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TRACKING A BENCHMARK INDEX IN PORTFOLIO OPTIMIZATION WITH TWO-STAGE MIXED INTEGER PROGRAMMING MODEL

(Menjejak Indeks Tanda Aras dalam Pengoptimuman Portfolio dengan Model Pengaturcaraan Integer Bercampur Dua-Tahap)

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ABSTRACT

The investors wish to achieve higher portfolio return than the benchmark index return at minimum tracking error (TE) in enhanced index tracking. This study aims to develop the optimal portfolio to track the benchmark sectorial index in Malaysia with two-stage mixed integer programming (MIP) model by minimizing the TE at the first stage followed by maximizing the portfolio mean return at the second stage. The data consists of Technology Index and the index components from Malaysia stock market. The results indicate that the two-stage MIP model gives higher mean return than the benchmark sectorial index at minimum TE. This study is significant because it helps to develop the optimal portfolio with two-stage MIP model to outperform the benchmark sectorial index without holding all index components.

Keywords: Enhanced index tracking; mixed-integer programming; optimal portfolio

ABSTRAK

Pelabur ingin mencapai pulangan portfolio yang melebihi pulangan indeks tanda aras dengan ralat penjejakan (TE) yang minimum dalam penjejakan indeks dipertingkat. Kajian ini bertujuan untuk membangunkan portfolio optimum untuk menjejak indeks sektor tanda aras di Malaysia dengan model pengaturcaraan integer bercampur (MIP) dua-tahap yang meminimumkan TE pada tahap pertama diikuti dengan memaksimumkan min pulangan portfolio pada tahap kedua. Data terdiri daripada Indeks Teknologi dan komponen indeks dari pasaran saham Malaysia. Keputusan menunjukkan bahawa model MIP dua-tahap memberi min pulangan yang lebih tinggi daripada indeks sektor tanda aras dengan TE yang minimum. Kajian ini adalah signifikan kerana ia membantu untuk membangunkan portfolio optimum dengan model MIP dua-tahap yang dapat mengatasi indeks sektor tanda aras tanpa memegang semua komponen indeks.

Kata kunci: Penjejakan indeks dipertingkat; pengaturcaraan integer bercampur; portfolio optimum

1. Introduction

In enhanced index tracking (EIT), the investors and fund managers aim to develop the optimal portfolio to achieve higher return than the benchmark index without holding all the index components by minimizing the tracking error (TE) (Beasley *et al.* 2003; Roll 1992). Minimization of tracking error and maximization of portfolio return are the two main elements in enhanced index tracking problem. Zero tracking error indicates that the portfolio is able to achieve the same return with the benchmark index return perfectly. Based on the literature review, the sum-weighted model was introduced in EIT to maximize the excess return as well as minimize the TE in tracking various benchmark indices (Beasley *et al.* 2003). A goal programming model was presented in EIT to maximize the mean return and minimize the TE of portfolio (Wu *et al.* 2007). In addition, two-stage mixed integer programming (MIP) model

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was proposed in EIT by minimizing the TE at the first stage followed by maximizing the portfolio mean return at the second stage to track the main market index (Lam *et al*. 2017a). Various EIT models have been developed and studied in the past to outperform the benchmark index. (Bedoya & Birge 2014; Canakgoz & Beasley 2009; Guastaroba & Speranza 2012; Lam *et al*. 2015a; Mezali & Beasley 2013).

According to the past research, there is lack of study in tracking Malaysia's sectorial index. The past studies only focused on tracking the main indices in various stock markets. Therefore, this study aims to develop the optimal portfolio to track and outperform the Technology Index in Malaysia with the two-stage MIP model. The two-stage MIP model (Lam *et al*. 2017a) is adopted to track the sectorial index return by minimizing the TE at the first stage followed by maximizing the portfolio mean return at the second stage. Technology sector plays an important role in information and communication technology based on applications and innovations that enhance organizational performance and economic growth of a country.

2. Materials and Methods

2.1. *Data*

The data comprises monthly returns of stocks that make up the Technology Index in Malaysia from January 2012 to December 2017. The optimal portfolio is developed with the two-stage MIP model for EIT. In this study, the optimal solution is computed using LINGO software (Lam *et al.* 2015b; Lam *et al.* 2017b; Lam *et al.* 2017c).

2.2. *Two-Stage MIP Model*

In this paper, a two-stage MIP model (Lam *et al.* 2017a) is adopted to track the benchmark sectorial index return by minimizing the TE at the first stage followed by maximizing the portfolio mean return at the second stage. The optimization model is formulated as follows:

First stage:

$$\text{Minimize } E = \sqrt{\frac{1}{T}\sum_{i=1}^{T}\left(R_{Pt} - R_{It}\right)^2} \tag{1}$$

Subject to

$$\sum_{i=1}^{N} Z_i = K \tag{2}$$

$$Z_i \in \{0,1\} \tag{3}$$

$$L_i Z_i \le x_i \le U_i Z_i \tag{4}$$

$$\sum_{i=1}^{N} x_i = 1 \tag{5}$$

$$x_i \ge 0 \tag{6}$$

Second Stage:

Maximize $r_P = \sum_{i=1}^{N} r_i x_i$ (7)

