much attention among researchers worldwide including in Malaysia. High ozone exposure may lead to adverse effect on human such as asthma, coughing, pneumonia, bronchitis and other respiratory problems. Most of the studies done are based on statistical descriptions of the ozone trend, seasonal pattern and the relationship between ozone and other factors such as chemical precursors, intensity of ultra-violet (UV) radiation and biomass burning (Awang *et al.* 2013; Azmi *et al.* 2010; Ismail *et al.* 2010).

In this study, the ozone time series are investigated for the presence of fractal behaviour. The existence of fractal behaviour may lead to the possibility of implementing the fractal approach to examine the properties of a time series (Shang *et al.* 2007). Fractal analysis can be used to check the property of ozone time series and thus the findings may provide an alternative method for environmental study and also air quality monitoring.

Fractals are objects which have similar appearance when viewed at different scales which are known as self-similar. By using fractal techniques, the scale invariance and scaling properties of a time series may be explored. If the scaling behaviour of an object may be characterized by a single scaling exponent, then the fractal behaviour of that object may be described by using monofractal model. Whereas when multiple scaling exponents are needed to describe the scaling behaviour, then a multifractal model is used. Studies done by Lee (2002) and Lee *et al.* (2003b) showed that fractal analysis may be used as a tool to characterize, analyse and compare the temporal characteristics of air pollution concentrations. Lee *et al.* (2003a) and Ho *et al.* (2004a) showed that scale invariance existed in air pollution data when analysing with fractal methods. Fractal are also used in other fields such as wind speed, aftershock sequence of stock market, rainfall, earthquake, fisheries and medical field (Bojić *et al.* 2010; Ho *et al.* 2004b; Montes *et al.* 2012; Olsson *et al.* 1993; Telesca *et al.* 2004).

The aim of this study is to investigate the presence of fractal behaviour in the ozone concentration observed at six air monitoring stations in Peninsular Malaysia. The existence of fractal behaviour will be investigated using the power spectrum method and the empirical probability distribution. The paper is organized as follows. The first part of this paper describes the details of the data used in this study. Then the second part explains the methods employed in the study to investigate the presence of fractal behaviour in a time series and followed by the result and discussion. Finally, the conclusion is summarized in the final part.

## **2. Data and Methods**

### **2.1. Data**

Daily mean hourly ozone concentrations data used for this study were obtained from the Air Quality Division of the Department of Environment (DOE), Malaysia. The Air Pollutant Index (API) is used to report the status of the air quality and its effect to health.

Table 1: List of Stations

Code	Name of the Stations	State	Type of the Station	Data Set
St1	Shah Alam	Selangor	Urban	1998 – 2006 (9 years)
St2	Petaling Jaya	Selangor	Industrial	2004 – 2006 (3 years)
St3	Klang	Selangor	Urban	1998 – 2006 (9 years)
St4	Nilai	Negeri Sembilan	Industrial	1998 – 2006 (9 years)
St5	Tanjung Malim	Perak	Sub urban	2000 – 2006 (7 years)
St6	Jerantut	Pahang	Background	1998 – 2006 (9 years)

For the purpose of this study, six stations were chosen from different monitoring sites located at east coast of Peninsular Malaysia. The list of stations considered is shown in Table 1. The Shah Alam (St1), Petaling Jaya (St2), Klang (St3) and Nilai (St4) monitoring stations are surrounded by residential and industrial areas and are also influenced by congested traffic conditions. The Tanjung Malim (St5) monitoring station is surrounded by residential areas and large forested areas while Jerantut (St6) monitoring station acts as the background station which is surrounded by agricultural areas.

The data set used for St1, St3, St4 and St6 are over a period of 9 years from 1998 to 2006. While for St2, only 3 years of data are available that is from 2004 to 2006 and for St5, the data are over 7 years which is from 2000 to 2006. The missing values were treated using K-Nearest Neighbour (KNN) method where missing values are interpolated using nearest neighbour value. This method has been proved to be among the best method of imputation for air quality data sets in Malaysia (Nuryazmin *et al.* 2015).

## 2.2. Methods

From the daily mean hourly ozone time series data, we plot the time series data for the six stations considered in this study as shown in Figure 1. The fluctuation behaviour can be visualized graphically from this plot.

