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Three-Axes Rotation Algorithm for the Relaxed 3L-CVRP

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ABSTRACT

The purpose of this work is to present a developed three-axes rotation algorithm to improve the solving methodology for the relaxed 3L-CVRP (Three-Dimensional Capacitated Vehicle Routing Problem). Although there are reported works on solving approaches for the relaxed 3L-CVRP that consider product rotation to optimize load capacity, rotation on the three axes has not been thoroughly studied. In this aspect, the present work explicitly explores the three-axes rotation and its impact on load capacity optimization. In order to improve the relaxed 3L-CVRP problem, a two-phase solution was developed. The first phase consists of finding the solution for the CVRP problem, using a demand previously obtained with a heuristic developed to convert the 3L-CVRP demand into CVRP demand. The second phase is to obtain the loading of the vehicle using a heuristic developed to load the items using rules to obtain the rotation of the items. The proposed approach was able to improve the load assignment in 48.1% of well-known 3L-CVRP instances when compared to similar approaches on the relaxed 3L-CVRP. The outcomes of this research can be applied to transportation problems where package rotation on the z-axis is an option, and there are not fragile items to load in the vehicles.

Keywords: 3L-CVRP; constraint relaxation; three-axes rotation; load capacity optimization

INTRODUCTION

The Capacitated Vehicle Routing Problem (CVRP), proposed by (Dantzig & Ramser 1959), is a Non-deterministic Polynomial-time complexity class problem (NP-hard) in the transportation field. The CVRP consists of defining routes for every vehicle to minimize transportation costs or time with the restriction of capacity based on the weight or volume of the items. The problem has been addressed by different researchers using metaheuristic alternatives (Caballero-Morales, Martínez-Flores & Sánchez-Partida 2018; Hosseinabadi, Rostami, Kardgar, Mirkamali & Abraham 2017; Mazidi, Fakhrahmad & Sadreddini 2016) to obtain near-to-optimal solutions to large instances (>150 nodes or locations to be served by the vehicle) within a reasonable time. In contrast, exact algorithms can only provide solutions for instances with less than 137 nodes (Liu, Li, Luo & Chen 2013). Orrego (Orrego Cardozo, Ospina Toro, & Toro Ocampo, 2016) presents various metaheuristics used to solve CVRP. Recently, this type of problem evolved considering the transport of items of different sizes (dimensions), known as the Three-Dimensional Capacitated Vehicle Routing Problem (3L-CVRP) introduced by Gendreau (Gendreau, Iori, Laporte & Martello 2006). This problem is a combination of the CVRP and the Three-Dimensional Bin Packaging Problem (3D-BPP). The 3D-BPP has been solved to optimality for instances with a maximum transportation load of 60 items(Martello, Pisinger, & Vigo, 2000).

Because the 3L-CVRP takes into account the dimensions of the items, not all items are suitable to be loaded within the vehicle. The loading task increases in complexity as more constraints are considered (i.e., LIFO, support, fragility). Because of the complexity of the transportation scenario, some or all constraints are relaxed. It leads to Relaxed 3L-CVRP.

The present work extends on the solving aspect of the Relaxed 3L-CVRP by proposing an algorithm to improve the loading task. It is performed by extending the three-axes rotation of the items and performing constraint relaxation on fragility, support area, and LIFO. Relaxation was performed as in the reported reviewed works, and it was found that three-axes rotation can improve load assignment and support area. Particularly, load assignment was improved in 48.1% of well-known 3L-CVRP instances when compared to similar approaches.

