# Reynolds Number Calculation Method for Aerobic Biological Porous Packed Reactors

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## ABSTRACT

The degree of mixing in aerobic biological porous packed reactors can be quantitatively determined by Reynolds Numbers calculation. The dimensionless parameter accounts for all the inertial and viscous forces in the fully and partially packed reactors. Two identical reactors were built; each reactor was 14 cm in diameter and 100 cm in height, providing an empty bed volume of 15 L. At approximate organic loading rates (OLRs) of 2, 3, 4, and 6 kg COD/$m^3$.d, the Reynolds numbers in the partial-bed were 40 300, 36 100, 25 500 and 40 400 respectively, whilst those of the full-bed were 19 500, 20 500, 11 700 and 26 400. The quantity of media therefore had a significant effect on the extent of mixing in the filter. Porosity of the aerobic porous packed reactors is a bed characteristic resulting from the balance between the effect of biomass accumulation due to growth and biomass loss due to shear stress. The stress arises from increased organic loadings and increased upflow velocity due to aeration.

Keywords : Reynolds number, mixing, porous packed reactors, porosity

## *ABSTRAK*

*Darjah pengadukan di dalam reaktor biologi aerobik terpadat berliang boleh ditentukan secara kuantitatif melalui pengiraan Nombor Reynolds. Parameter tak berdimensi tersebut mengambil kira kesemua daya inersia dan likat di dalam reaktor yang dipadatkan sebahagian dan sepenuhnya. Dua reaktor yang serupa dibina; setiap satu bergarispusat 14 cm, ketinggian 100 cm, dan isipadu kerja 15 L. Pada kadar pembebanan organik 2, 3, 4, dan 6 kg COD/$m^3$.hari, Nombor Reynolds di dalam reaktor yang dipadatkan sebahagian ialah 40 300, 36 100, 25 500 and 40 400 manakala bagi reaktor yang dipadatkan sepenuhnya ialah 19 500, 20 500, 11 700 dan 26 400. Kuantiti media memberi kesan yang bererti terhadap darjah adukan di dalam reaktor. Keliangan reaktor aerobik terpadat merupakan ciri yang terhasil dari imbangan kesan pengumpulan biojisim disebabkan pertumbuhan dan kehilangan biojisim kerana tegasan ricih. Tegasan ricih ini berpunca daripada peningkatan beban organik dan peningkatan halaju ke atas oleh pengudaraan.*

*Kata kunci: Nombor Reynolds, adukan, reaktor terpadat berliang, keliangan*

## INTRODUCTION

Uniformity and effective contact between substrate and biomass in a fixed bed reactor system play an important role in its performance and behaviour (Nabizadeh et al. 2000). The degree of contact between an incoming substrate and a viable bacterial population has also been identified as one of the major factors affecting the performance of large- scale reactors (Smith et al. 1996). Other workers have also noted the importance of mixing in achieving an efficient substrate conversion (Murphy 1971; Monteith & Stephenson 1981; Guillard & Tragardh 1999; Levenspiel 1999). In other words, knowledge of the degree of mixing and the flow pattern within a reactor is vital before evaluation of the reactor's performance be made.

Quantification of the degree of mixing inside biological aerated filters is rather complex. Nabizadeh et al. (2000) stressed the effects of water flow, air supply, shape and morphological characteristics of packed media and filter porosity on the flow pattern in an aerated submerged fixed film reactor. Almost all of the parameters mentioned above change with an increase in organic loading rate, i.e. kg COD/$m^3$.d. The fluid hydrodynamic regime is characterised by various parameters. Of particular importance are the flow velocity of the fluid phase and the variation in pressure distribution. Properties of a fluid phase such as viscosity and density influence the flow velocity and eddy patterns within a liquid stream, as do liquid-solid and liquid-gas interfaces. Friction occurs, and free movement of eddies is hindered at the interfaces (Wilderer et al. 1995).

To account for all the inertial and viscous forces in the full- and partial-bed reactors, the Reynolds number ($N_{RE}$), a dimensionless parameter was selected. Although there is an abundance of literature on the topic of packed bed reactor hydraulics (Muslu 1986; Smith et al. 1996; Show & Tay 1999; Nabizadeh et al. 2000), only a few studies have been conducted at the very high $N_{RE}$ typical of a biological aerated filter. Mickley et al. (1965) carried out a hydraulic study at $N_{RE}$ 4780-7010 while Yevseyev et al. (1991) conducted one at $N_{RE}$ 1000-20 000 and Fukuchi and Ishii (1982) performed another at $N_{RE}$ 300-300 000.