In Figure 1, it can be seen that the ozone concentration for all the six stations exhibit fluctuation behaviour. There seems to exist high variation in all the six ozone time series but none can be said regarding the existence of fractals.

Next, the autocorrelation, ACF plot for the six stations are also examined and shown in Figure 2. The plots are useful to determine the correlation in the values of a time series data given the past or present values.

From Figure 2, it can be seen that the ACF plots show a very slow exponentially decay with an increasing lag. The slow decay in ACF values shows that the ozone concentration in time series is dependent of each other which are known as persistence. Based on Rodriguez-Iturbe *et al.* (1989), these phenomena may be related to the fractal properties. The slow decay in ACF values also shows that self-similarity are present in the ozone time series. Self-similarity is the property of fractal where an object looks and behaves the same regardless of any scales.

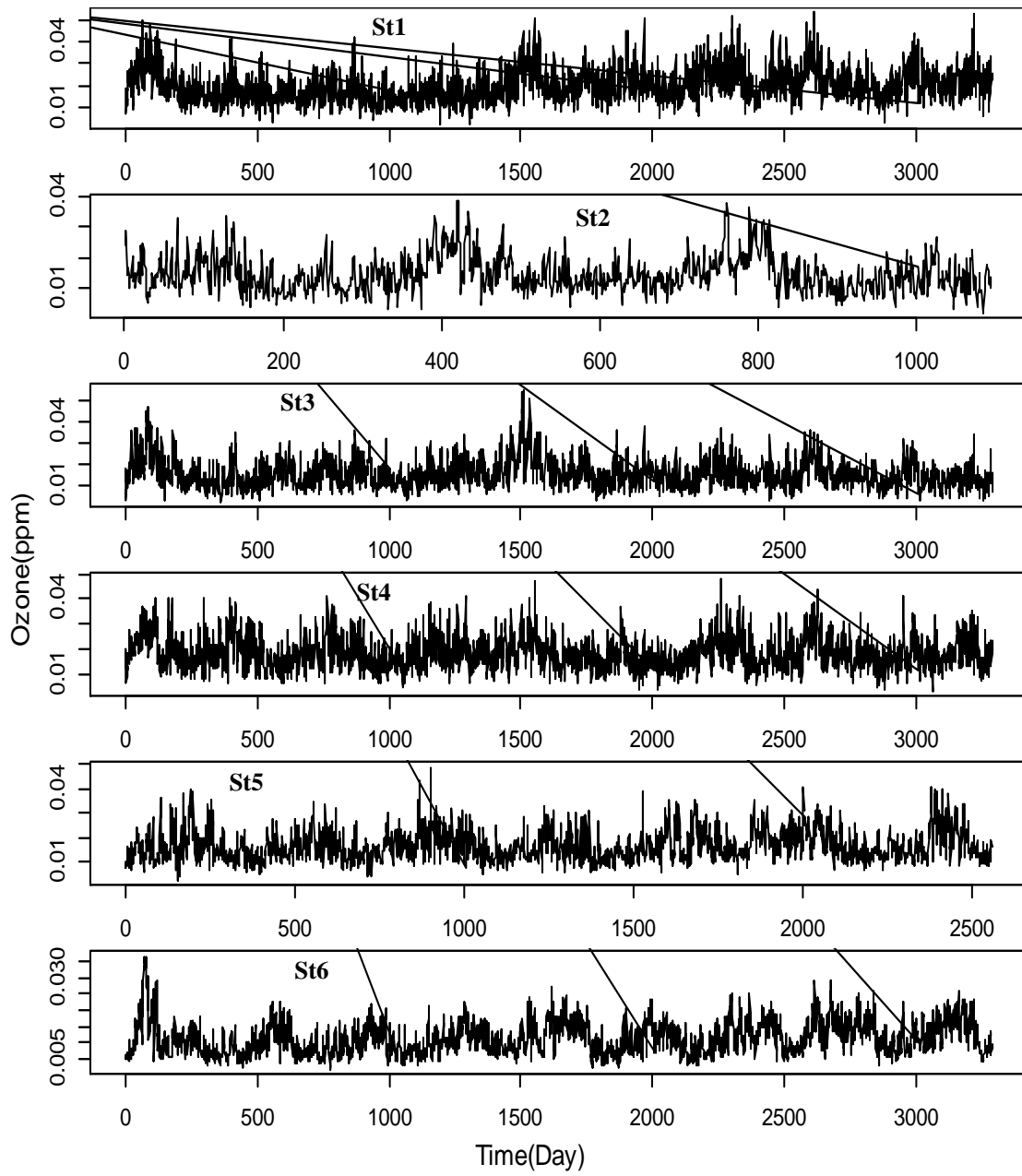


Figure 1: Time series plot of ozone concentration for the six stations

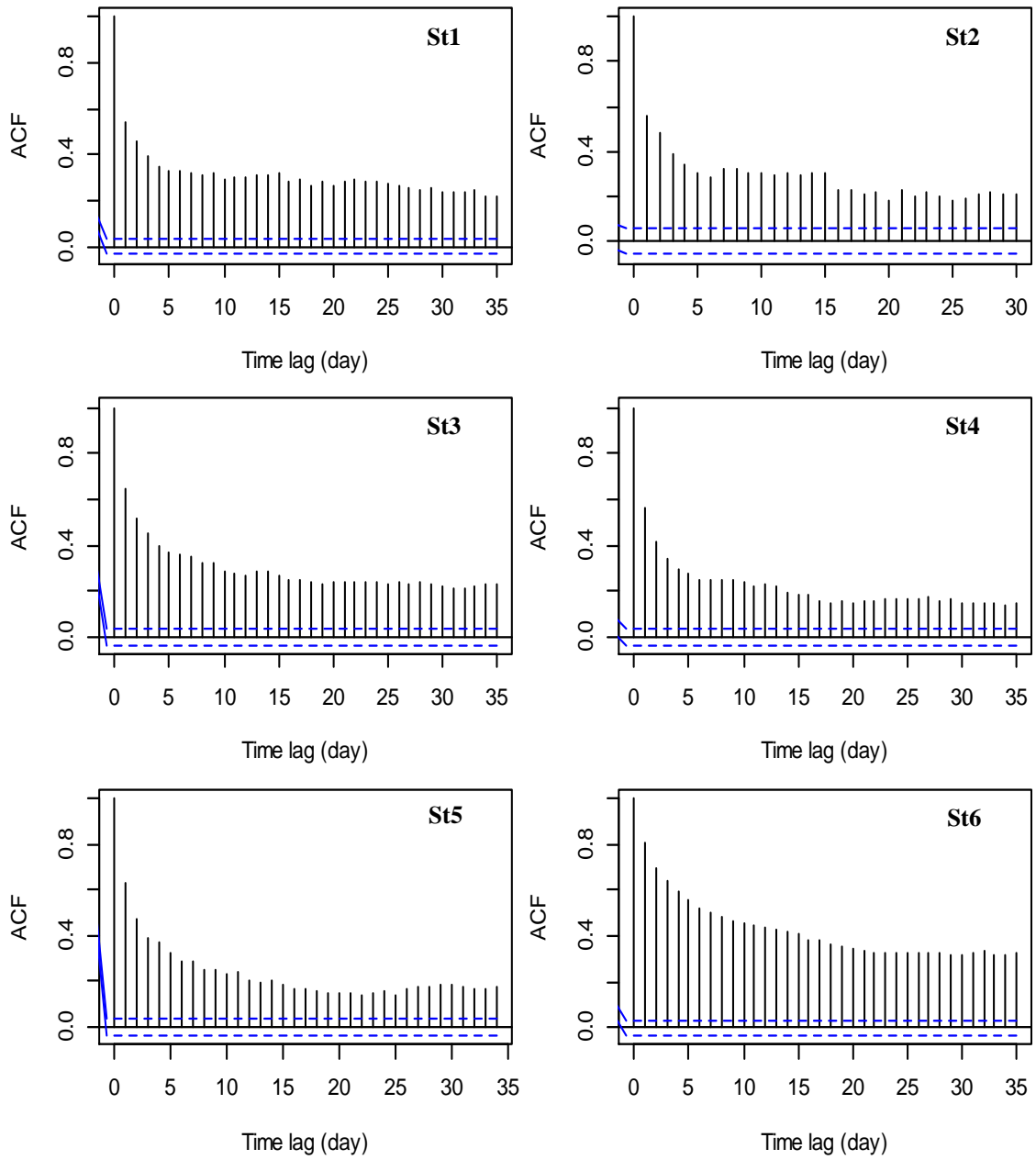


Figure 2: The ACF plot for the six stations

### 2.2.1. Power Spectrum

The power spectrum is one of the common and standard tool to investigate fractal behaviour in nonlinear time series data (Kai *et al.* 2008; Lovejoy & Mandelbrot 1985; Olsson *et al.* 1993; Shang *et al.* 2007). The power spectrum function,  $S(f)$  is given by

$$S(f) \propto f^{-\delta}. \quad (1)$$

where  $f$  is the frequency and  $\delta$  is the spectral exponent. From Eq. (1),  $\delta$  can be determined from the slope of the plot  $\log[S(f)]$  versus  $\log[f]$ . In this study,  $f$  represent the frequency for the daily mean hourly concentration.