The advances of the present work are described as follows: in 3L-CVRP Section the technical background of the 3L-CVRP is presented; then in the next section recent works on the 3L-CVRP and Relaxed 3L-CVRP are presented and discussed; after the proposed algorithm is described; immediately the obtained results and the discussion of the

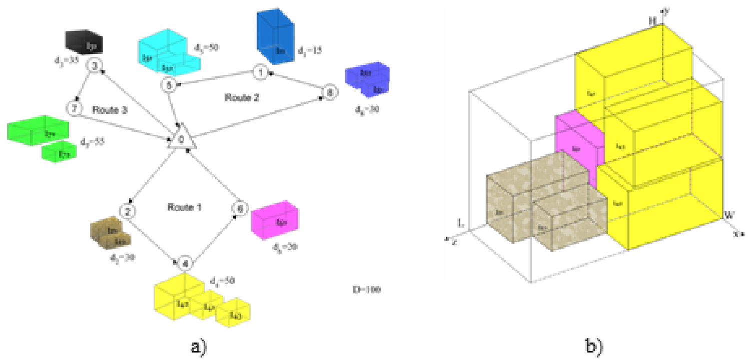

FIGURE 1. Example of a partial solution for a 3L-CVRP instance.

TABLE 1. Overview of solving approaches for the 3L-CVRP

Work	Type of problem	Solving Approach	Loading constraints	Visual Assessment
Gendreau(Gendreau et al., 2006)	3L-CVRP/Relaxed 3L-CVRP	Tabu Search (TS) to solve the routing problem with two heuristics (bottom left algorithm, touching perimeter algorithm) to solve the loading problem. Additional optimization of routes by using 4-opt.	Orientation, LIFO, Stability, Fragility	No
Fuellerer(Fuellerer et al., 2010)	3L-CVRP/Relaxed 3L-CVRP	Ant Colony Optimization (ACO) to solve the routing problem and two heuristics (bottom-left-fill algorithm, touching perimeter algorithm) to solve the loading problem.	Orientation, LIFO, Stability, Fragility	No
Wang(Wang, Guo, Chen, Zhu, & Lim, 2010)	3L-CVRP	TS to solve the routing problem with two heuristics (Deepest-Bottom-Left-Fill and Maximum Touching Area) for the loading problem.	Orientation, LIFO, Stability, Fragility	No
Borfeldt(Bortfeldt, 2012)	3L-CVRP/Relaxed 3L-CVRP	TS to solve the routing problem and Tree Search Algorithm with Extreme Points to solve the loading problem.	Orientation, LIFO, Stability, Fragility	No
Junqueira(Junqueira, Oliveira, Carravilla, & Morabito, 2013)	3L-CVRP	Integer Linear Programming (ILP)	Orientation, LIFO, Stability, Fragility	Yes
Zhu(Zhu, Qin, Lim, & Wang, 2012)	3L-CVRP	TS to solve the routing problem and two heuristics (Deepest-Bottom-Left-Fill (DBLF), Maximum Touching Area (MTA)) to solve the loading problem.	Orientation, LIFO, Stability, Fragility	No
Ruan(Ruan, Zhang, Miao, & Shen, 2013)	3L-CVRP/Relaxed 3L-CVRP	Honey Bee Mating Optimization (HBMO) to solve the routing problem and six heuristics (Back_Left_Low, Left_Back_Low, Max_Touching_Area_W, Max_Touching_Area_No_Walls_W, Max_Touching_Area_L, Max_Touching_Area_No_Walls_L) to solve the loading problem.	Orientation, LIFO, Stability, Fragility	No
Lacomme(Lacomme, Toussaint, & Duhamel, 2013)	3L-CVRP / Relaxed 3L-CVRP	Greedy Randomized Adaptive Search Procedure (GRASP) and Evolutionary Local Search (ELS) to solve the routing problem. The x-y axes are considered to place the items by only adding the height of the items in the same x-y coordinates; if the loading is feasible, then the items are pulled to the extremes, and the position in the z-axis is calculated.	None	Yes
Wei(Wei, Zhang, & Lim, 2014)	3L-FCVRP	Variable Neighborhood Search (VNS) to solve the routing problem and the Open Space-Based First Fit heuristic to solve the loading problem.	Orientation, LIFO, Stability, Fragility	No

