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AGGREGATION OF CRITERIA WEIGHTS FOR MULTI-PERSON DECISION MAKING WITH EQUAL OR DIFFERENT CREDIBILITY

(Pengaggregatan Wajaran Kriterium bagi Pembuatan Keputusan Berbilang-Orang dengan Kewibawaan yang Sama atau Berbeza)

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ABSTRACT

Multi-criteria (MC) problems involve making decision over alternatives that are characterized by several criteria. These criteria represent basis of evaluation in MC evaluation models or goal aspiration in MC optimization models. In most of MC models, criteria weights must be predetermined before the problem can be solved. These weights are interpreted differently but mostly as relative importance of criteria. There are many weighting methods available, but are generally categorized as subjective or objective methods. The subjective methods involve evaluator(s) to evaluate the relative importance of the criteria. Even though multi-person may involve in evaluating the criteria, the final weights must be represented as only one set of weights. Many aggregation methods have been proposed to compose the evaluations. However, these evaluators may have different degree of credibility since they may come from different background or may have different degree of superiority. The aim of this paper is to propose a different concept of weights that would represent the degree of credibility of the evaluators. Furthermore, several aggregation approaches are suggested on how to include these 'new' weights in order to produce new criteria weights that also take the credibility of the evaluators into considerations. A numerical example is used to show how these weights of credibility can be used to solve a MC problem in particular to determine the criteria relative importance. This new concept of weight signifies a different insight to the domain of MC decision making (MCDM).

Keywords: aggregation approaches; criteria weight; evaluators' credibility

ABSTRAK

Masalah berbilang kriterium (BK) melibatkan pembuatan keputusan ke atas beberapa alternatif yang bercirikan beberapa kriterium. Kriterium ini mewakili asas penilaian dalam model penilaian BK atau aspirasi tujuan dalam model-model pengoptimuman BK. Dalam kebanyakan model BK, wajaran kriterium selayaknya ditentukan terlebih dahulu sebelum masalah tersebut dapat diselesaikan. Wajaran ini ditafsirkan secara berbeza tetapi kebanyakannya sebagai kepentingan relatif kriterium tersebut. Terdapat banyak kaedah mendapatkan wajaran kriterium, tetapi umumnya dikategorikan sebagai kaedah subjektif dan kaedah objektif. Kaedah subjektif melibatkan penilai yang menilai kepentingan relatif kriterium. Walaupun berbilang-orang yang menilai kriterium, wajaran akhir akan diwakilkan sebagai satu set wajaran. Banyak kaedah pengaggregatan dicadangkan untuk menggabung penilaian tersebut. Namun, penilai terlibat mungkin mempunyai darjah kewibawaan yang berbeza kerana mereka datang daripada latar belakang yang berbeza atau mereka mempunyai darjah keunggulan yang tidak sama. Tujuan makalah ini ialah untuk mencadangkan konsep wajaran yang baharu yang mewakili darjah kewibawaan penilai. Seterusnya, beberapa pendekatan pengaggregatan dicadangkan untuk memasukkan wajaran yang 'baharu' ini bagi menghasilkan wajaran kriterium yang mengambil kira kewibawaan para penilai. Satu contoh numerik digunakan untuk menunjukkan bagaimana wajaran kewibawaan ini digunakan dalam menyelesaikan satu masalah BK tertentu bagi mendapatkan kepentingan relatif kriterium. Konsep yang baharu ini menerangkan satu wawasan yang unik dalam bidang pembuatan keputusan BK.

Kata kunci: pendekatan pengaggregatan; wajaran kriterium; kewibawaan penilai

1. Introduction

Weights of criteria are important in multi-criteria decision making methods. Even though these weights carry many different meanings (Choo *et al*. 1999), these weights can influence the final decision particularly in evaluation or selection problems. This paper defines the criteria weights as the relative importance of the criteria. There are many methods available to determine the weights, but the methods are mostly classified into two main approaches: subjective and objective methods. Subjective methods involve evaluator(s) of different backgrounds and experiences, while objective weights are found by manipulating intrinsic information in the criteria.

