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COMPLEX FUZZY SOFT GROUP

(Kumpulan Lembut Kabur Kompleks)

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ABSTRACT

In this paper, we define complex fuzzy soft group and introduce some new concepts such as complex fuzzy soft functions and homogeneous complex fuzzy soft sets. Then we investigate some characteristics of complex fuzzy soft group.The relationship between complex fuzzy soft group and fuzzy soft group is also investigated. It is found that every complex fuzzy soft group yields two fuzzy soft groups. Finally we define the image and inverse image of a complex fuzzy soft group under complex fuzzy soft homomorphism and we study their elementary properties.

Keywords: complex fuzzy soft group; homogeneous complex fuzzy soft set; complex fuzzy soft homomorphism

ABSTRAK

Dalam makalah ini, ditakrifkan kumpulan lembut kabur kompleks dan perkenalkan beberapa konsep baharu seperti fungsi lembut kabur kompleks dan set lembut kabur kompleks yang homogen. Kemudian diselidiki beberapa cirian bagi kumpulan lembut kabur kompleks. Hubungan di antara kumpulan lembut kabur kompleks dengan kumpulan lembut kabur juga diselidiki. Ditemui bahawa setiap kumpulan lembut kabur kompleks menghasilkan dua kumpulan lembut kabur. Akhirnya ditakrifkan imej dan imej songsang bagi kumpulan lembut kabur kompleks di bawah homomorfisma lembut kabur kompleks serta dikaji sifat-sifat permulaannya.

Kata kunci: kumpulan lembut kabur kompleks; set lembut kabur kompleks yang homogen; homomorfisma lembut kabur kompleks

1. Introduction

Zadeh introduced the concept of fuzzy sets in 1965 (Zadeh 1965). A fuzzy set A in U is defined by a membership function $\mu_A : U \rightarrow [0,1]$, where U is nonempty set, called universe. The concept of fuzzy set spreads in many pure mathematical fields, such as topology (Lowen 1976), algebra (Hungerford 1974) and complex numbers (Buckley 1989). In 1971, Rosenfeld (1971) introduced the concept of fuzzy subgroup. In 1979, Anthony and Sherwood redefined fuzzy subgroup of a group using the concept of triangular norm. In 2002, Ramot *et al*. (2002) introduced the concept of the complex fuzzy sets. The novelty of the complex fuzzy set lies in the range of values its membership function may attain. In contrast to a traditional fuzzy membership function, this range is not limited to $[0,1]$, but extended to the unit circle in the complex plane. Nadia (2010) introduced the concept of complex fuzzy soft set in 2010. Dib (1994) introduced the concept of fuzzy space. Al-Husban and Abdul Razak (2016) introduced the concept of complex fuzzy group based on complex fuzzy space. They used the approach of Yossef and Dib (1992) and generalised it to complex realm by following the approach of Ramot *et al*. (2002) and Tamir *et al*. (2011). In 2017, independently Alsarahead and Ahmad (2017) introduced the concept of complex fuzzy subgroup which is a combination between two mathematical fields on fuzzy set, that are complex fuzzy set and fuzzy subgroup.We used Rosenfeld's approach and generalise it to the complex realm by following Ramot's approach.

The generalisation of fuzzy soft group to complex fuzzy soft group is not yet done either using Dib's or Rosenfeld's approach. Here in this paper, we define the complex fuzzy soft groups using Rosenfeld's approach and introduce some new concepts like complex fuzzy soft functions and homogeneous complex fuzzy soft sets. Then we investigate some of the characteristics of complex fuzzy soft groups. Finally we define the image and inverse image of complex fuzzy soft groups under complex fuzzy soft homomorphism and we studied their elementary properties.

2. Preliminaries

In this section, we present the basic definitions and results of $\pi-$fuzzy sets, complex fuzzy soft sets and fuzzy soft group which are necessary for subsequent discussions.

Definition 2.1.(Ramot *et al*. 2002) A complex fuzzy set, defined on a universe of discourse U, is characterised by a membership function $\mu_A(x)$ that assigns any element $x \in U$, a complex-valued grade of membership in A. By definition, the values $\mu_A(x)$ may receive all lie within the unit circle in the complex plane, and are thus of the form $r_A(x)e^{i\omega_A(x)}$, where $i=\sqrt{-1}$, $r_A(x)$ and $\omega_A(x)$ are both real-valued, and $r_A(x)\in[0,1]$, $\omega_A(x)\in[0,2\pi]$. The complex fuzzy set may be represented as the set of ordered pairs

$$A=\{(x,\mu_A(x)):x\in U\}.$$

Definition 2.2. (Alsarahead & Ahmad 2017) Let $A=\{(x,\mu_A(x)):x\in U\}$ be a fuzzy set. Then the set $A_\pi=\{(x,\mu_{A_\pi}(x)):x\in U\}$ is said to be a $\pi-$fuzzy set where $\mu_{A_\pi}(x)=2\pi\mu_A(x)$.

