A STUDY ON THE EFFECTS OF TRENDS DUE TO INERTIA ON EWMA AND CUSUM CHARTS
(Suatu Kajian Kesan Trend Akibat Inertia ke atas Carta EWMA dan CUSUM)

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ABSTRACT

Unlike a Shewhart chart, the exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) charts are memory control charts (also known as time weighted control charts) that are used for a quick detection of small shifts in the process mean. Control charts that combine information from present and past samples, like the EWMA and CUSUM charts have the ability to detect process changes based on past information. A trend could exist on the EWMA and CUSUM charts, where all the sample points fall in one particular direction on the charts, for example, only above or below the center line. For this case, if a shift in the process mean occurs in the opposite sides of the chart, then such a shift cannot be detected quickly. This phenomenon is known as the inertia problem. A simulation study is conducted using Statistical Analysis System (SAS) to study and compare the effects of inertia on EWMA and CUSUM charts. It is found that the EWMA chart is affected by the inertia problem but not the CUSUM chart.

Keywords: EWMA chart; CUSUM chart; inertia; average run length (ARL)

ABSTRAK

Berbeza daripada carta Shewhart, carta purata bergerak berpemberat eksponen (EWMA) dan hasil tambah longgokan (CUSUM) adalah carta kawalan beringatan (juga dikenali sebagai carta kawalan berpemberatkan masa) yang digunakan untuk pengesanan cepat anjakan kecil dalam min proses. Carta kawalan yang menggabungkan maklumat daripada sampel baru dan lama, seperti carta EWMA dan CUSUM mempunyai kebolehan untuk mengesan anjakan dalam proses berdasarkan maklumat lama. Suatu tren mungkin wujud pada carta-carta EWMA dan CUSUM, yang mana semua titik sampel jatuh pada suatu arah tertentu di atas carta, misalnya, hanya di atas atau di bawah garisan tengah. Untuk kes ini, jika anjakan dalam min proses berlaku dalam arah yang bertentangan di atas carta, maka anjakan sedemikian tidak dapat dikesan dengan cepat. Fenomena ini dikenali sebagai masalah inersia. Suatu kajian simulasi dijalankan dengan menggunakan Sistem Analisis Berstatistik (SAS) untuk mengkaji dan membandingkan kesan inersia ke atas carta EWMA dan CUSUM. Adalah didapati bahawa carta EWMA dipengaruhi oleh masalah inersia tetapi bukan carta CUSUM.

Kata kunci: carta EWMA; carta CUSUM; inersia; purata panjang larian (ARL)

1. Introduction

In statistical process control (SPC) implementation, Shewhart control charts are useful in the phase I process, where the process is likely to be out-of-control due to large shifts (Montgomery 2005). The patterns of the sampling points on Shewhart charts give information concerning the behaviour of assignable causes. Shewhart charts are slow in detecting small shifts as they use only the information contained in the last sampling point.

The exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) charts are time weighted control charts that are suggested as superior alternatives to the Shewhart chart when the detection of small shifts are desired. The EWMA and CUSUM

charts have comparable average run length (ARL) performances (Montgomery 2005). The ARL of a chart is defined as the expected number of sample points that must be plotted on a chart before the first out-of-control is signaled. Both the EWMA and CUSUM charts are more commonly used for individual observations, i.e., the sample size, $n = 1$, instead of subgrouped data, where $n > 1$. The EWMA chart was proposed by Roberts (1959) while the CUSUM chart originated from Page (1954).

Some of the recent works on EWMA charts are as follow: Serel and Muskowitz (2008) presented an EWMA cost minimization model to design a joint control scheme for the mean and variance based on economic and statistical performance criteria. Apley and Lee (2008) developed analytical expressions for the sensitivity of EWMA charts applied to residuals and to autocorrelated data. Eyvazian *et al.* (2008) proposed an exponentially weighted moving sample variance chart to monitor process variance when the sample size is one. Shu (2008) extended the adaptive EWMA chart for process location to monitoring process dispersion. Sheu *et al.* (2007) suggested a Poisson generally weighted moving average chart which improves upon the ARL performance of the Poisson EWMA chart. Castagliola *et al.* (2007) discussed the construction of a variable sampling interval S^2 EWMA chart for monitoring of the sample variance of a process. An EWMA chart for detecting joint shifts in both the mean and variance for autocorrelated data was proposed by Thaga and Yadavalli (2007). Chou *et al.* (2006) developed the economic design of variable sampling intervals EWMA charts to determine the values of six test parameters.