## MATERIALS AND METHODS

Two identical reactors were built; each reactor was 14 cm in diameter and 100 cm in height, providing an empty bed volume of 15 L. A 2.8 litre freeboard or headspace was provided at the top of each reactor. The reactors were constructed from PVC, a non-transparent material, to prevent the growth of phototropic organisms.

The control reactor was filled with 10.9 L cascade rings (373 pieces) whilst the second reactor was only partially packed with 5.5 L cascade rings (186 pieces). The media were stationary and held in place by rigid polypropylene mesh with 15 mm diameter holes placed at the top and bottom of the packing. The Cascade mini rings used had approximate voidage of 93% and specific surface area of 140 $m^2/m^3$. A synthetic waste prepared in the laboratory was used to provide a consistent organic substrate for all loadings. Detailed composition of the substrate is indicated in Table 1.

**Table 1.** Compositions of synthetic feed

| Chemicals | Amount |
|---|---|
| Whey powder | 16 g/L |
| Glucose | 13.5 g/L |
| Meat extract | 30 ml/ at 200 g/L |
| $NH_4H_2PO_4$ | 1.915 g/L |
| $FeCl_3$ | 0.0225 g/L |
| $MoO_3$ | 0.0035 g/L |
| $NiSO_4.6H_2O$ | 0.006 g/L |

The recycle line provided a superficial liquid velocity of 2 $m^3/m^2$.h throughout the experiment. Process air was supplied by the laboratory air into the air inlet pipe. To ensure that oxygen availability did not become a limiting factor, the Dissolved Oxygen (DO) values were maintained at between 4 and 6 mg/L. Backwashing of the filters was carried out on an elapsed time basis with a frequency of once every two or three days.

Computation of Reynolds number at increased organic loadings was performed at approximate OLRs of 2, 3, 4 and 6 kg COD/ $m^3$.d. Since the COD concentration of the raw wastewater was kept constant throughout this phase, the step increases in OLR were achieved through increases in the raw wastewater flowrate into the reactors. Flows to both reactors were carefully and frequently adjusted so that they were similar and comparable. The aeration rate was also increased accordingly, to satisfy the oxygen requirements of the biological system at the higher rates of respiration and to maintain the dissolved oxygen concentration at 4-6 mg/L. The porosity was measured at each steady state condition just before a backwashing operation took place. In this way, the porosity measured should have been the lowest in a 2-3 day operational cycle.

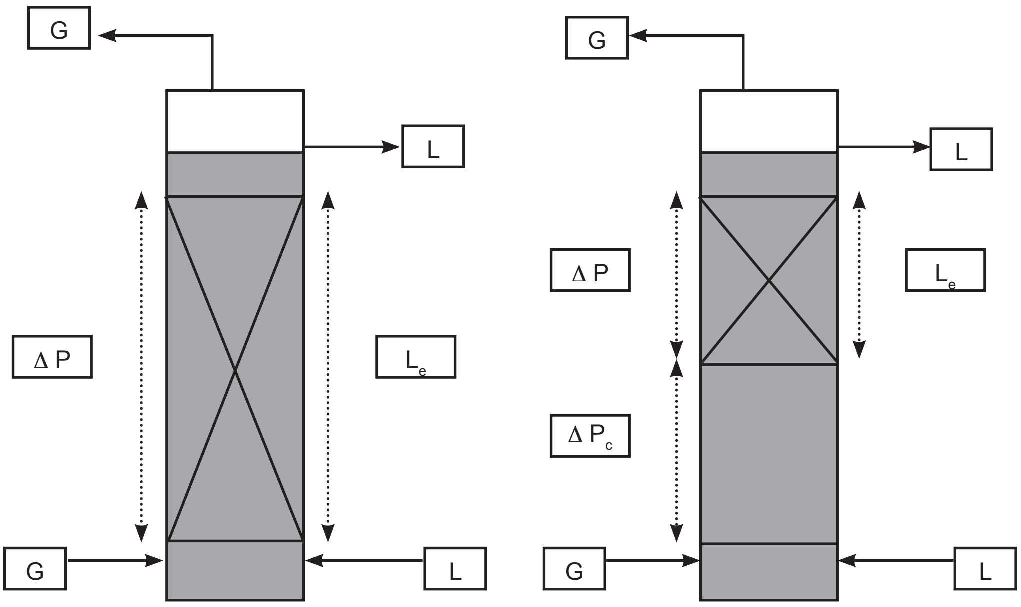
*Theoretical Background for Calculation of Reynolds' Number in Aerobic Porous Packed Reactors*