In group decision making context, even though multi-person may take part in making judgment particularly on considering the criteria weights, only a final set of weights is used in the final step of the evaluation process. Here, many aggregation methods are utilized to aggregate the weights that are given by more than one person. The traditional average methods such as the arithmetic or geometric average methods remain popular. But many new methods emerged, for example the ordered weighted average (OWA) method which was introduced by Yager (1988). If OWA is integrated with the concept of fuzzy majority (Kacprzyk 1986), a new way of interpreting the criteria weights based on degree of consensus is possible.

The evaluator(s) or sometimes called as the experts in the selected area that take part in the evaluation may have different credibility or superiority that may affect the evaluation. The evaluators with higher credibility could be considered as more trustworthy as compared to the less experienced evaluators. However, the credibility issue is not given attention by researchers. This paper aims to address the issue and suggests how to include the degree of credibility in criteria weight determination problem. The following sections discuss the criteria weighting methods, aggregation of criteria weights for multi-person decision making with equal credibility, aggregation of criteria weights for multi-person decision making with different credibility, the related numerical example and conclusions.

2. Criteria Weighting Methods

Methods to determine individual criteria weights are often divided into two main approaches which are the subjective method and objective methods. Some researchers (Ma *et al*. 1999; Desa *et al*. 2015) used aggregated weights that combine both subjective and objective weights to balance up between evaluator(s)' judgment and data driven effect.

2.1 *Subjective methods*

Analytical Hierarchy Process (AHP) is one of the most popular subjective methods which was developed by Saaty (1980; 1990), besides its inconsistency issues and rank reversal problem. More classic methods are rating method, direct point allocation (Robert & Goodwin 2002), ratio method (Edwards 1977), and the swing method (Von Winterfeldt & Edwards 1986). Among the rank-based methods (Barron & Barrett 1996) are rank-sum (RS), rank reciprocal (RR) and rank-centroid (RC) methods.

2.2 *Objective methods*

In order to avoid the subjectivity of human judgment, researchers may choose the objective criteria weighting methods where these methods are data–driven type. Among the methods are entropy (Zeleny 1982), Criteria Importance through Inter-Criteria (CRITIC) by Diakoulaki *et al*. (1995), standard deviation, and coefficient of variation (Kasim 2014).

2.3 *Fuzzy Measures*

The subjective and objective methods as discussed in previous two sections focus on the individual weights of the criteria. However, the concept of fuzzy measures is related to compound weights. Besides considering the individual criteria weights, these fuzzy measures represent the interaction measures between criteria or among criteria which was introduced by Sugeno (1985). One type of fuzzy measure is called as λ-fuzzy measure (Kasim 2014) and another type is called as k^0-measure (Krishnan *et al*. 2017).

3. Aggregation of Criteria Weights for Multi-Person Decision Making with Equal Credibility

When more than one evaluator gave weights of criteria by using any subjective method as discussed in section 2.1, these weights that represent the relative importance of the criteria must be aggregated since usually one set of weights is needed to complete the whole evaluation process. Let $\boldsymbol{w_j^l}$, be the weight for criterion *j*, $\boldsymbol{j = 1, \dots, n}$, *e*valuated by evaluator *l*, $\boldsymbol{l = 1, \dots, p}$. Let the aggregated weight for criterion *j*, is denoted as $\boldsymbol{w_j}$. If the evaluators have the same credibility, we assume that they are having the same superiority. For example, if all the evaluators are with the same payroll scheme or with the same position in certain organization.