Recent works on CUSUM charts include the following: A CUSUM chart based on both adaptive and variable sampling interval (VSI) features are developed by Luo *et al.* (2009). Wu *et al.* (2009) discussed the construction of an upward CUSUM chart in the presence of inspection error. Zantek (2008) proposed an analytic method for computing the run-length distribution of the CUSUM of a statistic. Shu *et al.* (2008) presented a Markovian-type mean estimating procedure in the conventional CUSUM scheme to update its reference value in an adaptive way. MacEachern *et al.* (2007) provided a robust-likelihood CUSUM chart that discounts outliers and yet has the ability to detect large shifts quickly. Testik (2007) quantified the effect of estimated process mean on the conditional and marginal performance of the Poisson CUSUM chart. Zhang and Wu (2007) investigated the weighted loss function (WLC) CUSUM chart with a variable sample size feature. Wu *et al.* (2007) proposed a variable sample size and sampling interval WLC CUSUM chart.

The layout of this paper is as follows: Section 2 and 3 explain the EWMA and CUSUM charts, respectively. Section 4 studies and compares the effects of inertia on EWMA and CUSUM charts. Conclusions and suggestions for future works are drawn in Section 5.

2. An EWMA Control Chart

The EWMA chart was introduced by Roberts (1954). Crowder (1987) and Lucas and Saccucci (1990) provided excellent discussions on the EWMA chart. The EWMA chart is more desirable than the Shewhart chart in the detection of small shifts. Let X_1, X_2, …, represent a sequence of measurements from a $N(\mu, \sigma^2)$ distribution. The EWMA chart is based on the following statistic (Montgomery 2005):

$$Z_i = \lambda X_i + (1-\lambda)Z_{i-1}, \text{ for } i = 1, 2, \ldots . \tag{1}$$

Note that $0 < \lambda \leq 1$ is a smoothing constant and the starting value is $Z_0 = \mu_0$. Here, μ_0 is the desired mean value when the process is in-control. When μ_0 is unknown, the average of the

preliminary data can be used as the starting value, i.e., $Z_0 = \overline{X}$. The limits of the EWMA chart are

$$\text{UCL} = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \tag{2a}$$

and

$$\text{LCL} = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}, \tag{2b}$$

where L is the factor that controls the width of the limits. An out-of-control is signaled when Z_i exceeds the limit in Equation (2a) or (2b). Several theoretical studies in selecting the λ and L combination based on some ARL properties of the EWMA chart were made by Crowder (1987) and Lucas and Saccucci (1990). The term $\left[1-(1-\lambda)^{2i}\right]$ approaches unity as i becomes larger. Thus, the asymptotic limits of an EWMA chart are as follow:

$$\text{UCL} = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}} \tag{3a}$$

and

$$\text{LCL} = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}}. \tag{3b}$$

Crowder (1989) provided plots for selecting the λ and L combination based on a desired ARL_0 value for the case of asymptotic limits.

3. A CUSUM Control Chart

The CUSUM chart was proposed by Page (1954). Several versions of CUSUM charts exist but the tabular CUSUM is the most commonly used. The tabular CUSUM works by accumulating deviations from the target mean, μ_0 that are above the target with a statistic C^+ and accumulating deviations from μ_0 that are below the target with another statistic C^-. The statistics C^+ and C^- are called one-sided upper and lower CUSUMs, respectively, and are computed as follow (Montgomery 2005):

$$C_i^+ = \max[0, X_i - (\mu_0 + K) + C_{i-1}^+], \text{ for } i = 1, 2, \dots \tag{4a}$$

and

$$C_i^- = \max[0, (\mu_0 - K) - X_i + C_{i-1}^-], \text{ for } i = 1, 2, \dots, \tag{4b}$$

where $C_0^+ = C_0^- = 0$. The reference value, K is usually chosen as one-half the magnitude of the shift, i.e.,

$$K = \frac{|\mu_1 - \mu_0|}{2}. \tag{5}$$

Here, μ_1 is the out-of-control mean, where a quick detection is needed. If either C_i^+ or C_i^- exceeds the decision interval, H, the process is considered as out-of-control. Gan (1991) gave plots for selecting H and K values based on a desired ARL_0 value. In addition, by defining