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Tao (Tao & Wang, 2015)	3L-CVRP/Relaxed 3L-CVRP	TS to solve the routing problem and two heuristics (least waste algorithm, touching perimeter algorithm) to solve the loading problem.	Orientation, LIFO, Stability, Fragility	No
Escobar(Escobar-Falcón, Álvarez-Martínez, Granada-Echeverri, Escobar, & Romero-Lázaro, 2015)	3L-CVRP/Relaxed 3L-CVRP	Branch-and-Cut (B&C) algorithm to solve the routing problem and GRASP algorithm to solve the loading problem.	Orientation, LIFO, Stability, Fragility	Yes
Mahvash (Mahvash, Awasthi, & Chauhan, 2015)	3L-CVRP	Column-Generation(CG) algorithm combined with pricing problem to solve the routing problem and extreme points to solve the loading problem	Orientation, LIFO, Stability, Fragility	No

TABLE 2. Dimensions of items (boxes) to be loaded into the vehicle

Customer	Box	Length	Width	Height
20	1	36	10	10
20	2	29	10	8
20	3	29	8	9
1	1	34	10	13
13	1	14	9	11
13	2	24	7	7
13	3	15	10	8
7	1	15	11	17
7	2	22	6	12
22	1	18	13	11
22	2	22	8	11
22	3	19	12	17

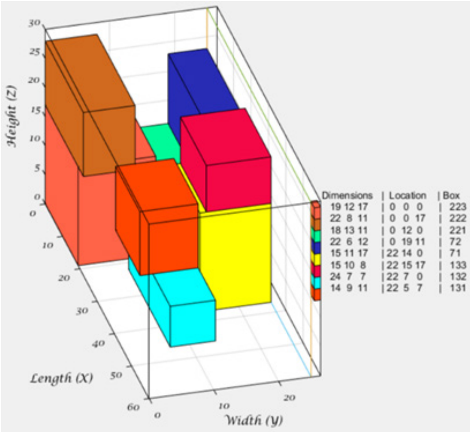


FIGURE 2. Example of a fully constrained 3L-CVRP loading solution

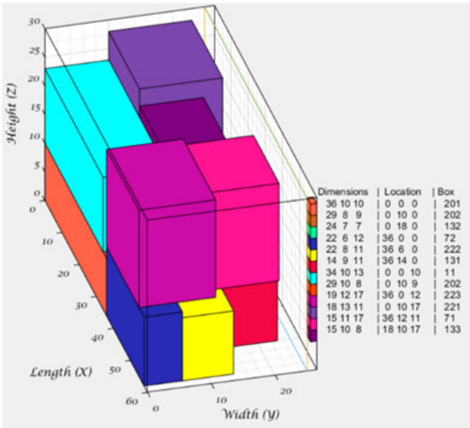


FIGURE 3. Example of relaxed (unconstrained) 3L-CVRP loading solution

Algorithm. 3L-CVRP demand into CVRP demand

```
Procedure demand
Input parameters
   Instance (demand weight, capacity, dimensions of items, quantity of items)
   ok=true
Output parameters
  Instance with fixed parameters
begin
   while ok
      load the items in vehicles
      if items can be loaded in vehicles
         ok=false
      else
         decrease the capacity of the vehicle
         if items can be loaded in vehicles
            ok=false
         else
          for i=1 to 3 do
            modify the weight of the demand of customers according to the quantity and dimensions of items
            they required
          endfor
      endif
   endif
  endwhile
```