If a time series spectrum obeys the power law, it shows the absence of a characteristic time scales on the range of the power law, i.e. the scaling behaviour. Thus, this indicates that the fractal behaviour exists as the fluctuations at all scales within the range are related to each other.

### 2.2.2. Empirical probability distribution

Empirical probability distribution function (PDF) can be used to describe the fluctuation of fractal nature of the threshold of a time series at a specific time scales (Shang *et al.* 2007).

Let  $X$  represent a time series data and for a chosen threshold value  $x$ , the tail of the distribution of  $X$  follows a power law of the form

$$\Pr(X > x) \propto x^{-D}. \quad (2)$$

where  $D$  is the probability exponent.  $D$  can be obtained from the slope of plot  $\log(\Pr(X > x))$  versus  $\log(X)$ . If the time series data shows a hyperbolic tail distribution, then there exists fractal behaviour. However, this feature is not a necessary feature of a multifractal process (Fraedrich & Larnder 1993).

The value of  $D$  can determine the fractal behaviour whether the data is monofractal or multifractal. When  $D < 2$ , it indicates that a monofractal model is sufficient to describe the data while when  $D > 2$ , a multifractal model is needed.

## 3. Results and Discussion

Figure 3 shows the power spectrum of the daily mean hourly ozone concentration for the six stations. The spectrum appears to consist of two regions of power law within a certain range in the high-frequency and low-frequency region for all the six stations. As to these six time series, for the high-frequency region, the spectral exponent  $\delta$  are 0.63 (St1), 0.62 (St2), 1.13 (St3), 0.81 (St4), 1.19 (St5), and 1.17 (St6); while for the low-frequency region, the spectral exponent  $\delta$  are 0.17 (St1), 0.24 (St2), 0.18 (St3), 0.40 (St4), 0.39 (St5), and 0.48 (St6). These results indicate that scaling behaviour exist in the ozone concentration time series and thus, may be considered as an indication of fractal behaviour. However, it should be noted that the procedure of estimating the spectral exponent and the scaling region depend on individual judgment and hence extreme caution is needed in identifying the existence of fractal behaviour using this method (Shang *et al.* 2007).