Definition 2.3. (Alsarahead & Ahmad 2017) Let G be a group and $A_\pi=\{(x,\mu_{A_\pi}(x)):x\in G\}$ be a $\pi-$fuzzy set of G. Then A_π is said to be a $\pi-$fuzzy subgroup if the following hold

(i) $\mu_{A_\pi}(xy)\ge\min\{\mu_{A_\pi}(x),\mu_{A_\pi}(y)\}$ for all x,y in G.

(ii) $\mu_{A_\pi}(x^{-1})\ge\mu_{A_\pi}(x)$ for all x in G.

Proposition 2.4. (Alsarahead & Ahmad 2017) *A $\pi-$fuzzy set A_π is a $\pi-$fuzzy subgroup if and only if A is a fuzzy subgroup.*

Definition 2.5. (Nadia 2010) Let U be the initial universe and E be the set of parameters. Let S^U denote the set of all complex fuzzy sets of U, $A\subset E$ and $f:A\mapsto S^U$. A pair (f,A) is said to be a complex fuzzy soft set over U.

Let U be a universe set and (f,A) be a complex fuzzy soft set over U. Then (f,A) yields two fuzzy soft sets over U as follows

(i) The fuzzy soft set $(\overline{f},A)$, where $\overline{f}:A\mapsto \overline{s}^U$ and $\overline{s}^U$ is the set of all fuzzy sets of the form $\{(x,r_{f(a)}(x)):x\in U,a\in A\}$ such that $\mu_{f(a)}(x)=r_{f(a)}(x)e^{i\omega_{f(a)}(x)}$ is the membership function of the complex fuzzy set $f(a)$.

(ii) The $\pi-$fuzzy soft set $(\underline{f}, A)$, where $\underline{f}: A \mapsto \underline{s}^U$ and $\underline{s}^U$ is the set of all $\pi-$fuzzy sets of the form $\{(x, \omega_{f(a)}(x)) : x \in U, a \in A\}$ such that $\mu_{f(a)}(x) = r_{f(a)}(x)e^{i\omega_{f(a)}(x)}$ is the membership function of the complex fuzzy set $f(a)$.

Definition 2.6. (Aygünoğlu & Aygün 2009) A fuzzy soft set (f, A) is said to be a fuzzy soft group if and only if $f(a)$ is a fuzzy subgroup for all $a \in A$.

Definition 2.7. A $\pi-$fuzzy soft set $(\underline{f}, A)$ is said to be a $\pi-$fuzzy soft group if and only if $f(a)$ is a $\pi-$fuzzy subgroup for all $a \in A$.

Definition 2.8. (Nadia 2010) The intersection of two complex fuzzy soft sets (f, A) and (g, B) over U, denoted by $(f, A) \cap (g, B)$, is the complex fuzzy soft set (h, C), where $C = A \cap B$, and $h(e) = f(e) \cap g(e)$ for all $e \in C$.

Definition 2.9. (Nadia 2010) The union of two complex fuzzy soft sets (f, A) and (g, B) over U, denoted by $(f, A) \cup (g, B)$, is the complex fuzzy soft set (h, C), where $C = A \cup B$ and for all $e \in C$,

$$h(e) = \begin{cases} f(e) & e \in A - B \\ g(e) & e \in B - A \\ f(e) \cup g(e) & e \in A \cap B \end{cases}$$

Definition 2.10. (Alsarahead & Ahmad 2017) Let $A = \{(x, \mu_A(x)) : x \in G\}$ and $B = \{(x, \mu_B(x)) : x \in G\}$ be two complex fuzzy subsets of G, with membership functions $\mu_A(x) = r_A(x)e^{i\omega_A(x)}$ and $\mu_B(x) = r_B(x)e^{i\omega_B(x)}$, respectively. Then

(i) a complex fuzzy subsets A is said to be a homogeneous complex fuzzy set if for all $x, y \in G$

$$r_A(x) \le r_A(y) \text{ if and only if } \omega_A(x) \le \omega_A(y),$$

(ii) a complex fuzzy subsets A is said to be homogeneous with B, if for all $x, y \in G$ $r_A(x) \le r_B(y)$ if and only if $\omega_A(x) \le \omega_B(y)$.

3. Complex Fuzzy Soft Groups

Definition 3.1. Let (f, A) and (g, B) be two complex fuzzy soft sets over a universe set U. Then

(i) A complex fuzzy soft set (f, A) is said to be a homogeneous complex fuzzy soft set if and only if $f(a)$ is a homogeneous complex fuzzy set for all $a \in A$,

(ii) A complex fuzzy soft set (f, A) is said to be a completely homogeneous complex fuzzy soft set if and only if $f(a)$ is a homogeneous with $f(b)$ for all $a, b \in A$,

(iii) A complex fuzzy soft set (f, A) is said to be homogeneous with (g, B) if and only if $f(a)$ is a homogeneous with $g(a)$ for all $a \in A \cap B$.

