

## Structural Analysis of Geothermal Decline Rate

(Analisis Struktur Kadar Penurunan Geotermal)

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### ABSTRACT

*The aim of semivariogram modeling is to infer the structure of spatial continuity of the measurements. Practical experiences show that semivariogram modeling is an important step in spatial interpolation. The usual empirical semivariogram is sensitive to extreme data and shows a noised pattern. Some robust empirical semivariogram was proposed. This paper reports the application of pairwise relative empirical semivariogram to Kamojang geothermal decline rate. Using the same data, the usual empirical semivariogram and pairwise semivariogram are compared. Comparative study shows that the empirical pairwise relative semivariogram is able to infer the structure of spatial continuity of the process.*

*Keywords: Semivariogram; pairwise semivariogram; decline rate*

### ABSTRAK

*Pemodelan semivariogram bertujuan meneliti kontinuiti spasial daripada pengukuran. Pemodelan semivariogram merupakan langkah penting dalam interpolasi spasial. Semivariogram peka terhadap data ekstrim dan memperlihatkan pola tidak teratur (noised). Beberapa semivariogram kekar (robust) telah diusulkan. Makalah ini melaporkan aplikasi semivariogram pairwise relatif empirikal pada kadar penurunan geotermal Kamajong. Dengan menggunakan data yang sama, semivariogram biasa dengan semivariogram pairwise dibandingkan. Perbandingan menunjukkan bahawa semivariogram pairwise relatif empirik mampu memperlihatkan kontinuiti struktur proses tersebut.*

*Kata kunci: Semivariogram; pairwise semivariogram; kadar penurunan*

### INTRODUCTION

Spatial analysis has been introduced in geothermal reservoir analysis to conduct spatial interpolation (Saptadji et al. 2005), and has been combined with geological interpretation to assess the degree of variation of permeability. Based on the theory of small fluctuations, Yortsos and Al-Afaleg (1997) present a permeability semivariogram from pressure transients of multiple wells. Hewett (1986) shows that transport properties of fluid flow in heterogenous porous media are determined by the structure of spatial correlations in the permeability distribution. Fairley et al. (2003) presented a spatial description of permeability in an active fault zone. The fault hydraulic structure was inferred from spatial analysis of temperatures in 243 geothermal springs. The study concluded that the fault zone is predominately low permeability interspersed with few high permeability channels. In order to reveal spatial characteristics of temperature distribution, a semivariogram study was conducted on temperature profiles in the Yanaizu-Nishiyama geothermal field (Shoji 2000). Shoji expected that temperature distribution in a geothermal field is accurately predicted by the combination of trend analysis and spatial interpolation.

At Kamojang geothermal field, seventy-seven wells have been drilled since 1974, and fifty-three are located in the main production area. The other are drilled for both

standby as maintenance 140 MW and for future expansion of 60 MW. Flow rate at points in a borehole is correlated with formation permeability (Kamah et al. 2005). Decline rate  $D$  is a geological parameter related to reservoir production, and has a significant contribution to reservoir flow simulation. Decline rate analysis of Kamojang field has been published, but the semivariogram analysis has not yet been reported.

The usual empirical semivariogram is sensitive to the skewness of data distribution. Some robust empirical semivariogram was proposed in order to find a better spatial correlation representation (Deutsch & Journel 1997). This paper investigates the applicability of pairwise relative semivariogram  $\hat{\gamma}_{PR}$  to Kamojang decline rate. The data, listed in Table 1, was reported by Sasradipoera et al. (2000). Based on type curve matching, back pressure equation and Arps equation, Sasradipoera et al. concluded that static pressure of wells located near the injection well is lower than that are located far away from injection well.

### DECLINE CURVES

Decline curves are one of the most extensively used methods in the evaluation of reservoir production. Proposed sixty years ago, the empirical Arps equation represents the relationship between production rate  $q_t$  and time  $t$  (Arps 1945; Li & Horne 2003).