$H = h\sigma$ and $K = k\sigma$, Hawkins (1993) gave combinations of (h, k) values (see Table 1) based on an ARL_0 of 370, which corresponds to a false alarm rate of 0.0027. In general, k is chosen relative to the size of the shift, where a quick detection is desired, i.e., $k = 0.5\delta$. Here, δ is the size of the shift in standard deviation units. A widely used value in practice is $k = 0.5$. After k is chosen, one should choose h to give the desired ARL_0 performance.

Table 1: Combinations of (h, k) values for a two-sided tabular CUSUM chart based on $ARL_0 = 370$

h	8.01	4.77	3.34	2.52	1.99	1.61
k	0.25	0.5	0.75	1.0	1.25	1.5

4. A Study on the Effects of Inertia on EWMA and CUSUM Charts

A simulation study is conducted using Statistical Analysis System (SAS) version 9 to compute the ARL values of the EWMA and CUSUM charts in the presence of inertia. Two different cases are considered, i.e., when the process is in-control and out-of-control. The sample size, $n = 1$ is used as the EWMA and CUSUM charts are usually used for individual measurements. The in-control process is assumed to follow a standard normal distribution, i.e., $\mu_0 = 0$. Similar results will be obtained for a normal distribution having other values of mean and variance. For the design of upper-sided EWMA and upper-sided CUSUM charts, the in-control ARL (ARL_0) of 500 is considered. This is equivalent to setting the false alarm rate as 0.002. For the upper-sided EWMA chart based on the EWMA statistic in Equation (1) and UCL in Equation (3a), the values of the smoothing constant considered are $\lambda \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$. The corresponding values of L are determined to be $L \in \{2.52, 2.78, 2.85, 2.87, 2.87\}$, respectively, for the asymptotic UCL, using the method described in Crowder (1989).

For the upper-sided CUSUM chart, based on the CUSUM statistic in Equation (4a), the values of $K \in \{0.2, 0.4, 0.6, 0.8, 1\}$ are considered. The corresponding values of $H \in \{8.32, 5.20, 3.75, 2.90, 2.32\}$ are found from Gan (1991).

Various levels of inertia are considered by using different levels of headstarts. The headstarts used for both the upper-sided EWMA and upper-sided CUSUM charts are $Z_0/C_0^+ \in \{0, -0.2w, -0.3w, ..., -0.9w\}$, where

$$w = \text{UCL} - \mu_0 \tag{6}$$

and

$$w = H - \mu_0, \tag{7}$$

for the upper-sided EWMA and upper-sided CUSUM charts, respectively. The ARL performances for the two charts in detecting increasing shifts in the mean are studied. Here, the mean shifts from its target value, μ_0 to the out-of-control value $\mu_1 = \mu_0 + \delta\sigma$, where $\delta \in \{0, 0.05, 0.10, 0.20, 0.5, 1, 2, 4\}$ are considered. Note that the lower-sided EWMA and lower-sided CUSUM charts are not considered as similar results will be obtained due to the symmetrical properties of the two charts.

Due to space constraint, only the ARL profiles for the upper-sided EWMA chart when $\lambda = 0.1$, 0.5 and 0.9 and for the upper-sided CUSUM chart when $K = 0.2$, 0.6 and 1, are shown in Tables 2 and 3, respectively.