Definition 3.2. Let G be a group and (f, A) be a homogeneous complex fuzzy soft set over G. Then (f, A) is said to be a complex fuzzy soft group shortly (CFSG) over G if and only if the following hold

(i) $\mu_{f(a)}(xy) \geq \min\{\mu_{f(a)}(x), \mu_{f(a)}(y)\}$ for all $a \in A$ and $x, y \in G$,

(ii) $\mu_{f(a)}(x^{-1}) \geq \mu_{f(a)}(x)$ for all $a \in A$ and $x \in G$.

Example 3.3. Let N be the set of all positive integers and $G = Z_{12}$. Define a mapping $f : N \to (S^{Z_{12}})$, where for any $n \in N$,

$$\mu_{f(n)}(x) = \begin{cases} \frac{1}{n} e^{i\pi} & \text{if } x \in \{0,6\}, \\ 0 & \text{if } x \in Z_{12} - \{0,6\}. \end{cases}$$

Then (f, N) is a complex fuzzy soft group over Z_{12}.

Proof. Clearly for all $n \in N$, $f(n) = \left\{\left(x, \mu_{f(n)}(x)\right)\right\}$ is a complex fuzzy subgroup of Z_{12}. Thus (f, N) is a complex fuzzy soft group over Z_{12}.□

The next result shows the relationship between complex fuzzy soft groups and fuzzy soft groups analogue to results by Al-Husban and Abdul Razak (2016) and also Alsarahead and Ahmad (2017).

Theorem 3.4. *Let G be a group and (f, A) be a homogeneous complex fuzzy soft set over G. Then (f, A) is a complex fuzzy soft group of G if and only if:*

(i) The fuzzy soft set $(\overline{f}, A)$ is a fuzzy soft group.

(ii) The $\pi -$fuzzy soft set $(\underline{f}, A)$ is a $\pi -$fuzzy soft group.

Proof. Let (f, A) be a CFSG and $x, y \in G$. Then for all $a \in A$ we have

$$\begin{aligned} r_{f(a)}(xy)e^{i\omega_{f(a)}(xy)} &= \mu_{f(a)}(xy) \\ &\geq \min\left\{\mu_{f(a)}(x), \mu_{f(a)}(y)\right\} \\ &= \min\left\{r_{f(a)}(x)e^{i\omega_{f(a)}(x)}, r_{f(a)}(y)e^{i\omega_{f(a)}(y)}\right\} \\ &= \min\left\{r_{f(a)}(x), r_{f(a)}(y)\right\} e^{i\min\left\{\omega_{f(a)}(x), \omega_{f(a)}(y)\right\}} \\ &\quad \text{(since } (f, A) \text{ is homogeneous).} \end{aligned}$$

So $r_{f(a)}(xy) \geq \min\left\{r_{f(a)}(x), r_{f(a)}(y)\right\}$ and $\omega_{f(a)}(xy) \geq \min\left\{\omega_{f(a)}(x), \omega_{f(a)}(y)\right\}$. On the other hand

$$r_{f(a)}(x^{-1})e^{i\omega_{f(a)}(x^{-1})} = \mu_{f(a)}(x^{-1})$$

$$\geq \mu_{f(a)}(x)$$
$$= r_{f(a)}(x)e^{i\omega_{f(a)}(x)}$$

which implies $r_{f(a)}(x^{-1}) \geq r_{f(a)}(x)$ and $\omega_{f(a)}(x^{-1}) \geq \omega_{f(a)}(x)$. So $(\overline{f}, A)$ is a fuzzy soft group and $(\underline{f}, A)$ is a $\pi-$fuzzy soft group.

Conversely, let $(\overline{f}, A)$ be a fuzzy soft group and $(\underline{f}, A)$ be a $\pi-$fuzzy soft group, then for all $a \in A$ we have

$$r_{f(a)}(xy) \geq \min\left\{r_{f(a)}(x), r_{f(a)}(y)\right\}, \omega_{f(a)}(xy) \geq \min\left\{\omega_{f(a)}(x), \omega_{f(a)}(y)\right\},$$
$$r_{f(a)}(x^{-1}) \geq r_{f(a)}(x) \text{ and } \omega_{f(a)}(x^{-1}) \geq \omega_{f(a)}(x).$$

Now,

$$\mu_{f(a)}(xy) = r_{f(a)}(xy)e^{i\omega_{f(a)}(xy)}$$
$$\geq \min\left\{r_{f(a)}(x), r_{f(a)}(y)\right\}e^{i\min\left\{\omega_{f(a)}(x), \omega_{f(a)}(y)\right\}}$$
$$= \min\left\{r_{f(a)}(x)e^{i\omega_{f(a)}(x)}, r_{f(a)}(y)e^{i\omega_{f(a)}(y)}\right\} \text{ (homogeneity)}$$
$$= \min\left\{\mu_{f(a)}(x), \mu_{f(a)}(y)\right\}.$$

On the other hand

$$\mu_{f(a)}(x^{-1}) = r_{f(a)}(x^{-1})e^{i\omega_{f(a)}(x^{-1})}$$
$$\geq r_{f(a)}(x)e^{i\omega_{f(a)}(x)}$$
$$= \mu_{f(a)}(x).$$

