Time-Varying Hedging using the State-Space Model in the Malaysian Equity Market

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ABSTRACT

Theoretical and practice of financial hedging have expanded over the last 25 years. Research in this area is numerous and one of them is identifying the time-varying optimal hedge ratio. In this study, the time-varying hedge ratio is analysed using the State Space model (Kalman Filter) on daily Kuala Lumpur Composite Index (KLCI) and Kuala Lumpur Future Index (KLFI) from April 2005 to March 2008. Comparison between the static and time-varying hedge ratio and forecast performance is done to analyse the efficiency of the time-varying estimates. Our results show that for forecasting purposes the State Space model has the ability to forecast better when 30 days of forecast horizon are used. The volatility of the time varying hedge ratio is relatively low, but the static estimate of the hedge ratio overestimates the amount of the KLFI futures contract needed to hedge the KLCI. This may prove to be an unnecessary cost for fund managers in hedging using KLFI.

INTRODUCTION

The rapid expansion of derivatives markets over the last 25 years has led to a corresponding increase in interest on the theoretical and practical aspect of hedging. Hedging financial risks, involve the use of derivatives as a mean to reduce exposures that are associated with the volatility of the financial variables. A hedging transaction is expected to lock in future values and eliminate the effect of volatility. However, a complete elimination of risk will not be a practical strategy in any investment and thus, a reasonable amount of tolerable risk should be allowed for the purpose of capturing the uncertain favorable movement in the future. This can be done by estimating the size of the short position that must be held in the futures market, as a proportion of the long position held in the spot market, that maximizes the agent’s expected utility, defined over the risk and expected return of the hedged portfolio. In doing so, the optimal hedge ratio is invariably calculated and an adjustment is then made according to individual tolerable risk level in an investment. This leads to the problem of estimating the optimal hedge ratio (OHR).

Optimal hedge ratio is basically based on the coefficient of the regression between the change in the spot prices and the change in price of the hedging instrument. The problem arises when it is recognized that the coefficient is time varying and investors need to readjust the hedge ratio or rebalance the proportion between the cash and the derivative instruments. It has been recognized that time varying coefficient (TVC) model outperforms the static coefficient (SC). Thus, this study proposes and demonstrates a time varying procedure based on the Kalman Filter as suggested by Hatemi and Roca (2006). This is in line with Harvey (1997) findings, which conjectured that the Kalman Filter approach has better statistical and forecasting properties.

The dynamic model using the Kalman Filter approach will be adopted and analysed using data from the Kuala Lumpur Composite Index (KLCI), which represents the cash market (underlying asset) and the Kuala Lumpur Futures Index (FKLI) as the derivative asset. The rest of the paper is organised as follows. The next section discusses the literature reviews based on related issue. This is followed by a section on methodology that discusses the procedure of the Kalman Filter. This
procedure will enable us to find the time varying hedge ratio that will be compared to the static hedge ratio using the conventional least square regression. The next section is the analyses of the data in this study where we first test for stationarity and cointegration of the KLIC and KLF. Then the hedge ratios are obtained using the static and the time varying model. Finally the last section provides the conclusion of the paper.

LITERATURE REVIEW

Since risk in this context is usually measured as the volatility of portfolio returns, an intuitively plausible strategy might be to choose the hedge ratio that minimizes the variance of the returns of a portfolio containing the stock and futures position, which is known as the optimal hedge ratio. To estimate such a ratio, early work simply used the slope of an ordinary least squares regression of stock on futures prices. Primarily, estimating the hedge ratio generally falls under Ordinary Least Square (OLS), Error Correction Models (ECM) and the Autoregressive Conditional Heteroscedasticity (ARCH)-based models; (see Cechetti et al. 1988; Myers and Thompson 1989; Bailie and Myers 1991; Kroner and Sultan 1991; Lien and Luo 1993; Park and Switzer 1995, among others).