FIGURE 4. Adaptation heuristic for 3L-CVRP demand into CVRP demand

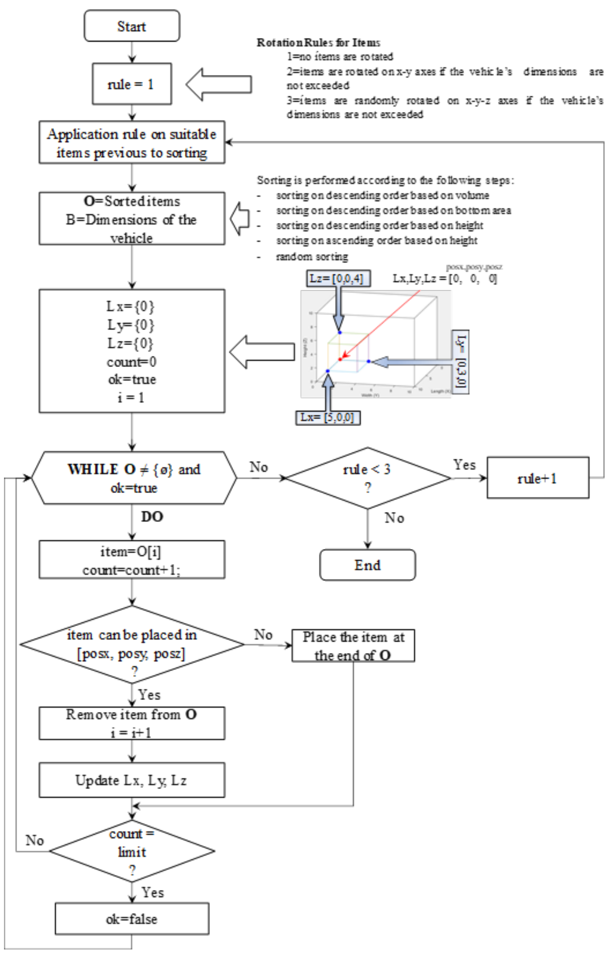


FIGURE 5. Loading heuristic for Relaxed 3L-CVRP.

After the items are sorted, the arrays Lx, Ly, and Lz are initialized ({0,0,0}). These arrays contain the advancing coordinates (posx, posy, posz) on the axes *x*, *y* and *z,* respectively. Also, the variables *count*, *ok* and *i* are initialized (*count*=0, *ok*=true, *i*=1).

While there are items in **O** and *ok* is true, the items will be loaded into the vehicle, and the variable *count* will be increased by one until a limit established will be reached. The first item loaded into the vehicle is placed in coordinates (0,0,0); the subsequent items are loaded in the available space within the vehicle. The available space is performed sequentially through the *x*-*y*-*z* axes as follows (always the space with lower values on posx, posy, posz are first considered):

1. Available space is searched on the *x*-axis. If space is available, the item is loaded and removed from **O,** and Lx, Ly, and Lz are updated. Otherwise, available space is searched on the *y*-axis.
2. Available space is searched on the *y*-axis. If space is available, the item is loaded and removed from **O,** and Lx, Ly, and Lz are updated. Otherwise, available space is searched on the *z*-axis.
3. Available space is searched on the *z*-axis. If space is available, the item is loaded and removed from **O,** and Lx, Ly, and Lz are updated. Otherwise, the item is moved to the end of the list **O** to be loaded into the final available spaces.

If the variable *count* has reached the limit established, the variable *ok* is changed from true to false to end the loop

RESULTS & DISCUSSION

Implementation of the algorithm described in Figure 5 was performed in MATLAB R2015a in a DELL laptop with Intel Core i-7 CPU 8750H at 2.20 GHz with 16 GB RAM. Tests

TABLE 3. Comparison of performance of the proposed algorithm for the Relaxed 3L-CVRP.