So $f(a)$ is a complex fuzzy subgroup, thus (f, A) is a complex fuzzy soft group.□

Theorem 3.5. *Let $\{(f_i, A_i): i \in I\}$ be a collection of CFSGs of a group G such that (f_j, A_j) is homogeneous with (f_k, A_k) for all $j, k \in I$. Then $\cap_{i\in I}(f_i, A_i)$ is a CFSG.*

Proof. Let $\cap_{i\in I}(f_i, A_i) = (h, C)$ where $C = \cap_{i\in I} A_i$. Then we have $f_i(c)$ is a complex fuzzy subgroup for all $i \in I$ and $c \in C$, so $r_{f_i(c)}(x)$ is a fuzzy subgroup and $\omega_{f_i(c)}(x)$ is a $\pi-$fuzzy subgroup (Theorem 3.4). Now, for all $x, y \in G$ we have

$$\mu_{h(c)}(xy) = \mu_{\cap_{i\in I} f_i(c)}(xy)$$
$$= r_{\cap_{i\in I} f_i(c)}(xy)e^{i\omega_{\cap_{i\in I} f_i(c)}(xy)}$$
$$= \min_{i\in I}\left\{r_{f_i(c)}(xy)\right\}e^{i\min_{i\in I}\left\{\omega_{f_i(c)}(xy)\right\}}$$
$$\geq \min_{i\in I}\left\{\min\left\{r_{f_i(c)}(x), r_{f_i(c)}(y)\right\}\right\}e^{i\min_{i\in I}\left\{\min\left\{\omega_{f_i(c)}(x), \omega_{f_i(c)}(y)\right\}\right\}}$$
$$= \min\left\{\min_{i\in I}\left\{r_{f_i(c)}(x)\right\}, \min_{i\in I}\left\{r_{f_i(c)}(y)\right\}\right\} e^{i\min\left\{\min_{i\in I}\left\{\omega_{f_i(c)}(x)\right\}, \min_{i\in I}\left\{\omega_{f_i(c)}(y)\right\}\right\}}$$

$$= \min\left\{ \min_{i\in I}\left\{ r_{f_i(c)}(x)\right\} e^{i\min_{i\in I}\left\{\omega_{f_i(c)}(x)\right\}},\ \min_{i\in I}\left\{ r_{f_i(c)}(y)\right\} e^{i\min_{i\in I}\left\{\omega_{f_i(c)}(y)\right\}} \right\}$$

(since (f_j, A_j) is homogeneous (f_k, A_k) with for $j,k \in I$.

$$= \min\left\{ \mu_{\cap_{i\in I} f_i(c)}(x), \mu_{\cap_{i\in I} f_i(c)}(y) \right\}$$

$$= \min\left\{ \mu_{h(c)}(x), \mu_{h(c)}(y)\right\}.$$

On the other hand

$$\mu_{h(c)}(x^{-1}) = \mu_{\cap_{i\in I} f_i(c)}(x^{-1})$$

$$= r_{\cap_{i\in I} f_i(c)}(x^{-1}) e^{i\omega_{\cap_{i\in I} f_i(c)}(x^{-1})}$$

$$= \min_{i\in I}\left\{ r_{f_i(c)}(x^{-1})\right\} e^{i\min_{i\in I}\left\{\omega_{f_i(c)}(x^{-1})\right\}}$$

$$\geq \min_{i\in I}\left\{ r_{f_i(c)}(x)\right\} e^{i\min_{i\in I}\left\{\omega_{f_i(c)}(x)\right\}}$$

$$= \mu_{\cap_{i\in I} f_i(c)}(x)$$

$$= \mu_{h(c)}(x) . \square$$

Theorem 3.6. *Let (f, A) and (g, B) be disjoint CFSGs. Then $(f, A)\cup(g, B)$ is CFSG.*

Proof. Let $(f, A)\cup(g,B) = (h,C)$. Since (f, A) and (g, B) are disjoint CFSGs, then $A\cap B = \varphi$, therefore for all $c \in C = A\cup B$. If $c\in A$, then $\mu_{h(c)}(x) = \mu_{f(c)}(x)$, if $c \in B$, then $\mu_{h(c)}(x) = \mu_{g(c)}(x)$, clearly $\mu_{f(c)}(x)$ and $\mu_{g(c)}(x)$ are complex fuzzy subgroups, so $(f, A)\cup(g,B) = (h,C)$ is CFSG.$\square$

Lemma 3.7. *Let G be a group and (f, A) be a homogeneous complex fuzzy soft set over G. Then (f, A) is a complex fuzzy soft group over G if and only if for all $a \in A$*

$$\mu_{f(a)}(xy^{-1}) \geq \min\left\{\mu_{f(a)}(x), \mu_{f(a)}(y)\right\}.$$

Proof. Let (f, A) be a complex fuzzy soft group, then for all $a\in A$ we have $\mu_{f(a)}(xy) \geq \min\{\mu_{f(a)}(x), \mu_{f(a)}(y)\}$ and $\mu_{f(a)}(x^{-1}) \geq \mu_{f(a)}(x)$. Therefore $\mu_{f(a)}(xy^{-1}) \geq \min\left\{\mu_{f(a)}(x), \mu_{f(a)}(y^{-1})\right\} \geq \min\left\{\mu_{f(a)}(x), \mu_{f(a)}(y)\right\}$.