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It is shown in Table 2 that the problem of inertia on the EWMA chart is more pronounced for a smaller value of λ. For example, the out-of-control ARL (ARL$_1$) profiles for $\delta = 0.05$ and $Z_0 \in \{0, -0.1w, -0.2w, ..., -0.9w\}$ are ARL$_1 \in \{301.1, 303.8, 305.6, ..., 315.9\}$, ARL$_1 \in \{383.7, 384.2, 384.6, ..., 386.1\}$ and ARL$_1 \in \{419.4, 419.4, 419.4, ..., 419.9\}$ when $\lambda = 0.1, 0.5$ and 0.9, respectively. It is obvious that as the value of Z_0 increases, or similarly, when the level of inertia increases, the difference in the ARL$_1$ values between say, $Z_0 = 0$ and $Z_0 = -0.9w$, is more pronounced for a smaller λ. Thus, the speed of an upper-sided EWMA chart in detecting an increasing shift in the mean decreases as the level of inertia increases and the severity of this problem is greater for a smaller λ.

Table 2: ARL profiles for the upper-sided EWMA chart

λ	δ	Z_0									
		0	−0.1w	−0.2w	−0.3w	−0.4w	−0.5w	−0.6w	−0.7w	−0.8w	−0.9w
0.1 (L=2.52)	0.00	500.0	502.0	503.3	504.7	506.5	507.3	509.7	511.2	512.2	513.2
	0.05	301.1	303.8	305.6	307.3	309.7	311.2	312.2	313.6	315.2	315.9
	0.10	195.9	198.2	200.6	201.9	203.2	204.7	205.9	207.1	208.2	209.1
	0.20	93.5	94.8	96.0	97.4	98.3	99.5	100.4	101.1	101.8	102.4
	0.50	24.2	25.1	25.8	26.6	27.3	28.0	28.6	29.2	29.7	30.3
	1.00	8.9	9.4	9.9	10.4	10.8	11.2	11.6	12.0	12.4	12.7
	2.00	3.9	4.2	4.4	4.7	4.9	5.2	5.4	5.6	5.9	6.1
	4.00	2.1	2.1	2.3	2.4	2.5	2.7	2.8	3.0	3.1	3.2
0.5 (L=2.85)	0.00	500.0	500.2	500.4	501.0	501.1	501.5	501.8	502.1	502.1	502.6
	0.05	383.7	384.2	384.6	384.8	385.1	385.1	385.2	385.3	385.9	386.1
	0.10	305.0	305.2	305.5	305.6	306.0	306.1	306.2	306.3	306.4	306.4
	0.20	188.8	189.1	189.7	189.9	190.0	190.3	190.3	190.5	190.6	190.9
	0.50	55.3	55.5	55.7	55.8	56.0	56.1	56.2	56.4	56.5	56.6
	1.00	12.8	13.0	13.2	13.3	13.5	13.7	13.7	13.9	14.0	14.0
	2.00	3.1	3.3	3.4	3.5	3.6	3.6	3.7	3.8	3.9	4.0
	4.00	1.3	1.3	1.4	1.4	1.5	1.6	1.6	1.7	1.8	1.8
0.9 (L=2.87)	0.00	500.0	500.0	500.1	500.1	500.1	500.2	500.3	500.6	500.7	500.7
	0.05	419.4	419.4	419.4	419.8	419.8	419.8	419.8	419.9	419.9	419.9
	0.10	351.2	351.2	351.2	351.2	351.2	351.2	351.2	351.2	351.2	351.5
	0.20	250.7	250.7	250.8	250.8	250.8	250.8	250.8	250.8	250.8	250.8
	0.50	98.1	98.1	98.1	98.3	98.3	98.3	98.5	98.6	98.6	98.7
	1.00	26.6	26.7	26.7	26.7	26.8	26.8	26.9	27.0	27.0	27.1
	2.00	4.4	4.5	4.5	4.5	4.6	4.6	4.6	4.7	4.7	4.7
	4.00	1.1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2

In Table 3, we observe that inertia has little effect on the ARL$_1$ performance of the upper-sided CUSUM chart, irrespective of the value of K when δ is small or moderate. This is because for an arbitrary K, with a small or moderate δ, the difference in the ARL$_1$ values between say, $C_0^+ = 0$ and $C_0^+ = -0.9w$, is negligible. However, the effect of inertia on the upper-sided CUSUM chart, in terms of the percentage of difference between the ARL$_1$ values, of say $C_0^+ = 0$ and $C_0^+ = -0.9w$, increases with δ.