Johansen’s (1995) suggested that the OLS method does not perform as well as the ECM or the ARCH-based models due to the nature of time-series data that can be summarised in stylised facts of most financial data. It also does not capture the time varying nature of the hedge ratio and the cointegration effect between the cash asset and its derivative. The presence of cointegration between the two assets requires the use of ECM for parameter estimation and this method also has the ability to show the long-run and short-run relationship between the two assets. Ghosh (1995), Chou et al. (1996), Ghosh and Clayton (1996), Lien (1996), Sim and Zurbruegg (2001), Moosa (2003), among others, use the ECM estimation to study the optimal hedge ratio. They found that the ECM method yields better results compared to OLS. This is due to the misspecification of OLS when cointegration is present between asset and its derivative, which results in downward bias of the parameters and subsequently the optimal hedge ratio. However, the ECM still does not take into account the time varying nature of optimal hedge ratio.

Recently, empirical works powerfully supported the time-varying volatility discovered in many economic and financial time series. After considering the deterministic volatility functions (Dumas, Fleming & Whaley 1998), most researchers adopted the framework of the GARCH model. Particularly, the bivariate GARCH models were widely adopted to explain the behaviour of the spot and futures prices which produced the dynamic hedging strategy (Baillie & Myers 1991; Myer 1991; Lien & Luo 1993).

An improvement was made by adopting a bivariate generalised autoregressive conditional heteroskedasticity (GARCH) framework (Kroner & Sultan 1993; Lien & Luo 1993; Moschini & Myers 2002) or the stochastic volatility (SV) model (Anderson & Sorensen 1996; Lien and Wilson 2001). Although these studies are successful in capturing the time-varying covariance/correlation features, many of them focus on the myopic hedging problem.  On the contrary, models of Howard and D’Antonio (1991), Lien and Luo (1993, 1994), Geppert (1995), and Lien and Wilson (2001) examined the multiperiod minimum risk hedging strategy through various methods. Two general approaches were developed to estimate time-varying minimum-variance hedge ratios; one approach was to estimate hedge ratios by estimating the conditional second moments of spot and futures return series via a variety of GARCH (generalised autoregressive conditional heteroscedasticity) models and the other approach treated the hedge ratio as a time-varying coefficient and estimated the coefficient directly (Lee, Yoder, Mittelhammer & Mccluskey 2006).

Though all of the above models allowed hedge ratios to be time varying, a few authors allow optimal hedge ratios to be state-dependent. Alizadeh and Nomikos (2004) were the first to apply a Markov regime-switching model (MRS) for estimating time-varying minimum variance hedge ratios. They tested their model with the FTSE 100 and S&P 500 index data and found that the MRS can improve hedging performance in terms of variance reduction and utility maximisation. Sarno and Valente (2000, 2005a, 2005b) applied regime-switching models in the context of stock index futures markets and exchange rate risk management. Nevertheless, MRS has some restrictions; there is an upper and a lower bound on the time-varying hedge ratio and the hedge ratio estimated from MRS is not time varying if the transition probabilities are constant.

Lee et al. (2006) developed a more general Markov regime-switching model, the random coefficient autoregressive Markov regime-switching model (RCARRS), for estimating the state-dependent time-varying minimum variance hedge ratio. The RCARRS combines properties of both the MRS and random coefficient autoregressive model (RCAR) proposed by Bera, Garcia, and Roh (1997). Estimated hedge ratios from RCARRS are time varying even when the transition probabilities are constant, and the hedge ratios can fluctuate freely without upper and lower bounds. Based on point estimates of hedging portfolio variance reduction using aluminum and lead futures data, RCARRS outperforms both MRS and RCAR.

In contrast, Lee and Yoder (2006) extended Engle and Kroner’s (1995) BEKK-GARCH (Baba-Engle-Kraft-Kroner-GARCH) model with a bivariate regime-switching model (RS-BEKK) for estimating state-dependent time-varying optimal hedge ratios based on estimated conditional second moments of spot and futures time series. Their results suggested that RS-BEKK outperforms
the state-independent BEKK, although the relative improvement is not statistically significant.