Conversely, if $\mu_{f(a)}(xy^{-1}) \geq \min\left\{\mu_{f(a)}(x), \mu_{f(a)}(y)\right\}$, let $x = y$ to obtain $\mu_{f(a)}(e) \geq \mu_{f(a)}(x)$ for all $x\in G$, hence $\mu_{f(a)}(y^{-1}) = \mu_{f(a)}(ey^{-1}) \geq \min\left\{\mu_{f(a)}(e), \mu_{f(a)}(y)\right\} = \mu_{f(a)}(y)\, \square$

Definition 3.8. Let (f, A) be a complex fuzzy soft set over a universe U. Then for all $\alpha\in[0,1]$ and $\beta\in[0,2\pi]$, the set $(f, A)_{(\alpha,\beta)} = \{f(a)_{(\alpha,\beta)} : a \in A\}$ is called an $(\alpha,\beta)-$ level soft set of the complex fuzzy soft set (f, A) , where $f(a)_{(\alpha,\beta)} = \{x\in U : r_{f(a)}(x) \geq \alpha, \omega_{f(a)}(x) \geq \beta\}$ is an $(\alpha,\beta)-$ level set of the complex fuzzy set $f(a)$. Here, for each $\alpha \in [0,1]$ and $\beta \in [0,2\pi]$, $(f,A)_{(\alpha,\beta)}$ is a soft set in the classical case.

The next result shows the relationship between complex fuzzy soft groups and classical soft groups.

Theorem 3.9. *Let (f,A) be a homogeneous complex fuzzy soft set over a group G. Then (f,A) is a CFSG over a group G if and only if for all $a \in A$ and for arbitrary $\alpha \in [0,1]$ and $\beta \in [0,2\pi]$, with $f(a)_{(\alpha,\beta)} \neq \phi$ the $(\alpha,\beta)-$level soft set $(f,A)_{(\alpha,\beta)}$ is a soft group over G in classical case.*

Proof. Let (f,A) be a CFSG over a group G. Then for all $a \in A$, $f(a)$ is a complex fuzzy subgroup of G. For arbitrary $\alpha \in [0,1]$ and $\beta \in [0,2\pi]$ with $f(a)_{(\alpha,\beta)} \neq \phi$, let $a \in A$ and $x, y \in f(a)_{(\alpha,\beta)}$. Then we have $r_{f(a)}(x) \geq \alpha$ and $\omega_{f(a)}(x) \geq \beta$, also $r_{f(a)}(y) \geq \alpha$ and $\omega_{f(a)}(y) \geq \beta$.

Now,

$$\begin{aligned} r_{f(a)}(xy)e^{i\omega_{f(a)}(xy)} &= \mu_{f(a)}(xy) \\ &\geq \min\left\{\mu_{f(a)}(x), \mu_{f(a)}(y)\right\} \\ &= \min\left\{r_{f(a)}(x)e^{i\omega_{f(a)}(x)}, r_{f(a)}(y)e^{i\omega_{f(a)}(y)}\right\} \\ &= \min\left\{r_{f(a)}(x), r_{f(a)}(y)\right\} e^{i\min\left\{\omega_{f(a)}(x), \omega_{f(a)}(y)\right\}}. \end{aligned}$$

This implies

$$\begin{aligned} r_{f(a)}(xy) &\geq \min\left\{r_{f(a)}(x), r_{f(a)}(y)\right\} \\ &\geq \min\{\alpha, \alpha\} \\ &= \alpha \end{aligned}$$

and

$$\begin{aligned} \omega_{f(a)}(xy) &\geq \min\left\{\omega_{f(a)}(x), \omega_{f(a)}(y)\right\} \\ &\geq \min\{\beta, \beta\} \\ &= \beta. \end{aligned}$$

So $xy \in f(a)_{(\alpha,\beta)}$.

On the other hand we have $r_{f(a)}(x^{-1})e^{i\omega_{f(a)}(x^{-1})} = \mu_{f(a)}(x^{-1}) \geq \mu_{f(a)}(x) = r_{f(a)}(x)e^{i\omega_{f(a)}(x)}$. This implies $r_{f(a)}(x^{-1}) \geq r_{f(a)}(x) \geq \alpha$ and $\omega_{f(a)}(x^{-1}) \geq \omega_{f(a)}(x) \geq \beta$. So $x^{-1} \in f(a)_{(\alpha,\beta)}$ .Therefore $f(a)_{(\alpha,\beta)}$ is a subgroup of G .Thus $(f,A)_{(\alpha,\beta)}$ is a soft group over G.