TABLE 1. Decline rate of reservoir static pressure (Sasradipoera et al. 2000). The decline rates of wells located near injection well are lower compared to the wells located far from injection well

Wells located around K-15 injection well, average decline rate is .3113				
Well	P <sub>s</sub> ,intitial (ksc)	P <sub>s</sub> ,intitial (ksc)	Period (year)	Decline rate (t <sup>-1</sup> )
11	32.80	28.41	14.68	.2990
14	33.00	20.18	14.78	.8674
17	32.00	31.48	14.16	.0367
18	32.20	29.87	14.66	.1589
43	32.80	30.71	10.75	.1944
Wells located around K-32 injection well, average decline rate is .7294				
Well	P <sub>s</sub> ,intitial (ksc)	P <sub>s</sub> ,intitial (ksc)	Period (year)	Decline rate (t <sup>-1</sup> )
28	32.20	26.52	11.00	.6073
31	32.40	24.90	9.58	.7829
33	31.00	27.91	2.83	1.0919
34	32.90	22.05	10.49	1.0343
38	35.00	27.46	11.16	.6756
45	34.00	24.91	1.41	.7967
52	31.30	30.58	6.16	.1169
Wells located far from injection wells, average decline is 1.1228				
Well	P <sub>s</sub> ,intitial (ksc)	P <sub>s</sub> ,intitial (ksc)	Period (year)	Decline rate (t <sup>-1</sup> )
67	29.50	26.89	1.66	1.5723
25	33.00	28.94	9.14	.4442
39	31.60	20.33	8.97	1.2564
44	33.30	19.76	9.32	1.4528
51	31.50	29.24	6.14	.3681
26	34.00	21.41	11.48	1.0067
27	33.30	25.66	11.58	.6598
30	33.00	20.82	10.13	1.2024
35	33.50	23.46	6.77	1.4830
36	33.50	27.36	11.57	.5307
40	31.60	27.33	2.68	1.5933
42	33.00	17.47	8.72	1.7810
46	32.00	20.06	10.41	1.1470
62	29.00	26.17	2.33	1.2146
65	29.50	27.19	2.22	1.0410

$$q_t = \frac{q_0}{1 + bD_0t^{1/b}}$$

where  $q_t$  is the production rate at time  $t$  and  $q_0$  is the initial production rate,  $b$  is the reciprocal of decline curve exponent ( $1/b$ ), and  $D_0$  is the initial decline rate  $t^{-1}$ . A great number of studies on production decline analysis are based on Arps empirical equation. The Arps equation can

be reduced in two special cases:  $b = 0$  and  $b = 1$ ,  $b = 0$  represents an exponential decline, which is expressed as

$$\ln q_t = \ln q_0 + D_0 t$$

The parameters  $D_0$  are  $q_0$  estimated from least squares method.  $b = 1$  represents a harmonic decline, which can be represented as

$$\ln q_t = \ln q_0 - \frac{D_0}{q_0} Q t$$

where  $Q_t = \int_0^t q_t dt = \frac{q_0}{D_0} \ln(1 + D_0 t)$ . A hyperbolic decline,  $0 < b < 1$ , satisfies the relationships

$$\ln q_t = q_0 - \frac{1}{b} \ln(1 + b D_0 t)$$

and

$$Q_t = \frac{q_0^b}{1 - b D_0} q_t^{1-b} - q_t^{1-b}$$

Gentry (1972) explain a simple and effective method for graphically solving all three types of production decline.

The decline rate,  $D(t)$ , is defined as a derivative (Spivey 1986)

$$D(t) = - \frac{dq_t / dt}{q_t}$$

where  $q(t)$  is the flow rate at time  $t$ . Flow rate and decline rate satisfies the differential equation relationship

$$D(t) = (dq/dt)/q = -Kq^b, 0 < b < 1$$

Decline rates are available only at well locations. To improve reservoir description, spatial analysis is proposed that incorporates the decline rate and well locations to predict the decline rate at locations where no measurements are available and to produce a reservoir decline map via kriging interpolation. GeTools uses spatial interpolation to predict the reservoir characteristics at unsampled locations (Saptadji et al. 2005).