The Reynolds number in a packed bed is proportional to the interstitial velocity of the fluid, the hydraulic diameter, the density and absolute viscosity of the liquid (Nsofor & Adebiyi 2001). Reynolds number ($N_{RE}$)can be obtained by the following culculation:

$$N_{RE} = \frac{D_h V_i \rho}{\mu}$$

where

$N_{RE}$ = Reynolds number (dimensionless)
$D_h$ = hydraulic diameter (m)
$V_i$ = interstitial velocity (m/s)
$\rho$ = density of liquid (kg/m$^3$)
$\mu$ = dynamic viscosity coefficient (kg/ms)




G = gas flow (aeration)
L = liquid flow (feed and recycle)
$\Delta P$ = pressure drop through the packed bed
$\Delta P_c$ = pressure drop through the empty bed
$L_e$ = height of the packed bed

**Figure 1.** Schematic view of the full- (left) and partial-bed (right) reactors showing flows and parameters involved in Reynolds' number computation (Darby 1996)

Figure 1 illustrates the schematic view of flows and parameters involved in the calculation. Since fluid in a porous medium follows a tortuous path through channels of varying size, one method of describing the flow behaviour in the pores is to regard the flow path as a "noncircular conduit" (Darby 1996).

$$D_h = \frac{4A_i}{W_p} = 4\,\frac{(\varepsilon)\ (Bed\ Volume)}{(Specific\ surface\ area\ of\ media) \times (Bed\ volume)} \qquad (2)$$

where
$D_h$ = hydraulic diameter (m)
$A_i$ = cross sectional area (m$^2$)
$W_p$ = wetted parameter (m)
$\varepsilon$ = porosity of the bed

Porosity ($\varepsilon_{\text{clean bed}}$) of a packed bed can be assessed experimentally. For a clean bed, the formula is

$$\varepsilon_{clean\ bed} = \frac{[\ Total\ vol - (Number\ of\ media) \times (Specific\ vol\ of\ media)]}{Total\ volume}$$

The specific volume of the media was determined by measuring the increase of water volume in a cylindrical column due to the insertion of known pieces of media. Thus, the specific volume of a media is simply the increase in liquid volume divided by the number of media used.

In order to measure the porosity of the reactor in operation ($\varepsilon_{\text{operation}}$), the liquid that occupied the bed was decanted from successive layers of the reactor in order to lessen the effect of suspended solids accumulation on the attached biofilm. The volume of the liquid was then physically measured to improve the accuracy of the value when calculating the bed porosity:

$$\varepsilon_{operation} = \frac{(\text{Total volume}) - (\text{Media} + \text{Biofilm} + \text{Suspended biomass volume})}{Total\ volume}$$

The interstitial velocity in the packed bed of the full and partial bed reactors was obtained by the balance of pressure drop and total drag of fluid on solid boundaries. The pressure drop in the bed was the result of the difference between the upward friction force and downward force of gravity. In a biological aerated reactor, the upward force is a function of power dissipated by the rising air bubbles and volumetric flowrate of the fluid.

