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SIR INTEGRATED MODEL BASED ON RUNGE-KUTTA FOR POLIO VACCINATION ANALYSIS

(Model Integrasi SIR Berdasarkan Runge-Kutta bagi Analisis Vaksinasi Polio)

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ABSTRACT

Polio was one of the most lethal acute viral infectious diseases in the 20th century. A great deal of success has been achieved since the implementation of the polio vaccination strategy in 1988, and wild polio cases have decreased by more than 99%. However, in recent years, during the COVID-19 pandemic, vaccination strategies have encountered difficulties and challenges. At this stage, we are faced with the question of whether vaccination strategies should be continued. To determine the effect of vaccination strategies on the spread of polio at this stage, this paper proposes to use the SIR model based on world data collected in 2021 to simulate the 30-days transmission process of polio with and without vaccination. In addition, the results of this model also provide us with a reference disease response plan. The simulation results show that the vaccinated polio transmission model performs better than the uninoculated polio transmission model in terms of average estimated infection case, reproduction number, and infection rate, etc. At this stage, polio will not become an endemic disease if it occurs in vaccination areas. Nevertheless, if the case occurs in an unvaccinated area, the disease may develop into an endemic disease, and we must take immediate action (increase social distance, isolation,etc.) to effectively control its spread.

Keywords: poliomyelitis; vaccination strategy; Runge-Kutta; SIR; model

ABSTRAK

Polio adalah salah satu penyakit berjangkit virus akut yang paling berbahaya pada abad ke-20. Banyak kejayaan telah dicapai sejak pelaksanaan strategi vaksinasi polio pada tahun 1988, dan kes polio liar telah menurun lebih daripada 99%. Walau bagaimanapun, dalam beberapa tahun kebelakangan ini, semasa pandemik COVID-19, strategi vaksinasi telah menghadapi kesukaran dan cabaran. Pada peringkat ini, kita berhadapan dengan persoalan sama ada strategi vaksinasi perlu diteruskan. Untuk menentukan kesan strategi vaksinasi terhadap penyebaran polio pada peringkat ini, kertas kerja ini mencadangkan untuk menggunakan model SIR berdasarkan data dunia yang dikumpul pada tahun 2021 untuk mensimulasikan proses penularan 30 hari polio dengan dan tanpa vaksinasi. Di samping itu, hasil model ini juga memberikan rujukan untuk pelan tindak balas penyakit. Hasil simulasi menunjukkan bahawa model transmisi polio yang divaksin berprestasi lebih baik daripada model transmisi polio yang tidak divaksin dari segi anggaran purata kes jangkitan, bilangan pembiakan, dan kadar jangkitan, dsb. Pada peringkat ini, polio tidak akan menjadi penyakit endemik jika ia berlaku dalam kawasan vaksinasi. Namun begitu, jika kes itu berlaku di kawasan yang tidak divaksin, penyakit itu mungkin berkembang menjadi penyakit endemik, dan kita mesti mengambil tindakan segera (meningkatkan jarak sosial, pengasingan, dll.) untuk mengawal penyebarannya dengan berkesan.

Kata kunci: poliomielitis; strategi vaksinasi; Runge-Kutta; SIR; model

1. Introduction

Poliomyelitis (Polio) is a highly contagious viral disease. The virus is mainly transmitted through feces and oral secretions. Infected children under the age of five are most likely to develop flaccid paralysis, and the mortality rate associated with respiratory failure is approximately five to ten percent (World Health Organization 2022). After polio infection, the majority of polio survivors may suffer from the post-polio syndrome (PPS), which inflicts them many years later (Centers for Disease Control and Prevention 2022). Prior to the introduction of the polio vaccine, polio was one of the most serious acute viral infectious diseases of the 20th century (Mehndiratta *et al.* 2014).

Poliomyelitis has no cure at present, but significant progress has been made since vaccination strategies were implemented. In 1998, the World Health Assembly adopted a resolution calling for the global eradication of polio. In various countries, oral polio vaccines and inactivated polio vaccines (IPV) are widely used. The number of cases in more than 125 endemic countries has decreased from 350 000 in 1988 to 175 in 2019. The two wild poliovirus strains (types 2 and 3) have been officially certified for global eradication. In 2020, only Pakistan and Afghanistan were affected by Type 1 epidemics (World Health Organization 2022). As of June 2022, wild polio cases had decreased by more than 99% (World Health Organization 2022).

While the vaccination strategy for polio has achieved great success, there is still the possibility of disease outbreaks as long as the wild poliovirus has not been completely eradicated by humans. In August 2011, the Xinjiang Uygur Autonomous Region of China imported a case of wild poliovirus (WPV) from Pakistan; on 8 December 2019, Malaysia reported its first case of polio. Through the emergency polio vaccination program, these cases were successfully prevented.