Instance	(Gendreau et al., 2006)	(Fuellerer et al., 2010)	(Bortfeldt, 2012)	(Lacomme et al., 2013)	Proposed algorithm
1	297.65	297.65	297.65	297.65	297.37
2	334.96	334.96	334.96	335.67	299.64
3	362.27	362.27	381.36	362.27	371.36
4	430.89	430.89	430.89	430.89	372.98
5	395.64	406.50	397.16	379.43	425.62
6	495.85	495.85	498.07	495.85	452.95
7	742.23	732.52	741.80	725.43	951.08
8	735.14	735.14	735.14	735.14	726.67
9	630.13	630.13	631.82	630.13	621.95
10	717.90	711.45	739.94	687.57	920.23
11	718.24	718.25	723.44	718.24	719.31
12	614.60	612.63	623.10	610.05	584.34
13	2316.56	2391.77	2348.48	2306.04	2734.09
14	1276.60	1222.17	1234.54	1186.96	1458.01
15	1196.55	1182.86	1202.34	1161.20	1757.66
16	698.61	698.61	704.47	698.61	680.38
17	906.42	862.18	928.93	861.80	859.32
18	1124.33	1112.18	1108.37	1084.26	1214.00
19	680.29	671.60	678.59	670.44	679.85
20	529.00	515.39	520.55	510.95	527.65
21	1004.40	951.87	964.66	943.05	918.14
22	1068.96	1030.12	1041.92	1029.87	1010.62
23	1012.51	971.05	995.22	987.06	1041.14
24	1063.61	1057.39	1053.41	1056.33	1039.31
25	1371.32	1207.97	1238.83	1232.73	1276.87
26	1557.12	1453.39	1444.58	1415.15	1376.72
27	1378.52	1333.16	1342.23	1317.38	1356.29
Avg. cost	876.31	856.67	864.53	847.04	913.83

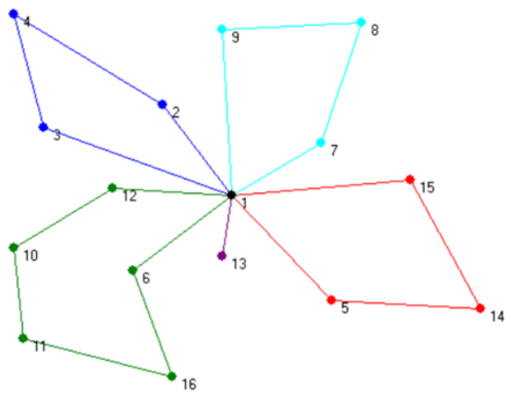
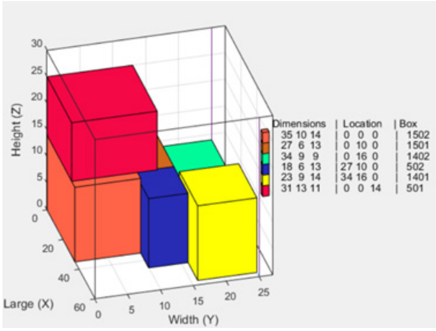


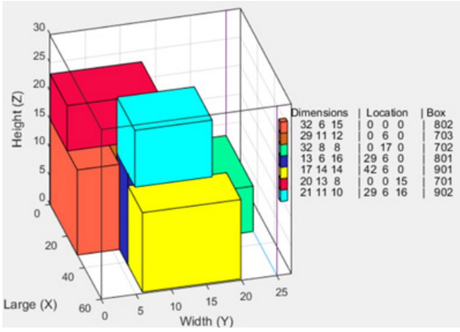
FIGURE 6. Benchmark relaxed routing solution for 3L-CVRP instance 2

TABLE 4. Benchmark relaxed loading solution for 3L-CVRP instance 2.