Power dissipated by the rising air bubbles can be estimated with the following equation (Metcalf & Eddy 1991), P:

$$P = KQ_a \ \text{In}\left(\frac{(h + 10.33)}{10.33}\right) \qquad (3)$$

where
$K$ = constant = 10.75
$Q_a$ = air flow rate at atmospheric pressure (m$^3$/min)
$h$ = air pressure at the point of discharge expressed in meter of water (m)

The pressure that acts downward is due to the hydrostatic headloss. As the flow rate of water or air up through the solid particles is increased, there is an increasing pressure drop between the bottom and top of the bed. For highly turbulent flow where inertial forces predominate (Re > 1000) experimental results may be correlated in terms of the Burke-Plummer equation (Gibilaro 2001). The most successful approach for correlating the pressure drop to the total drag of fluid on solid boundaries is the "tube bundle" theory. The porous medium is regarded as a series of tangled capillaries. The pressure drop ($\Delta$P) is calculated by applying the result for a single straight tube.

$$\Delta P = \frac{(\lambda)\ (L_e)\ (\rho)\ (V_i^2)}{2a} \qquad (4)$$

where
$\Delta P$ = pressure drop through the column (N/m$^2$)
$\lambda$ = pipe friction factor (no unit)
$a$ = tube radius (m)
$L_e$ = the equivalent length of the capillary tube (m)
$\varrho$ = density of fluid (kg/m$^3$)
$V_i$ = interstitial velocity (m/s)

Because of the tortuous path, the fluid is actually taking a longer route than the bed height. However, since the packed bed is approximated as a bundle of straight capillary tubes, $L_e$ equals to the height of reactor occupied by media.

The tube radius, *a*, is obtained by making use of the specific surface, *Sv*, of the porous media,

$$Sv = \frac{(\textit{Surface area of bed particles to which flow is exposed})}{\textit{Total volume occupied by media}} \quad (5)$$

$$= (2\pi a L_e)\left(\frac{n}{L_e(1-\varepsilon)}\right)$$

where $n = \dfrac{\textit{Number of capillaries}}{\textit{Area}}$

$$\varepsilon = \begin{matrix}\textit{Void fraction}\\ \textit{within the bed}\end{matrix} = \frac{\pi a^2 n L_e}{L_e} = n\,\pi a^2$$

Therefore,

$$Sv = \frac{\textit{Pore suface area}}{\textit{Tube volume}} = \frac{2\pi a n}{(1-\varepsilon)}$$

At a high $N_{RE}$ ($N_{RE}$ > 1000), the flow is dominated by inertial forces and "wall roughness". The flow in a porous medium can be considered extremely rough, with $\varepsilon/d \approx 1$ where d is the diameter of packed bed elements. Therefore, the flow at a sufficiently high Reynolds number should be fully turbulent, and the friction factor should be constant. This has been confirmed by observations by other researchers, with the value of the constant equal to approximately 1.75 (McCabe et al. 2000).

The Reynolds number in the unpacked portion of the partial bed reactor is simply calculated using the following equation (Reynolds & Richards 1996):

$$N_{RE} = \frac{D\,V_s\rho}{\mu} \quad (6)$$

where

$\rho$ = density of fluid (kg/m$^3$)
$V_s$ = superficial velocity (m/s)
$D$ = column diameter (m)
$\mu$ = dynamic viscosity coefficient (kg/ms)

The Burke Plummer equation (Gibilaro 2001) for turbulent flow in a column correlates the pressure drop and drag forces in the empty bed. The equation is described below:

$$\Delta P = \frac{(\lambda)\,(L)\,(p)\,(V_s^2)}{2\,D} \quad (7)$$

where

$\lambda$ = pipe friction factor (no unit)
$D$ = column diameter (m)
$\Delta P$ = pressure drop through the column (N/m$^2$)
$L$ = depth of empty bed (m)

## RESULTS AND DISCUSSION

Table 2 shows the experimental conditions applied and measured, at the times the Reynolds' numbers were calculated. The most significant finding for the calculation of Reynolds numbers in the packed beds was that values obtained for the partial bed were nearly double those of the full bed. The results obtained shows that the removal of the lower 50% of the media increased the region for dispersive mechanisms to occur, resulting in a larger mixed zone for a given aeration rate. The quantity of media therefore had a significant effect on the extent of mixing in the filter. The result is in accordance with the study on hydraulic pattern in an upflow anaerobic filter carried out by Tilche and Vieira (1991) and Smith et al. (1996). When less packing was present, gas bubbles generated below the packing hit the packing with a greater impact, thereby enhanced mixing.