Based on the arguments above, it is rather conspicuous that the ECM and the ARCH-based models are superior compared to the OLS method, nevertheless, no consensus have been arrived as yet and the results are mixed as to which is the best. Floros and Vougas (2004) made a comparison on the Greek stock and futures markets for 1990 to 2001 and found that ECM and Vector Error Correction Model (VECM) were superior over the OLS model. However, the GARCH model is superior over the ECM and VECM methods. On the contrary, Lim (1996) found that the ECM was a superior model when using Nikkie 225 futures contracts but the GARCH model was superior over the OLS model when data on LIFFE futures contracts were used.

Bystrom (2003) used the OLS method on the effectiveness of electricity futures contracts as a hedging tool in Norway between 1996 to 1999 and found that the OLS method was superior. Butterworth and Holmes (2001) also found that the OLS method performs better on the FTSE-mid250 futures contract when outliers were omitted from the analysis. Further support for the OLS over GARCH can be found in Lien et al. (2002) when data on currency futures, commodities futures and stock indices covering ten markets. This is supported by Holmes (1995), based on a study using FTSE-100 stock index futures.

With the mix results on the methods to find the optimal hedge ratio, further research is required to strengthen the understanding on the behavior of the optimal hedge ratio in risk reduction. As the methods to find the ratio can be divided into static and time varying, and since time varying estimation has better estimation properties, this study uses the Kalman Filter time varying estimation as proposed by Hatemi and Roca (2006) on the Malaysian stock and futures index.

METHODOLOGY

For hedging purposes, we need to know the hedging ratio \( \beta \), which is calculated by the following regression;

\[
S_t = \alpha_0 + \beta F_t + \mu_t, \tag{1}
\]

where \( S = \) spot price of asset
\( F = \) futures price of hedging instrument.

This is derived from \( V_h = Q_F S - Q_S F \),

where \( V_h = \) value of hedged portfolio
\( Q_F = \) quantity of spot asset
\( Q_S = \) quantity of futures instrument.

and \( \Delta V_h = Q_S \Delta S - Q_F \Delta F \)

Thus when \( \Delta V_h = 0 \), that is \( \frac{Q_S}{Q_F} = \frac{\Delta S}{\Delta F} \), we have perfect hedging.

Letting \( \beta = \frac{Q_S}{Q_F} \) so that \( \beta = \frac{\Delta S}{\Delta F} \), where \( \beta = \) optimal hedge ratio.

Thus the optimal hedge ratio (\( \hat{\beta} \)) is the coefficient of regression (1). In this case, the coefficient is a static one and it expected that the hedge ratio is dynamic in nature. Thus we need a time varying estimation of the hedge ratio as this will result in more accurate forecasting properties. According to Lucas (1976), investors may anticipate policy changes and rationally change their portfolio accordingly to reflect their expectation. Engle and Watson (1987) and Hatemi (2002) also support the dynamic nature of the hedge ratio that is due to the expectation and adjustment to unanticipated changes. Further, the static estimation of the hedge ratio may be downward bias due to the misspecification of the regression equation (1) as the dynamic nature of hedge ratio results in the non-whiteness of the error terms.

This study will use a time varying estimation of the hedge ratio by using Kalman Filter on estimation (1) and by consider the hedge ratio or the parameter estimation to follow an autoregressive process of order 1. Thus we have;

\[
S_t = \alpha_0 + \beta_t F_t + \mu_t \tag{2}
\]