PAIRWISE RELATIVE EMPIRICAL SEMIVARIOGRAM

A semivariogram is a measure of spatial correlation; it replaces the distance  $d$  by a dissimilarity distance  $\gamma(d)$  that is specific to the measurement under study. This indice measures the degree of dissimilarity between an unknown value at location  $s$ ,  $Z(s)$ , and a nearby measurement. The dissimilar measurements should receive lesser weight in the estimation of  $Z(s)$ . The usual empirical semivariogram is defined as half of the average squared difference between two measurements separated by vector  $d$ :

$$\hat{\gamma}(d) = \frac{1}{2|N(d)|} \sum_{i=1}^{N(d)} [Z(s_i) - Z(s_i + d)]^2$$

where  $N(d) = \{(i, j), s_i - s_j = d\}$  and  $|N(d)|$  is the number of distinct elements of  $N(d)$ . The usual empirical semivariogram  $\hat{\gamma}$  is affected by data sparsity when applied to skewed distributions.

Pairwise relative semivariogram  $\hat{\gamma}_{PR}$  is defined as traditional semivariogram where each pair is normalized by the squared average of the tail  $Z(s_i)$  and head values,  $Z(s_i + d)$ ,  $i = 1, 2, \dots, N(d)$

$$\hat{\gamma}_{PR}(d) = \frac{1}{2|N(d)|} \sum_{i=1}^{N(d)} \frac{Z(s_i) - Z(s_i + d)}{\left[ \frac{Z(s_i) + Z(s_i + d)}{2} \right]^2}$$

Figure 1 shows the usual semivariogram  $\hat{\gamma}$  plot of Kamojang well decline rate. A valid semivariogram model (Armstrong & Diamond 1984) cannot be fitted due to the irregularity of the plot.

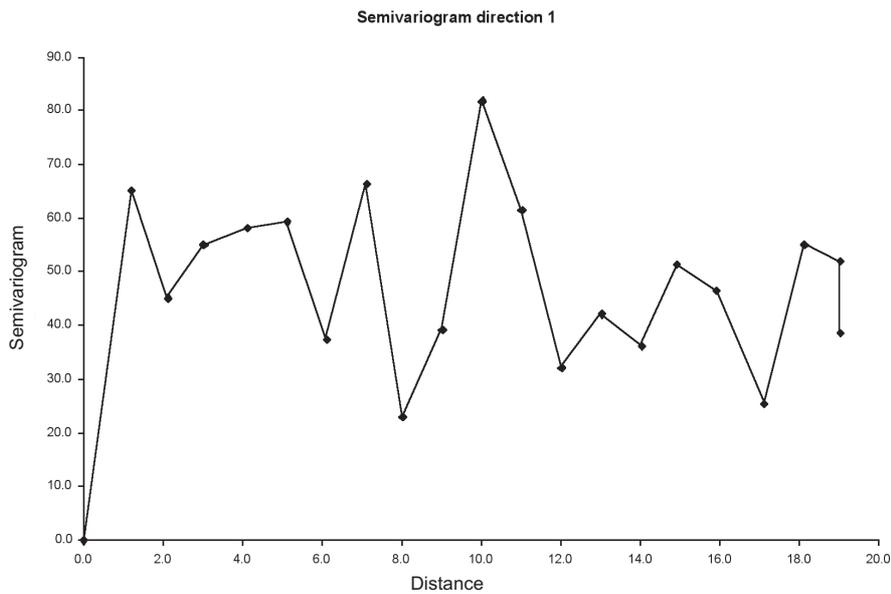


FIGURE 1. Traditional semivariogram  $\hat{\gamma}$  plot of Kamojang decline rate. A valid semivariogram model cannot be fitted due to irregularity of the plot

The exploratory analysis of the measurements aims to describe the spatial continuity of the process under investigation. If a semivariogram fails to produce a clear description, exploring the causes of disappointing results often leads to new insights into the data set. As the separation distance between pairs increase, the semivariogram also increase. An increase in the separation distance causes the semivariogram reaches a constant variance, called a sill. The distance at which the semivariogram reaches a constant is called the range. An attempt at improving the clarity of the experimental semivariogram through a comparison with different measure of spatial continuity should be investigated.

Plotting a histogram of the variable under study should precede any calculation of semivariogram. Figure 2 shows the histogram of decline rate data. The distribution is

asymmetric with skewness coefficient 1.5. Due to four extreme observations, the usual empirical semivariogram fails to produce a clear description of spatial correlation.