Conversely, for all $a \in A$ and for arbitrary $\alpha \in [0,1]$ and $\beta \in [0,2\pi]$, let $(f,A)_{(\alpha,\beta)}$ be a soft group over G. Let $x, y \in G$ and $a \in A$, assume $r_{f(a)}(x) = \lambda$, $r_{f(a)}(y) = \delta$, $\omega_{f(a)}(x) = \theta$ and $\omega_{f(a)}(y) = \eta$. Suppose $\alpha = \min\{\lambda, \delta\}$ and $\beta = \min\{\theta, \eta\}$ .This implies $x, y \in f(a)_{(\alpha,\beta)}$ .By hypothesis, $f(a)_{(\alpha,\beta)}$ is a subgroup of G, so $xy^{-1} \in f(a)_{(\alpha,\beta)}$. Thus

$r_{f(a)}(xy^{-1}) \geq \alpha = \min\{\lambda, \delta\} = \min\{r_{f(a)}(x), r_{f(a)}(y)\}$ and $\omega_{f(a)}(xy^{-1}) \geq \beta = \min\{\theta, \eta\} =$ $\omega_{f(a)}(xy^{-1}) \geq \beta = \min\{\theta, \eta\} = \min\{\omega_{f(a)}(x), \omega_{f(a)}(y)\}$. Therefore $\mu_{f(a)}(xy^{-1}) \geq \min$ $\{\mu_{f(a)}(x), \mu_{f(a)}(y)\}$.So by Lemma 3.7 (f, A) is a CFSG over a group G.□

4. Homomorphism of Complex Fuzzy Soft Groups

In this section, the image and inverse image of complex fuzzy soft groups under complex fuzzy soft homomorphism are defined and studied.

Theorem 4.1. (Aygünoğlu & Aygün 2009) *Let (f, A) and (g, B) be two fuzzy soft groups over a group G and H, respectively and (φ, ψ) be a fuzzy soft homomorphism from G to H. Then*

(i) The image of (f, A) under the fuzzy soft function (φ, ψ) is a fuzzy soft group over H.

(ii) The pre-image of (g, B) under the fuzzy soft function (φ, ψ) is a fuzzy soft group over G.

We are going to generalise this result to the case of complex fuzzy soft groups.

Definition 4.2. Let $\varphi: U \to V$ and $\psi: A \to B$ be two functions, where A and B are parameter sets for the crisp sets U and V, respectively. Then the pair (φ, ψ) is called a complex fuzzy soft function from U to V.

Definition 4.3. Let (f, A) and (g, B) be two completely homogeneous complex fuzzy soft sets over U and V, respectively. Let (φ, ψ) be a complex fuzzy soft function from U to V. Then

(i)The image of (f, A) under the complex fuzzy soft function (φ, ψ), denoted by $(\varphi, \psi)(f, A)$, is the complex fuzzy soft set over V defined by $(\varphi, \psi)(f, A) = (\varphi(f), \psi(A))$, with membership function

$$\mu_{\phi(f)(b)}(y) = \begin{cases} \underset{\phi(x)=y}{\vee} \ \underset{\psi(a)=b}{\vee} \mu_{f(a)}(x) & \text{if } \phi^{-1}(y) \neq \varphi \\ 0, & \text{otherwise} \end{cases}$$

where $b \in \psi(A)$ and $y \in V$.

(ii) The pre-image of (g, B) under the complex fuzzy soft function (φ, ψ), denoted by $(\varphi, \psi)^{-1}(g, B)$, is the complex fuzzy soft set over U defined by $(\varphi, \psi)^{-1}(g, B) = (\varphi^{-1}(g), \psi^{-1}(B))$ such that

$$\mu_{\varphi^{-1}(g)(a)}(x) = \mu_{g(\psi(a))}(\varphi(x))$$

where $a \in \psi^{-1}(B)$, $x \in U$.

Example 4.4. Let $U = \{u_1, u_2, u_3\}$ and $V = \{v_1, v_2\}$ be two universe sets. Let $A = \{a_1, a_2, a_3\}$, define (f, A) over U such that $f(a_1) = \{(u_1, 1e^{2\pi}), (u_2, 1e^{\pi}), (u_3, 1e^{\frac{\pi}{2}})\}$,

$f(a_2)=\{(u_1,\frac{1}{2}e^{\frac{\pi}{3}}),(u_2,\frac{1}{2}e^{\frac{\pi}{4}}),(u_3,\frac{1}{2}e^{\frac{\pi}{5}})\}$, $f(a_3)=\{(u_1,\frac{1}{3}e^{\frac{\pi}{6}}),(u_2,\frac{1}{3}e^{\frac{\pi}{7}}),(u_3,\frac{1}{3}e^{\frac{\pi}{8}})\}$, let $B=\{b_1,b_2\}$, define (g,B) over V such that $g(b_1)=\{(v_1,1e^{\pi}),(v_2,\frac{1}{2}e^{\pi})\}$,

$g(b_2)=\{(v_1,\frac{1}{4}e^{\frac{\pi}{3}}),(v_2,\frac{1}{4}e^{\frac{\pi}{4}})\}$, define $\varphi:U\to V$ and $\psi:A\to B$ such that $\varphi(u_1)=v_1$, $\varphi(u_2)=\varphi(u_3)=v_2$, $\psi(a_1)=b_1$ and $\psi(a_2)=\psi(a_3)=b_2$, then for $b_1\in\psi(A)$ we have