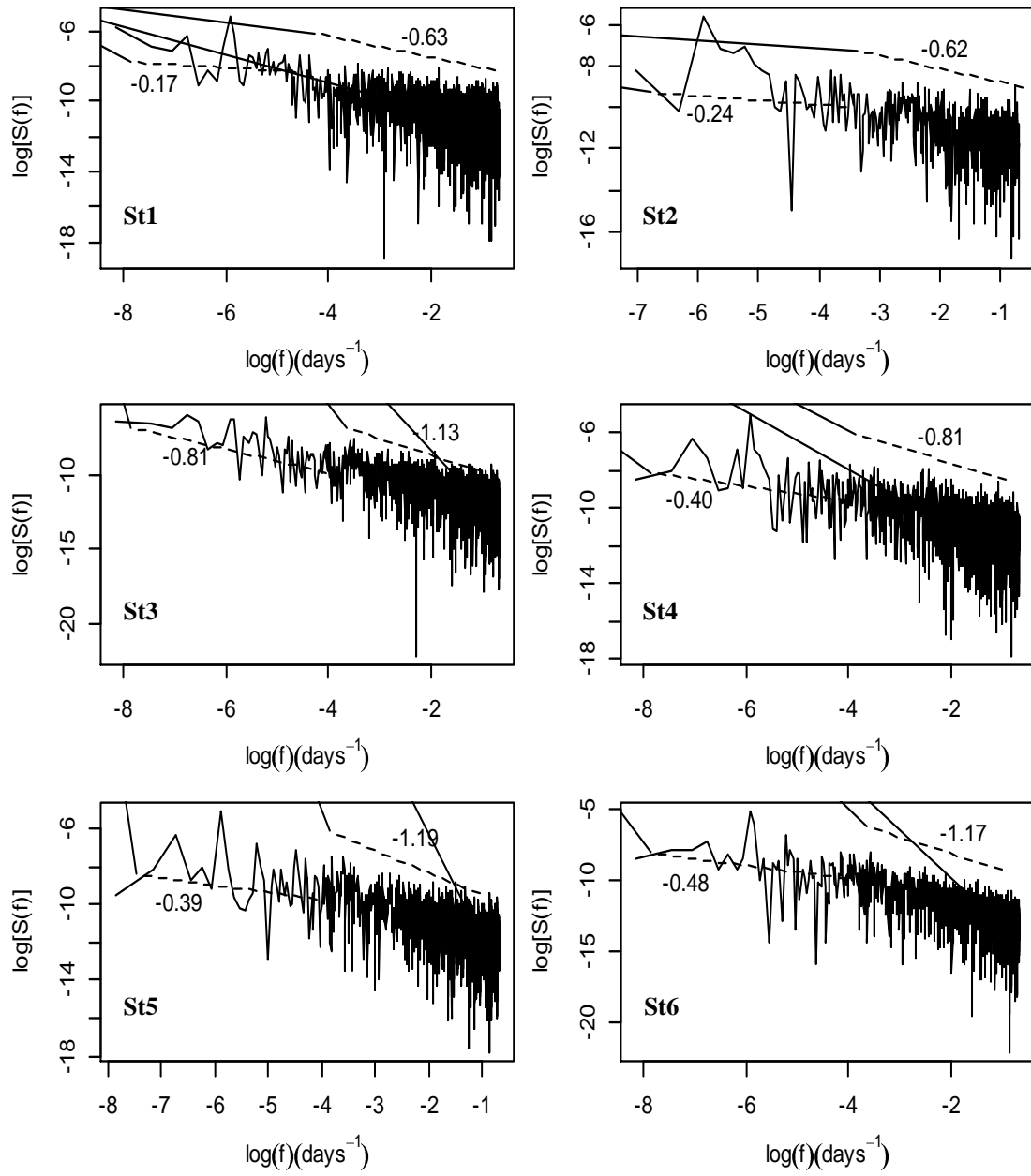


Figure 3: Power spectrum of the daily mean hourly ozone concentration

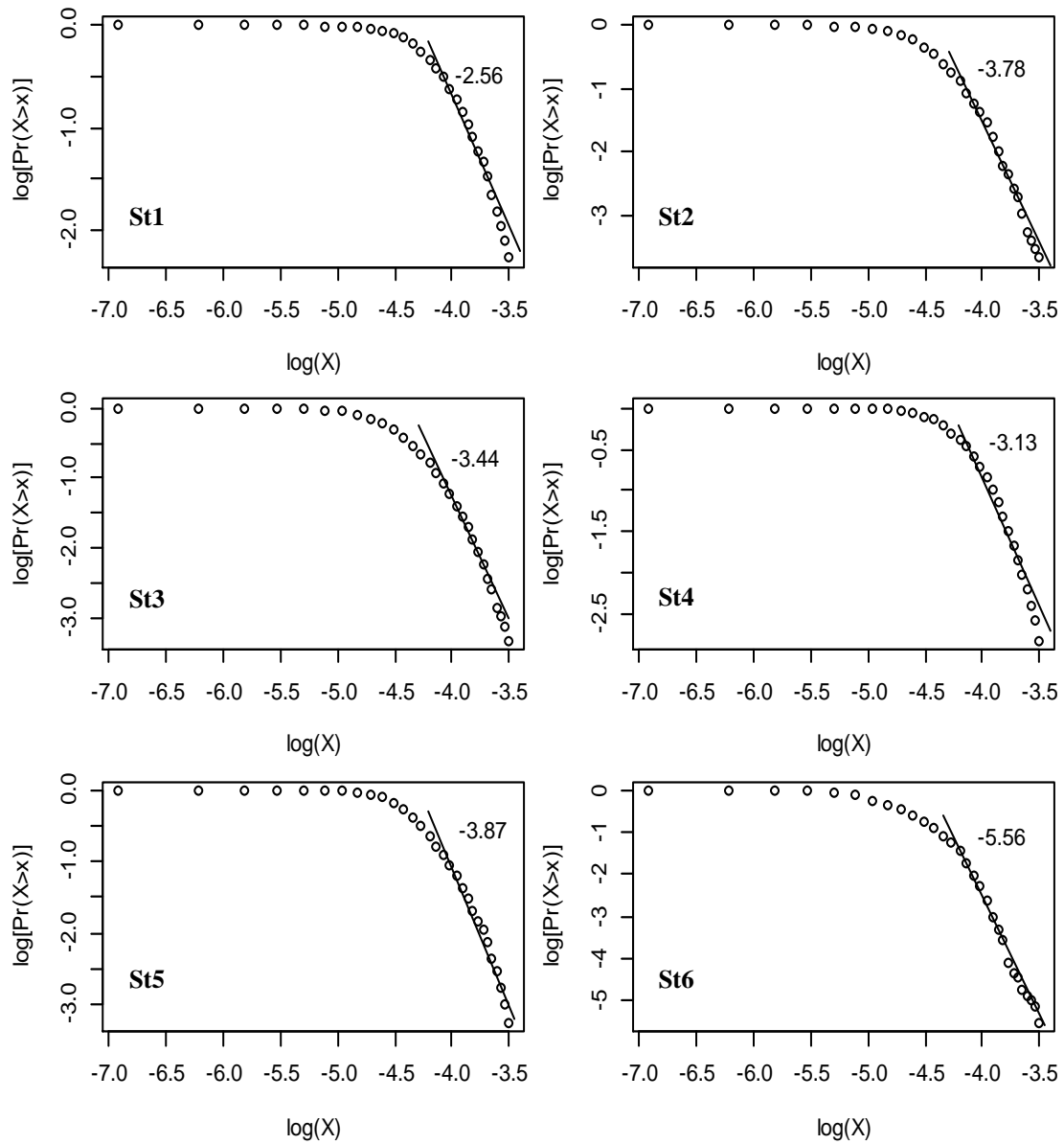


Figure 4: Empirical probability distribution function for the daily mean hourly ozone concentration

The empirical PDF obtained for the daily mean hourly ozone concentration for the six monitoring stations are shown in Figure 4. It can be observed that the plot exhibit a hyperbolic tail behaviour. Hence, fractal behaviour exists in the daily mean hourly ozone concentration for all the six stations. The values of the probability exponent  $D$ , estimated from the slope of the regression line are shown in Figure 4 and we found that  $D > 2$  for all the six stations. Since  $D > 2$ , this suggest that a multifractal model is needed for the ozone time series.



#### 4. Conclusion

In this study, we aim to investigate the presence of fractal behaviour in the daily mean hourly ozone concentration time series observed at six air monitoring stations in Peninsular Malaysia. Different types of monitoring stations are tested for the existence of fractal behaviour. The power spectrum and the empirical probability distribution (PDF) are used to investigate the presence of fractal behaviour. The existence of scaling behaviour in the power spectrum of the ozone time series indicates the presence of fractal behaviour. The empirical PDF also shows that fractal behaviour exists. From the probability exponent  $D$  values, a multifractal model is found to be needed in order to characterize the ozone behaviour. The result obtained is a preliminary finding on the existence of fractal behaviour in daily mean hourly ozone time series. The finding may lead to the possibility of implementing the fractal approach to examine the properties of the ozone concentrations based on the types of monitoring stations. A further investigation of fractal behaviour using other methods is still needed to identify and provide a strong proof regarding the existence of fractal.

#### Acknowledgments

The authors would like to thank Department of Environment Malaysia and Prof. Dr. Mohd Talib Latif from Universiti Kebangsaan Malaysia for providing information and data. This research is funded by Universiti Kebangsaan Malaysia research grant (UKM-AP-2011-19).

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*Fundamental Studies of Engineering Unit  
Faculty of Engineering and Built Environment  
Universiti Kebangsaan Malaysia  
43600 Bangi, Selangor DE  
MALAYSIA  
E-mail: nuryazmin@gmail.com \**

*School of Mathematical Sciences  
Faculty of Science and Technology  
Universiti Kebangsaan Malaysia  
43600 Bangi, Selangor DE  
MALAYSIA  
E-mail: azizj@ukm.my, noramuda@ukm.my*

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\*Corresponding author