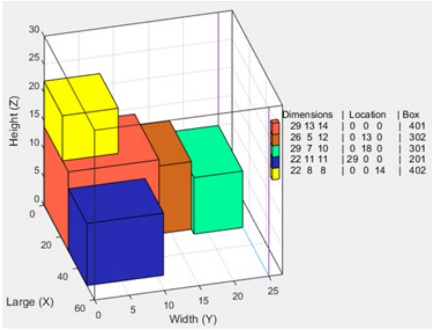
Route 1-5-14-15-1 with 5 Items



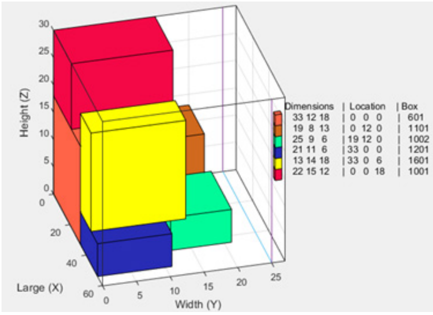
Route 1-7-8-9-1 with 6 Items



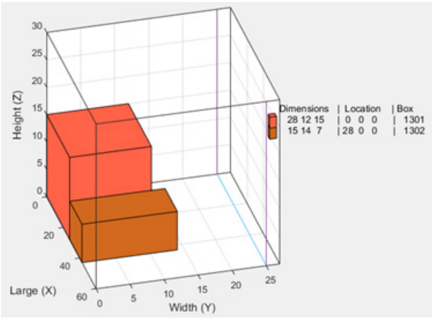
Route 1-2-4-3-1 with 5 Items



Route 1-12-10-11-16-6-1 with 6 Items



Route 1-13-1 with 2 Items



were performed with the 3L-CVRP instances reported by (Gendreau et al. 2006).

The heuristic proposed is based on the relaxation of all the constraints, including the rotation in z-axis, this option was not considered by other investors.

The proposed heuristic was probed with 27 instances obtaining better results in 48.1% (13/27) of the instances.

The instances were divided in three groups with nine elements in each one of them, to analyze the results, the first group has 15-25 customers and 26-50 items; the second group has 29-44 customers and 58-94 items; the last group has 50-100 customers and 99-198 items.

The results show that in the first group, the result of six instances are improved, in the second group the results of three instances are improved and in the last group the results of four instances are improved. The better results are obtained with less than 26 customers and less than 51 items, for this reason we can conclude that the algorithm works better with little instances.

The results also were analyzed to know how much is improved in each group, obtaining that for the first group the average improvement was 5.23%; for the second group the average improvement was 2.14%; and for the last group the average improvement was 2.18%. The group with less customers and less items obtained the better results.

Table 3 presents the comparison of the performance of the proposed algorithm for the Relaxed 3L-CVRP with the algorithms reported by (Gendreau et al. 2006; Fuellerer et al. 2010; Bortfeldt 2012; Lacomme et al. 2013).

The details of the solution, for instance two, are described in Figure 6 and Table 4.

CONCLUSION AND FUTURE WORK

In this work, an algorithm to solve the 3L-CVRP when the relaxation of constraints is needed and appropriate to enhance load capacity in transportation was developed. In practice, not all constraints are used, and their consideration depends on the type of items to be distributed.

For example, although the LIFO constraint allows items to be unloaded without additional movements, it is one of the constraints that can lead to significant empty spaces in the loading space. As more constraints are used, the utilization of the vehicle's capacity is decreased. Thus, emptier vehicles are obtained, requiring additional vehicles to serve a distribution network.

On the other hand, the support constraint can be relaxed when no fragile items are transported, especially when the items are loaded first over the *x-y* axes and finally on the *z*-axis because it improves the support of the items.

In an emergency or urgent request of items, it is necessary to evaluate what is more important, to avoid having extra movements (i.e., comply with LIFO constraints) or to transport the highest quantity of items with full utilization of the vehicle's cargo.

Although there are reported works on solving approaches for the relaxed 3L-CVRP that consider product rotation to optimize load capacity, rotation on the three axes has not been thoroughly studied. In this aspect, the present work explicitly explored the three-axes rotation and its impact on load capacity optimization.

This approach led to obtain better solutions than those reported in reviewed works for the Relaxed 3L-CVRP in 48.1% of well-known instances. However, more research is needed due to the complexity of 3L-CVRP. Thus, future research will be focused on the following actions:

1. Improve the algorithm to achieve improvements on the fully constrained 3L-CVRP. The use of Tabu-Search to improve diversification of the sorting options can be considered.
2. Adapt the algorithm for the 3L-CVRP with more than one depot.
3. Extend on the constraints for the 3L-CVRP.

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DECLARATION OF COMPETING INTEREST

None.

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