$\mu_{\varphi(f)(b_1)}(v_1)=\mu_{f(a_1)}(u_1)=1e^{2\pi}$

$$
\begin{aligned}
\mu_{\varphi(f)(b_1)}(v_2)&=\{\mu_{f(a_2)}(u_2)\vee\mu_{f(a_2)}(u_3)\}\vee\{\mu_{f(a_3)}(u_2)\vee\mu_{f(a_3)}(u_3)\}\\
&=\{\tfrac{1}{2}e^{\frac{\pi}{4}}\vee\tfrac{1}{2}e^{\frac{\pi}{5}}\}\vee\{\tfrac{1}{3}e^{\frac{\pi}{7}}\vee\tfrac{1}{3}e^{\frac{\pi}{8}}\}\\
&=\tfrac{1}{2}e^{\frac{\pi}{4}}.
\end{aligned}
$$

By the same way we can find $\mu_{\varphi(f)(b_2)}(x)$ where $x\in V$. Now, for $a_1\in\psi^{-1}(B)$

$\mu_{\varphi^{-1}(g)(a_1)}(u_1)=\mu_{g(\psi(a_1))}(\varphi(u_1))=\mu_{g(b_1)}(v_1)=1e^{\pi}$

$\mu_{\varphi^{-1}(g)(a_1)}(u_2)=\mu_{g(\psi(a_1))}(\varphi(u_2))=\mu_{g(b_1)}(v_2)=\frac{1}{2}e^{\pi}$

$\mu_{\varphi^{-1}(g)(a_1)}(u_3)=\mu_{g(\psi(a_1))}(\varphi(u_3))=\mu_{g(b_1)}(v_2)=\frac{1}{2}e^{\pi}$

By the same way we can find $\mu_{\varphi^{-1}(g)(a_2)}(x)$ and $\mu_{\varphi^{-1}(g)(a_3)}(x)$ where $x\in U$.

Lemma 4.5. *Let (f,A) and (g,B) be two completely homogeneous complex fuzzy soft sets over U and V, respectively. Let (φ,ψ) be a complex fuzzy soft function from U to V. Then*

(i) $\mu_{\varphi(f)(a)}(y)=r_{\varphi(f)(a)}(y)e^{i\omega_{\varphi(f)(a)}(y)}$.

(ii) $\mu_{\varphi^{-1}(g)(b)}(x)=r_{\varphi^{-1}(g)(b)}(x)e^{i\omega_{\varphi^{-1}(g)(b)}(x)}$.

Proof. (i)

$$
\begin{aligned}
\mu_{\varphi(f)(a)}(y)&=\bigvee_{\varphi(t)=y}\bigvee_{\psi(k)=a}\mu_{f(k)}(t)\\
&=\bigvee_{\varphi(t)=y}\bigvee_{\psi(k)=a}r_{f(k)}(t)e^{i\omega_{f(k)}(t)}\\
&=\left\{\bigvee_{\varphi(t)=y}\bigvee_{\psi(k)=a}r_{f(k)}(t)\right\}e^{i\left\{\bigvee_{\varphi(t)=y}\bigvee_{\psi(k)=a}\omega_{f(k)}(t)\right\}}
\end{aligned}
$$

(since (f,A) is completely homogeneous)

$$=r_{\varphi(f)(a)}(y)e^{i\omega_{\varphi(f)(a)}(y)}.$$

(ii)

$$
\begin{aligned}
\mu_{\varphi^{-1}(g)(b)}(x)&=\mu_{g(\psi(b))}(\varphi(x))\\
&=r_{g(\psi(b))}(\varphi(x))e^{i\omega_{g(\psi(b))}(\varphi(x))}
\end{aligned}
$$

$$= r_{\varphi^{-1}(g)(b)}(x)e^{i\omega_{\varphi^{-1}(g)(b)}(x)}.\square$$

Theorem 4.6. *Let* (f, A) *and* (g, B) *be two completely homogeneous CFSGs over a group* G *and* H*, respectively, and* (φ, ψ) *be a complex fuzzy soft homomorphism from* G *to* H*. Then*

(i)The image of (f, A) *under the complex fuzzy soft homomorphism* (φ, ψ) *is a CFSG over* H*.*

(ii) The pre-image of (g, B) *under the complex fuzzy soft homomorphism* (φ, ψ) *is a CFSG over* G*.*

Proof.(i) Since (f, A) is a complex fuzzy soft group, then by Theorem 3.4 for all $a \in A$ we have $r_{f(a)}(x)$ is a fuzzy subgroup and $\omega_{f(a)}(x)$ is a $\pi-$fuzzy subgroup. Thus by Theorem 4.1 and Proposition 2.4 the image of $r_{f(a)}(x)$ and $\omega_{f(a)}(x)$ are fuzzy subgroup and $\pi-$ fuzzy subgroup, respectively, therefore for all $x, y \in H$ we have:

$$r_{\varphi(f)(a)}(xy) \geq \min \left\{r_{\varphi(f)(a)}(x), r_{\varphi(f)(a)}(y)\right\}, r_{\varphi(f)(a)}(x^{-1}) \geq r_{\varphi(f)(a)}(x),$$

$$\omega_{\varphi(f)(a)}(xy) \geq \min \left\{\omega_{\varphi(f)(a)}(x), \omega_{\varphi(f)(a)}(y)\right\} \text{and } \omega_{\varphi(f)(a)}(x^{-1}) \geq \omega_{\varphi(f)(a)}(x)$$