Where the first equation in (2) is the transition equation and the second equation is the state equation that describe the time varying of the hedge ratio that follow an autoregressive process of order 1. The error terms \( \epsilon \) and \( \eta \) are assumed to be independent white noise processes. This state space model can be estimated using the Kalman Filter by considering the following specifications. Let \( y_t = \) a function of \( x \) where \( y_t \) is an \( N \times 1 \) vector and \( x_t \) be an \( N \times k \) matrices and the coefficients \( \beta_t \) is a \( k \times 1 \) vector. Further the coefficient \( \beta_t \) is assumed to follow an autoregressive process of order one. The specification is;

\[
y_t = \beta_t x_t + \epsilon_t \tag{3}
\]

\[
\beta_t = A \beta_{t-1} + \eta_t \tag{4}
\]

\( \epsilon = NID(0, \sigma_\epsilon) \) and \( \eta = NID(0, \eta_\epsilon) \)

\( E(\epsilon_t \eta_t) = 0 \) \( \forall t \) and \( s \).

With the assumptions above, it is possible to determine the parameters \( A \), \( Q \) and \( P \) and make inference about the time varying coefficient \( \beta_t \) given the observations of \( (x_t, y_t) \) by using the maximum likelihood estimations. The process is by applying the Kalman Filter for each period in time to the following equations;

\[
\hat{\beta}_{t+1} = A \hat{\beta}_{t+1} \\
\hat{\beta}_{t+1} = A \beta_{t+1} + \hat{\eta} \\
\hat{\epsilon}_t = (y_t - \hat{\beta}_t x_{t+1}) \\
\hat{\epsilon}_t = x_t P_{t+1} x_t + \sigma_\epsilon
\]
\[
\hat{\beta}_t = \hat{\beta}_{t-1} + P_t \epsilon_t + \frac{1}{f_t} \left( \frac{e_t}{f_t} \right)
\]

\[
P_t = P_{t|t-1} - P_{t|t-1} X_t' \left[ P_{t|t-1} \frac{1}{f_t} \right]
\]

Where \(\hat{\beta}_t\) is the maximum likelihood estimator of the coefficient at time \(t\), \(P_t\) denotes the variance of \(\hat{\beta}_t\), \(\epsilon_t\) is the one step prediction error with variance \(f_t\), the subscript \(t|t-1\) estimation of respective parameter at time \(t\) given information up to \(t-1\).

\[
\hat{\beta}_{t-1} = \hat{\beta}_{t-1} + Q
\]

The above explains the forward recursion in estimating the parameters. Harvey (1997b) shows that it is possible to do the above estimations by using backward estimation, thus using all observations in the first estimation. For a more detail and further analysis the the Kalman Filter based on maximum likelihood approach, the interested readers is referred to Harvey (1990, 1997a).

**ANALYSIS**

We apply the time varying estimation of the hedge ratio by using Kalman Filter (equation 2) on daily data from the Kuala Lumpur Composite Index (KLCI) and the Kuala Lumpur Futures Index (KLFI) from April 2005 to March 2008, a total of 743 observations. This period is used because it is a relatively volatile period, and hedging effect can be seen clearer in a volatile period. Equation (1) is used to estimate the static hedge ratio. Before estimating equations (1) and (2), unit root and cointegration tests are applied on KLCI and KLFI to avoid using non-stationary data that will result in spurious regression. The test uses Augmented Dickey-Fuller (ADF) and Johansen cointegration test respectively where the results are given in Table 1.

**TABLE 1. Test for unit-root and cointegration of KLCI and KLFI**

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>ADF Unit root tests (t-statistics)</th>
<th>Johansen Cointegration test (trace value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H0: I(1), H1: I(0)</td>
<td>KLCI: -1.0948(3)@</td>
<td>KLFI: -1.0900(0) @</td>
</tr>
<tr>
<td>H0: I(2), H1: I(1)</td>
<td>KLCI: -13.4165(2)**</td>
<td>KLFI: -28.5057(0)*</td>
</tr>
<tr>
<td>H0: No cointegration, H1: At least one cointegration eq.</td>
<td>KLFI: 79.5793**</td>
<td></td>
</tr>
<tr>
<td>H0: One cointegration eq., H1: More than one cointegration eq.</td>
<td>KLFI: 0.8301</td>
<td></td>
</tr>
</tbody>
</table>

@Number in bracket shows the optimal lag order using SIC
*Significant at 10%
**Significant at 5% (MacKinnon-Haug-Michelis, 1999)

Thus from the results above, KLCI and KLFI are integrated of first order and they are cointegrated. This allows us to run equations 1 and 2 for the purpose of comparing the static and dynamic hedge ratios. We proceed with the estimations of equations 1 and 2, and the results are given in Table 2.