3.1 *Simple arithmetic average and evaluators with equal credibility*

If the simple arithmetic average is used as the aggregation operator, the aggregated weight for criterion *j* is given as

$$\boldsymbol{w_j} = \frac{1}{p}\sum_l w_j^l \tag{1}$$

3.2 *Simple geometric average and evaluators with equal credibility*

If the simple geometric average is used as the aggregation operator, the aggregated weight for criterion *j* is given as

$$\boldsymbol{w_j} = \sqrt[l]{\prod_l w_j^l} \tag{2}$$

3.3 *Ordered weighted average (OWA) and evaluators with equal credibility*

If the OWA (Yager 1988; 1993), is used as the aggregation operator, the aggregated weight for criterion *j* is given as

$$\boldsymbol{w_j} = \sum_l b_j\left(w_j^l\right) \tag{3}$$

where b_j is a collection of weighting vector generated by OWA operator and $\left(w_j^l\right)$ is the weight of criterion *j*, *j* =1, …, *n* which are ordered in decreasing order.

4. Aggregation of Criteria eights for Multi-person Decision Making with Different Credibility

However, if the evaluators who took part in the evaluation process are having different credibility or superiority, for example they are having different position, this condition should be taken into consideration so that the final evaluation result is reliable. Normally, those who are at a better position have more power or more experience than those who are at the lower position. For example, a professor may represent three or four regular lecturers in making decision. Hence, in order to quantify the different credibility or superiority that may exist among the evaluators, a new set of weights has to be defined. Let $\boldsymbol{u_l}$ be a value associated with the degree of credibility of evaluator l, $\boldsymbol{l = 1, \dots, p}$, where $\boldsymbol{u_l > 0}$, and $\sum_l \boldsymbol{u_l = 1}$. These $\boldsymbol{u_l}$ should be attached to the evaluation or weight of criteria given by evaluator l. The following three subsections provide the corresponding three aggregations to (1), (2), and (3) respectively.

4.1 *Simple arithmetic average and evaluators with different credibility*

The suitable formula is

$$\boldsymbol{w_j} = \frac{1}{p}\sum_l u_l w_j^l \tag{4}$$

4.2 *Simple geometric average and evaluators with different credibility*

The matching mathematical expression is

$$\boldsymbol{w_j} = \sqrt[l]{\prod_l u_l w_j^l} \tag{5}$$

4.3 *Ordered weighted average (OWA) and evaluators with different credibility*

The corresponding OWA is given as

$$\boldsymbol{w_j} = \sum_l b_j\left(u_l w_j^l\right) \tag{6}$$

where b_j is a collection of weighting vector generated by OWA operator and $\left(w_j^l\right)$ is the weight of criterion j, $j = 1, \dots, n$ which are ordered in decreasing order.

5. A Numerical Example

Suppose there are 5 criteria where the relative importance of those criteria were assessed by three evaluators by using the subjective rank-sum (RS) method given by the following formula. If $\boldsymbol{r_{j,}\, j = 1, \dots, 5}$ is the rank given to criteria j. Then

$$\boldsymbol{w}_{(j)} = \frac{\boldsymbol{2(m+1-r_j)}}{\boldsymbol{m(m+1)}} \tag{7}$$

where $\boldsymbol{w}_{(j)}$ represent the weight of criteria with jth ranking in terms of importance. Based on that formula, $\boldsymbol{w_{(1)}}$=0.3333; $\boldsymbol{w_{(2)}}$ = 0.2667, $\boldsymbol{w_{(3)}}$ = 0.2000 $\boldsymbol{w_{(4)}}$ = 0.1333, and $\boldsymbol{w_{(5)}}$ = 0.0667. Suppose the following Table 1 summarizes the results of evaluation by three evaluators. Suppose the evaluators 1, 2, and 3 were given different degree of credibility as 0.1, 0.3, and 0.6 respectively. Evaluator 3 is assumed to have twice higher credibility than evaluator 2, and six times higher credibility than evaluator 1. However, suppose both evaluator 2 and evaluator 3 gave the same ranking even though they have different credibility.