Now, by Lemma 4.5

$$\begin{aligned}
\mu_{\varphi(f)(a)}(xy) &= r_{\varphi(f)(a)}(xy)e^{i\omega_{\varphi(f)(a)}(xy)} \\
&\geq \min\left\{r_{\phi(f)(a)}(x), r_{\phi(f)(a)}(y)\right\} e^{i\min\left\{\omega_{\phi(f)(a)}(x), \omega_{\phi(f)(a)}(y)\right\}} \\
&= \min\left\{r_{\phi(f)(a)}(x)e^{i\omega_{\phi(f)(a)}(x)}, r_{\phi(f)(a)}(y)e^{i\omega_{\phi(f)(a)}(y)}\right\} \\
&= \min\left\{\mu_{\phi(f)(a)}(x), \mu_{\phi(f)(a)}(y)\right\}.
\end{aligned}$$

Also,

$$\begin{aligned}
\mu_{\varphi(f)(a)}(x^{-1}) &= r_{\varphi(f)(a)}(x^{-1})e^{i\omega_{\varphi(f)(a)}(x^{-1})} \\
&\geq r_{\varphi(f)(a)}(x)e^{i\omega_{\varphi(f)(a)}(x)} \\
&= \mu_{\varphi(f)(a)}(x).
\end{aligned}$$

(ii) Since (g, B) is a complex fuzzy soft group, then by Theorem 3.4 for all $b \in B$ we have $r_{g(b)}(x)$ is a fuzzy subgroup and $\omega_{g(b)}(x)$ is a $\pi-$fuzzy subgroup. Thus by Theorem 4.1 and Proposition 2.4 the pre-image of $r_{g(b)}(x)$ and $\omega_{g(b)}(x)$ are fuzzy subgroup and $\pi-$ fuzzy subgroup, respectively, therefore for all $x, y \in G$ we have:

$$r_{\varphi^{-1}(g)(b)}(xy) \geq \min \left\{r_{\varphi^{-1}(g)(b)}(x), r_{\varphi^{-1}(g)(b)}(y)\right\}, r_{\varphi^{-1}(g)(b)}(x^{-1}) \geq r_{\varphi^{-1}(g)(b)}(x)$$

$$\omega_{\varphi^{-1}(g)(b)}(xy) \geq \min \left\{\omega_{\varphi^{-1}(g)(b)}(x), \omega_{\varphi^{-1}(g)(b)}(y)\right\} \text{and } \omega_{\varphi^{-1}(g)(b)}(x^{-1}) \geq \omega_{\varphi^{-1}(g)(b)}(x)$$

Now, by Lemma 4.5

$$
\begin{aligned}
\mu_{\varphi^{-1}(g)(b)}(xy) &= r_{\varphi^{-1}(g)(b)}(xy)e^{i\omega_{\varphi^{-1}(g)(b)}(xy)} \\
&\geq \min\left\{r_{\phi^{-1}(g)(b)}(x), r_{\phi^{-1}(g)(b)}(y)\right\}e^{i\min\left\{\omega_{\phi^{-1}(g)(b)}(x), \omega_{\phi^{-1}(g)(b)}(y)\right\}} \\
&= \min\left\{r_{\phi^{-1}(g)(b)}(x)e^{i\omega_{\phi^{-1}(g)(b)}(x)}, r_{\phi^{-1}(g)(b)}(y)e^{i\omega_{\phi^{-1}(g)(b)}(y)}\right\} \\
&= \min\left\{\mu_{\phi^{-1}(g)(b)}(x), \mu_{\phi^{-1}(g)(b)}(y)\right\}.
\end{aligned}
$$

Also,

$$
\begin{aligned}
\mu_{\varphi^{-1}(g)(b)}(x^{-1}) &= r_{\varphi^{-1}(g)(b)}(x^{-1})e^{i\omega_{\varphi^{-1}(g)(b)}(x^{-1})} \\
&\geq r_{\varphi^{-1}(g)(b)}(x)e^{i\omega_{\varphi^{-1}(g)(b)}(x)} \\
&= \mu_{\varphi^{-1}(g)(b)}(x). \square
\end{aligned}
$$

5. Conclusion

In this paper, we introduced the concept of complex fuzzy soft groups, We used Rosenfeld's approach and generalised it to the complex realm by following Ramot's approach. On one hand, the relationship between complex fuzzy soft groups and the classical soft groups was investigated. On the other hand, the relationship between complex fuzzy soft groups and fuzzy soft groups was also investigated.

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