The pairwise semivariogram of decline rates was calculated and presented in Figure 3. The plot shows a better spatial description compared with usual empirical semivariogram as shown in Figure 1. A spherical correlation model with range 150 m and sill 0.8 can be fitted to the empirical pairwise semivariogram. The interpretation of a semivariogram is related the knowledge of the production decline. The spatial correlation is significant for a distance less than 150 m. For a distance larger than 150 m, the production declines are uncorrelated. The range characterizes a transition phenomenon of zone of influence of a well decline rate.

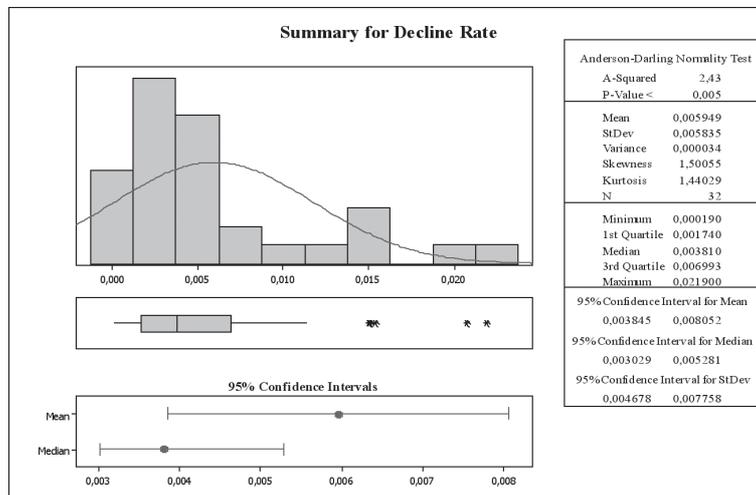


FIGURE 2. Histogram of decline rate of Kamojang geothermal field. The skewness 1.5 indicates that the decline rate distribution is positively skewed. Anderson-Darling normality test 2.43 with P-value < 0.005 indicates that normality assumption is rejected due to four extreme observations

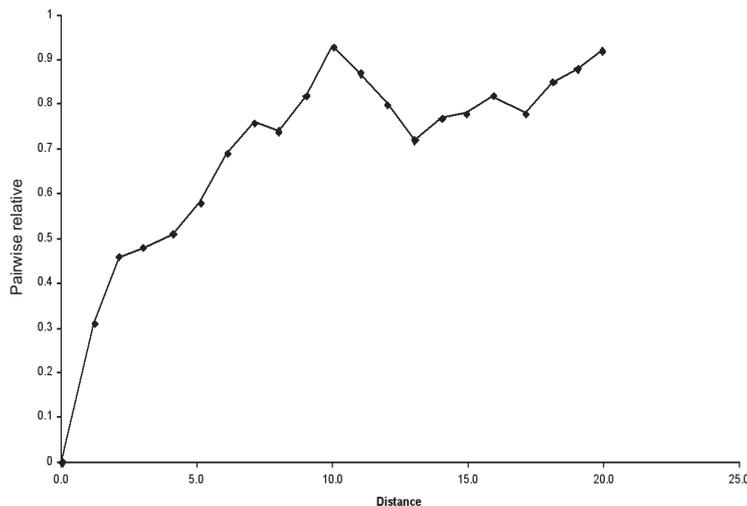


FIGURE 3. Pairwise relative semivariogram of decline rate. The plot shows a well spatial structured of decline rate. A spherical model can be fitted to the empirical pairwise semivariogram

## CONCLUSION

The usual empirical semivariogram fails to produce a regular pattern due to the asymmetric data distribution. Some robust empirical semivariogram was proposed and published in the literature. In this paper, a pairwise semivariogram was proposed to study the spatial correlation structure of flow rate decline of steam production. The correlation structure of geothermal production decline rate was investigated using usual semivariogram and pairwise relative semivariogram. The case study shows that the pairwise relative empirical semivariogram is able to infer the structure of spatial continuity of the process.

## Nomenclature

$\hat{\gamma}$  = usual empirical semivariogram,  $\hat{\gamma}_{PR}$  = pairwise relative empirical semivariogram

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