Table 1. A numerical example: Summary of criteria relative importance and their rankings

	Evaluator 1 (0.1)	Rank	Evaluator 2 (0.3)	Rank	Evaluator 3 (0.6)	Rank
Criterion 1	0.3333	1	0.2667	2	0.2667	2
Criterion 2	0.2000	3	0.2000	3	0.2000	3
Criterion 3	0.1333	4	0.3333	1	0.3333	1
Criterion 4	0.0667	5	0.0667	5	0.0667	5
Criterion 5	0.2667	2	0.1333	4	0.1333	4

Table 2. Aggregated relative importance of criteria

	Eq. 1	Eq. 2	Eq. 3	Eq. 4	Eq. 5	Eq. 6
Criterion 1	0.2889	0.3000	0.2734	0.2734	0.2969	0.3074
Criterion 2	0.2000	0.2000	0.2000	0.2000	0.2067	0.0889
Criterion 3	0.2666	0.2333	0.3133	0.3133	0.2538	0.3685
Criterion 4	0.0667	0.0667	0.0667	0.0667	0.0689	0.0760
Criterion 5	0.1778	0.2000	0.1467	0.1467	0.1736	0.1592

Table 3. Ranking of criteria based on aggregated weights

	Eq. 1	Eq. 2	Eq. 3	Eq. 4	Eq. 5	Eq. 6
Criterion 1	1	1	2	2	1	2
Criterion 2	3	3	3	3	3	4
Criterion 3	2	2	1	1	2	1
Criterion 4	5	5	5	5	5	5
Criterion 5	4	3	4	4	4	3

Table 2 shows the aggregated weights by the three aggregation methods as given in equations 1 to 6, while Table 3 gives the ranking based on the aggregated weights in Table 2. Based on Table 3, we can see that the overall ranking of criteria changed once the credibility of evaluators are considered. For example, column 2 and column 5 are ranking based on simple arithmetic average for evaluators with equal and different credibility respectively. For column 3 and column 6 are ranking based on simple geometric average for evaluators with equal and different credibility respectively. These two evaluations have many similarities of ranking except for criterion 5 since it has the same aggregated weight as criteria 2. The results criteria ranking by OWA methods are in column 5 and column 7 for evaluators with equal and different credibility respectively. The results show that the ranking for criterion 2, 3 and 4 are the same for both aggregations. However, ranking by Eq. 3 (which is based on OWA with evaluators of the same credibility) is surprisingly the same as the ranking by Eq.4 (which is based on the simple arithmetic average with evaluators of different credibility). This interprets the importance of the inclusion of weights representing the credibility of the evalutors in computing the final weights of the criteria. This is because by considering the credibility of the evaluators, the final overall ranking of the criteria are affected even though the numerical example is only of five criteria and three evaluators. In terms of the ranking of each individual criterion, all criteria had changes in their rankings when different aggregations were used or when the credibility of the evaluators were considered except for criterion 4 where its ranking is always at the fifth position. The influence of the new weights representing credibility of the evaluators may be more obvious if a different multi-criteria with more criteria and more evaluators was analysed. Further experiment should be done in future to study further how the credibility of different number of evaluators affects the criteria weights of different number of criteria. Another related example can be found in Kasim and Jemain (2013).

6. Conclusions

The paper introduces a new concept of weights that represent degree of credibility of evaluators in solving multi-criteria problem, particularly in the evaluation of the relative importance of criteria by multi-person. This new type of weights should be considered because in reality people who are evaluators in certain evaluation process are always of different credibility. Their evaluations should be treated differently according to their credibility. A numerical example illustrates the use of three aggregation methods to aggregate the evaluations on the relative importance of criteria given by evaluators with the same or different credibility. As expected, the overall rankings change when the degree of credibility of the evaluators are taken into consideration. There are changes in terms of individual ranking of each criterion when the different aggregation were used with or without considering the credibility of the evaluators. However the ranking of one of the criterion, that is criterion 4 has been retained at the fifth position. Thus, the different degree of credibility among the evaluators and different aggregation in any evaluation problems should be considered since these conditions influence the final results. This paper signifies a new insight in multi-person subjective judgment particularly in solving multi-criteria group decision making problem.

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