Discussion of the effect of OLR on $N_{RE}$ in aerobic packed bed reactors is not straightforward. An increase of OLR increases the respiration rate of microorganisms. The aeration requirement needs to be raised accordingly to maintain a fixed dissolved oxygen level at 4 6 mg/L. This also has effects on the porosity of the bed. The fluid upflow velocity in the system was contributed by the influent, recycle and the effect of aeration flows. Discussion of porosity values is crucial for the mixing study since the resulting Reynolds numbers of the packed bed trail the porosity pattern both in the full and partial bed reactors.

Although porosity in the bed was affected by the accumulation of biomass, Table 2 shows that the values did not increase linearly with the increase in organic loadings. It is postulated that shear stress caused by the airflow eroded some of the biomass attached to the media, resulting in increases of the porosity of the packed bed. It was also observed by previous workers that varying the aeration rate not only affects oxygen transfer efficiency and flow conditions but also shear in the reactors (Lee & Stensel 1986; Mann et al. 1995). Therefore, in this study, porosity is a bed characteristic resulting from the balance

**Table 2.** Operational conditions in the full- and partial-bed reactors

| | Full-bed reactor | | | | Partial-bed reactor | | | |
|---|---|---|---|---|---|---|---|---|
| OLR (kg COD/$m^3$.d) | 2 | 3 | 4 | 6 | 2 | 3 | 4 | 6 |
| Influent (L/d) | 13.39 | 19.57 | 24.48 | 32.4 | 13.39 | 19.57 | 24.48 | 32.40 |
| Recycle upflow velocity ($m^3/m^2$.h) | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| Air flowrate (L/min) @760 mm $H_2O$ atm and saturated air | 1.25 | 2.00 | 3.00 | 4.00 | 1.90 | 2.00 | 3.50 | 4.00 |
| Dissolved oxygen at top of the reactor (mg/L) | 5.43 | 5.55 | 5.47 | 5.32 | 5.26 | 4.72 | 4.60 | 4.95 |
| Porosity of the packed bed | 0.640 | 0.630 | 0.471 | 0.677 | 0.721 | 0.692 | 0.579 | 0.699 |

between the effect of biomass accumulation due to growth, and biomass loss due to shear stress, arising from increased organic loading rate, and increased upflow velocity. Discussion of porosity values is crucial for the mixing study since the resulting Reynolds numbers of the packed bed trail the porosity pattern both in the full and partial-bed reactors.

Results obtained for both the full and partial bed reactors are presented in Figure 2 and Figure 3 respectively. For the full-bed reactor, the porosity values were consistent most of the time with a mean value of 0.673 ± 0.0637. The greater values of shear stress induced by the higher air velocities due to the increasing air input to meet the respiration of the biomass at higher OLRs could have resulted in more sloughing of the biofilm attached to the media. At an OLR of 4 kg COD/$m^3$.d, however, the porosity dropped to the lowest value of 0.471. The reason for this anomaly could be the higher backwashing rate applied from this loading onwards, which affected the detachment and growth of biofilm in the reactor and finally the porosity of the system. Hall-Stoodley and Stoodley (2002) noted that there is a strong interaction between microbial detachment and attachment processes and cell growth in a biofilm system. Gas and liquid flow in biological reactors was found to be one of the major contributors in biofilm detachment, which influences the formation, structure and stability of biofilms (Liu & Tay 2002).

In the packed bed of the partial-bed reactor, the same trend of porosity could also be observed. Porosity values were almost maintained throughout the experiment with a mean value of 0.604 ± 0.0913, the lowest being 0.579 at OLR 4 kg COD/$m^3$.d. Again, this drop of porosity might have also resulted from the new backwashing regime in the reactor, which finally affects the sloughing off and the growth of biofilm. Without this backwashing effect, porosity in the packed bed of the full and partial bed reactors could have stayed the same, despite the fact that more biomass should have been generated by the increase in organic loadings. Higher growth in the system due to the new backwashing approach could have also affected the sludge retention time in the system. This explains the sudden increase of aeration needed in the partial-bed reactor. The resulting Reynolds numbers in the unpacked column of the partial-bed reactor were 309 000, 312 000, 338 000 and 342 000 at OLRs 2, 3, 4 and 6 kg COD/$m^3$.d respectively. The Reynolds numbers in the empty bed column of the partial-bed reactor were found to be very