Subject to

$$\sum_{i=1}^{N} Z_i = K \qquad (8)$$

$$Z_i \in \{0,1\} \qquad (9)$$

$$L_i Z_i \le x_i \le U_i Z_i \qquad (10)$$

$$\sum_{i=1}^{N} x_i = 1 \qquad (11)$$

$$E^* - \delta \le E \le E^* + \delta \qquad (12)$$

$$x_i \ge 0 \qquad (13)$$

where

R_{Pt} : OP return at time *t*,

R_{It} : benchmark index return at time *t*,

R_{it} : return of stock *i* at time *t*,

E : tracking error,

x_i : weight of stock *i*,

N : total number of stocks,

r_i : mean return of stock *i*,

K : number of stocks selected for tracking benchmark index return,

T : number of periods,

r_P : mean return of the OP,

Z_i : binary integer,

L_i : lower limit on stock i,

E^* : optimal value of TE determined in the first stage,

U_i : upper limit on stock i,

δ : tolerance for TE.

The performance of the optimal portfolio of two-stage MIP model is compared to the benchmark index based on the excess return and TE (Beasley *et al*. 2003; Meade & Salkin 1990; Wu *et al*. 2007).

3. Empirical Results

Table 1 presents the results of stock selection in tracking the benchmark Technology Index based on the optimal solution of two-stage MIP model.

Table 1: Stock selection in tracking benchmark Technology Index based on the optimal solution of two-stage MIP model

Stock	Stock Selection (Binary Integer)
CENSOF	0
CUSCAPI	0
D&O	0
DATAPRP	0
DIGISTA	0
DNEX	0
EFORCE	1
ELSOFT	0
FRONTKN	0
GHLSYS	0
GRANFLO	1
GTRONIC	0
HTPADU	0
ITRONIC	1
JCY	0
KESM	1
KEYASIC	0
MMSV	0
MPI	1
MSNIAGA	1
MYEG	1

Continued…

... Table 1 (Continued)

MMSV	0
MPI	1
MSNIAGA	1
MYEG	1
NOTION	0
OMESTI	0
PENTA	1
THETA	1
TRIVE	0
TURIYA	0
UNISEM	0
VITROX	0
WILLOW	1

As presented in Table 1, the binary integer of 1 indicates that the stock is selected in tracking the benchmark index with two-stage MIP model. On the other hand, the binary integer of 0 implies that the stock is not selected in tracking the benchmark index with two-stage MIP model. The result shows that EFORCE, GRANFLO, ITRONIC, KESM, MPI, MSNIAGA, MYEG, PENTA, THETA and WILLOW are selected in tracking the benchmark index based on the optimal solution of two-stage MIP model. Table 2 displays the portfolio composition of two-stage MIP model.

Table 2: Portfolio composition of two-stage MIP model

Stock	Weights (%)
EFORCE	2.83
GRANFLO	22.80
ITRONIC	1.53
KESM	29.19
MPI	5.40
MSNIAGA	18.92
MYEG	7.80
PENTA	1.00
THETA	9.53
WILLOW	1.00

Based on Table 2, KESM (29.19%) gives the highest weight in the portfolio of two-stage MIP model followed by GRANFLO (22.80%) and MSNIAGA (18.92%). PENTA (1.00%) and WILLOW (1.00%) are identified as the least component in the portfolio of two-stage MIP model. This implies that the portfolio consists of various stocks with different weights in tracking the sectorial Technology Index as supported by the past studies that focus on the main index (Lam *et al.* 2015a; Lam *et al.* 2017a).

Table 3 shows the performance comparison of the Technology Index and the two-stage MIP model.

Table 3: Performance comparison of the Technology Index and the two-stage MIP model

	Benchmark Technology Index	Optimal Portfolio (Two-Stage MIP Model)
Number of stocks	30	10
Mean return	0.0097	0.0107
Excess return	-	0.0010
Tracking error	-	0.0699

According to Table 3, the portfolio of two-stage MIP model comprises 10 stocks in tracking Technology Index that consists of 30 stocks. This implies that there is only 33% of Technology Index components are required to develop the portfolio which is in line with the past studies (Canakgoz & Beasley 2009; Guastaroba & Speranza 2012). The result shows that the portfolio of two-stage MIP model outperforms the Technology Index with excess return 0.0010 at minimum TE (0.0699). Other than focusing on the main indices according to the literatures (Beasley *et al*. 2003; Canakgoz & Beasley 2009; Guastaroba & Speranza 2012; Wu *et al*. 2007), the results of this study indicate that the two-stage MIP model is able to track the sectorial index as well, namely Technology Index without holding all index components.

4. Conclusion

This paper aims to develop the portfolio in tracking the benchmark Technology Index with two-stage MIP model. The result shows that EFORCE, GRANFLO, ITRONIC, KESM, MPI, MSNIAGA, MYEG, PENTA, THETA and WILLOW are selected in tracking the Technology Index. In conclusion, the two-stage MIP model outperforms the Technology Index by holding 33% of index components. It is suggested that this study should be extended to other countries for future research by considering different time periods and benchmark indices. In addition, future researchers may improve the existing two-stage MIP model by considering the transaction cost for enhanced index tracking problem so that it is more practical to the investors and fund managers.

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