In both estimations above, it is found that the hedge ratios are significant at 5% significant level, indicating that the KLFI can be used to hedge against the KLCI. The two results from the estimations are further tested to find out which of the two models perform better. In doing so, we set the static model (equation 1) as the null hypotheses and the time varying model (equation 2) as the alternative. As suggested by Hatemi-J (2002), the test statistic for the above hypotheses can be obtained from the likelihood ratio, given by

\[
\text{LR} = -2 \ln \left( \frac{\hat{L}_R}{\hat{L}_U} \right) = \chi^2_t,
\]

where and are the values for the likelihood functions for the restricted model (equation 1) and the unrestricted model (equation 2) respectively. The log likelihood values are \(-3712.44\) and \(-3660.94\) for equations 1 and 2 respectively. Thus estimated value of LR is found to be 103.01 and the critical value is given by 6.63 at 1% significant level. Thus we reject the null hypotheses of static model and conclude that the time varying model (equation 2) is a better model in estimating the optimal hedge ratio.

To further investigate the performance of the time varying model, we rerun equations 1 and 2 leaving the last 30 days observations for the purpose of forecasting. This forecast window is choose to match with that used by Hatemi and Roca (2006). It is found that the average forecasting error for the static and the time varying models are 2.73 and 0.12 respectively. This further indicates that the time varying model do forecast better than the static model.

Finally, we show the graph of the time varying hedge ratio using the Kalman Filter in Graph 1 below to track the values. It is interesting to note that the time varying hedge ratio is below the static hedge ratio for most of the times in the period of the study. This means that the static hedge ratio overestimates the amount of the KLFI futures contract needed to hedge the KLCI. Further, it also means that the return on the KLFI is greater than the return of...
the KLCI in that period. With this time varying hedge ratio, investors need to frequently rebalance their portfolio to hedge their cash assets. This may prove to be costly and they may have to find other derivatives to hedge, however the volatility of the time varying hedge ratio is relatively low.

CONCLUSION

This paper looks at the static and time varying hedge ratio calculated using the Kuala Lumpur Futures Index (KLFI) to hedge the Kuala Lumpur Composite Index KLCI. Daily data from the KLCI and the KLFI from April 2005 to March 2008 is used. It is shown from the analysis that the hedge ratio using the latter method performs better. The time varying estimates of the hedge ratio uses the Kalman Filter procedures which is to have a more favorable statistical properties compared to the static estimates. From the results, it is also found that the static method consistently over-hedged the KLCI, due to better returns in the KLFI, thus incurs unnecessary cost for that purpose. This can seen from graph 1 where the time varying graph is consistently below the static graph.

This however contradicts the results found in Hatemi and Roca (2006) using Australian market data where it is found that there are periods where the cash market is under-hedged before 1994 and over-hedged thereafter when compared to the static hedge ratio due to better returns in the futures market. Although the static method over-hedged the KLCI, the volatility of the hedge ratio is relatively low. This means that the need to readjust or rebalancing in the hedge ratio for the Malaysian market is relatively less compared to the Australian market, thus implying the hedging activities in less risky. On the forecasting error, the time varying estimations perform much better than the static model, where the forecasting errors are 2.73 and 0.12 respectively. With these results, the time varying method, using the Kalman Filter procedures, seems to be the more appropriate method for calculating the hedge ratio.

REFERENCES