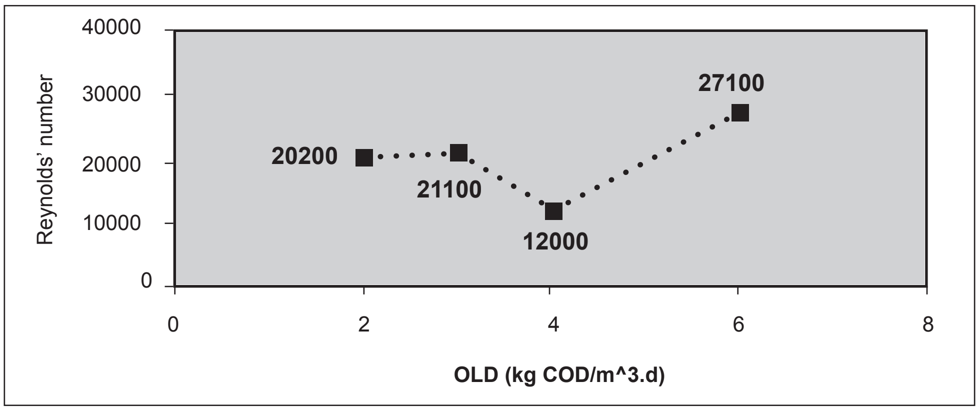



**Figure 2.** Reynolds' numbers for the packed bed of the full-bed reactor

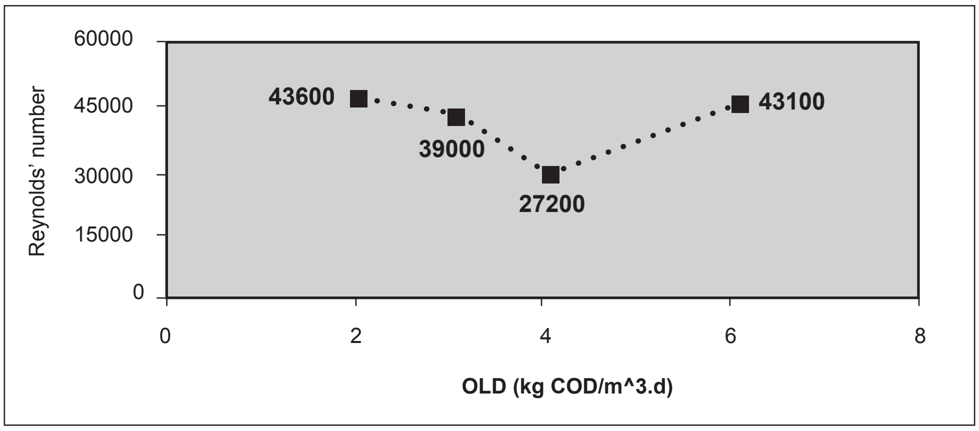



**Figure 3.** Reynolds' numbers for the packed bed of the partial-bed reactor

high, with values six to nine times higher than those of its packed bed. This indicates that media presence played a significant role in reducing mixing intensity in the vessel. In the absence of media, the upflow movement of fluid was limited only by boundary walls. Thus, Reynolds numbers increased with the increase of upflow velocities. Smith et al. (1996) also observed the removal of the lower 50% of the media resulted in a larger mixed zone for a given gas production with the greatest degree of mixing occurring at the highest liquid and gas velocity.

## CONCLUSIONS

The degree of mixing inside aerobic biological porous packed reactors was influenced by the effects of water flow, air supply, and filter porosity. A dimensionless parameter, the Reynolds number, can account for all the inertial and viscous forces in the full and partial bed reactors. Porosity of the aerobic porous packed reactors is a bed characteristic resulting from the balance between the effect of biomass accumulation due to growth and biomass loss due to shear stress. The stress arises from increased organic loadings and increased upflow velocity due to aeration. The quantity of media had a significant effect on the extent of mixing in the filter. In terms of hydraulic characteristics, removing half of the packing media increases the mixing intensity of the reactor. Reynolds Numbers in the packed bed of the partial bed reactor are nearly double the Reynolds Numbers in the full-bed reactor. At approximate organic loading rates of 2, 3, 4, and 6 kg COD/$m^3$.d, the Reynolds Numbers in the partial bed were 40 300, 36 100, 25 500 and 40 400 respectively, whilst those of the full bed were 19 500, 20 500, 11 700 and 26 400.

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