The usefulness of the EWMA and CUSUM charts lie in their abilities in detecting small shifts, where both charts have equal ARL$_1$ performances. The values of $0 < \lambda \leq 0.25$ and $0 <$

$K < 0.5$, are usually used in practice, as smaller values of λ and K enable the EWMA and CUSUM charts, respectively, to respond quickly to a small shift. In view of this, it can be concluded that the use of an upper-sided CUSUM chart, instead of an upper-sided EWMA chart, is recommended if past experience indicates that a process is likely to be affected by inertia, as Table 2 shows that the latter is easily affected by inertia for a small value of λ while Table 3 indicates that the effect of inertia on the former is negligible, irrespective of the value of K.

Table 3: ARL profiles for the upper-sided CUSUM chart

K	δ	C_0^+									
		0	−0.1w	−0.2w	−0.3w	−0.4w	−0.5w	−0.6w	−0.7w	−0.8w	−0.9w
	0.00	500.0	500.5	500.6	500.6	500.6	500.6	500.6	500.6	500.6	500.6
	0.05	299.3	300.2	300.2	300.2	300.2	300.2	300.2	300.2	300.2	300.2
	0.10	188.1	188.8	188.8	188.8	188.8	188.8	188.8	188.8	188.8	188.8
0.2	0.20	89.7	90.4	90.6	90.6	90.6	90.6	90.6	90.6	90.6	90.6
(*H*=8.32)	0.50	26.0	26.8	27.0	27.1	27.1	27.1	27.1	27.1	27.1	27.1
	1.00	11.2	11.8	12.1	12.2	12.2	12.2	12.2	12.2	12.2	12.2
	2.00	5.3	5.7	6.0	6.2	6.2	6.3	6.3	6.3	6.3	6.3
	4.00	2.7	3.0	3.2	3.4	3.5	3.6	3.7	3.7	3.7	3.7
	0.00	500.0	500.6	500.9	501.0	501.1	501.1	501.1	501.1	501.1	501.1
	0.05	360.3	361.0	361.1	361.2	361.2	361.2	361.2	361.2	361.2	361.2
	0.10	262.6	263.2	263.3	263.7	263.7	263.7	263.8	263.8	263.8	263.8
0.6	0.20	142.4	142.7	142.9	143.2	143.3	143.3	143.3	143.3	143.3	143.3
(*H*=3.75)	0.50	35.0	35.4	35.5	35.6	35.7	35.7	35.7	35.7	35.7	35.7
	1.00	9.3	9.7	9.9	10.1	10.2	10.2	10.3	10.3	10.3	10.3
	2.00	3.4	3.6	3.8	4.0	4.1	4.2	4.3	4.3	4.4	4.4
	4.00	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.4	2.5
	0.00	500.0	500.6	500.6	500.6	500.6	500.6	500.6	500.6	500.6	500.6
	0.05	391.2	391.7	392.1	392.1	392.2	392.2	392.2	392.2	392.2	392.2
	0.10	304.7	305.0	305.1	305.2	305.2	305.2	305.3	305.3	305.3	305.3
1.0	0.20	193.7	194.3	194.8	195.0	195.1	195.2	195.2	195.2	195.2	195.2
(*H*=2.32)	0.50	56.1	56.6	56.8	57.0	57.2	57.2	57.2	57.2	57.2	57.2
	1.00	12.3	12.5	12.7	13.0	13.1	13.2	13.3	13.3	13.3	13.3
	2.00	3.1	3.3	3.4	3.6	3.7	3.8	3.9	3.9	4.0	4.0
	4.00	1.3	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.0

5. Conclusions

This paper compares the ARL_1 performances of the upper-sided EWMA and upper-sided CUSUM charts. Since both EWMA and CUSUM charts have comparable performances in the detection of a small shift, a question may arise as to which chart to use when inertia is present in the underlying process. This paper provides the answer to this potential, yet practical question which addresses any doubts faced by practitioners. This study can be extended to the multivariate EWMA (MEWMA) and multivariate CUSUM (MCUSUM) charts, as most data are multivariate in nature. Future works could also consider the inertia problem of the EWMA and CUSUM charts and that of the MEWMA and MCUSUM charts when the underlying distribution is nonnormal, i.e., for either skewed or heavy tailed distributions as the normality assumption is often violated in most real situations.

Acknowledgments

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