In recent years, however, the shortage of world health resources and the anti-vaccine campaign have made it difficult to implement vaccination strategies. A significant number of countries have transferred facilities and resources originally used to combat polio to deal with the COVID-19 pandemic (Uwishema *et al.* 2022). Presently, the African polio eradication laboratory uses 50% of its diagnostic capacity for covid-19 testing in 15 African countries, and 25% of the polio staff at the World Health Organization spends more than 80% of their time responding to covid-19. In spite of the commendable efforts to bring these resources to bear on covid-19, the circulating vaccine derived poliovirus (CVDPV) remains a major concern (Uwishema *et al.* 2022). Recent years, there have been more and more anti-vaccine campaigns around the world, increasing the obstacles to vaccination implementation, making vaccination issues more and more concerning (Burki 2020; Benecke & E DeYoung 2019; Dubé 2015).

Thus, the question of whether we should continue to implement the polio vaccination strategy has become a new issue at this point. It should be noted, however, that neither Google Academic nor Scopus contain a scientific discussion or report on this issue.

Scientists use mathematical models to predict the prevalence of infectious diseases, deduce the conditions for non-transmission of infectious diseases, and formulate appropriate prevention and control measures accordingly. It is therefore the purpose of this study to simulate and compare the transmission process of polio in vaccinated and unvaccinated individuals, respectively, using the SIR model. Based on the results of the experiments, the impact of vaccination strategies on the transmission of polio at the current stage will be evaluated in order to provide a scientific basis for future decision-making.

2. Basic Concept

2.1. *SIR models*

2.1.1. *A demographics-based SIR model*

The Susceptible-Infected-Removed (SIR) epidemic model is a classic compartment model. This model is widely used to predict the progression of infectious diseases (Altmann 1995; Singh & Gupta 2022), such as rubella, measles, mumps, and COVID-19 (Kovalnogov *et al.* 2021).

The classic SIR model divides the population (N) into three groups or compartments: susceptibles (S), infecteds (I), and recovereds (R). The number of individuals S, I and R at time t is $S(t)$, $I(t)$ and $R(t)$. The population is $N(t) = S(t) + I(t) + R(t)$ at time t. In this model, the birth and mortality rates are assumed to be equal to M since the population is assumed to be constant (Widyaningsih *et al.* 2018). Each compartment of , S , I and R has death so that the number of individuals in each successive compartment is reduced by MS, MI and MR. Disease transmission occurs only when susceptible and infected individuals are directly in contact. Assume that β is the infection rate, and Γ is the recovery rate.

Thus, a demographics-based SIR model by Hethcote is

$$\begin{cases} \frac{dS}{dt} = M - \frac{\beta SI}{N} - MS \\ \frac{dI}{dt} = \frac{\beta SI}{N} - \Gamma I - MI \\ \frac{dR}{dt} = \Gamma I - MR \end{cases} \quad (1)$$

with initial conditions $S(0) = S_0, I(0) = I_0, R(0) = R_0, M, \beta, \gamma > 0$. Model (1) is a first-order nonlinear differential equation system.

2.1.2. *A demographics-based SIR model with vaccination*

In order to evaluate the impact and role of vaccination on polio transmission, vaccination factors have been added to the demographics-based SIR model. This model assumes that some people (primarily newborns) have been vaccinated, but not all, and the vaccination rate is V. The number of vaccinated individuals was MV, and were classified as the recovery group. Thus, the number of susceptible individuals decreased by MV, while the number of recovered individuals increased by MV.

This demographics-based SIR model with vaccination is formulated as follows:

$$\begin{cases} \frac{dS}{dt} = M(1 - V) - \frac{\beta SI}{N} - MS \\ \frac{dI}{dt} = \frac{\beta SI}{N} - \Gamma I - MI \\ \frac{dR}{dt} = MV + \Gamma I - MR \end{cases} \quad (2)$$

with $S(0) = S_0,\ I(0) = I_0,\ R(0) = R_0, M, \beta, \gamma,\ V > 0$.

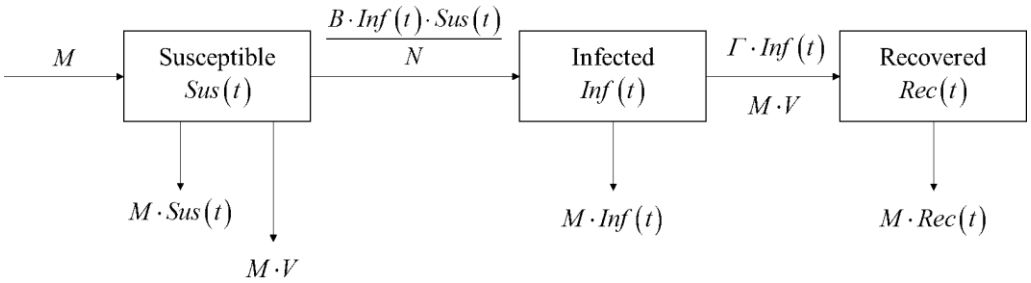


Figure 1: Illustration of demographics-based SIR model with vaccination

2.2. *Basic reproduction number*

The basic reproduction number ($\boldsymbol{R_0}$), was used to count offspring in the field of demography (Heesterbeek 2002). When $\boldsymbol{R_0}$ was adopted for use by epidemiologists, numerous definitions were proposed. Dietz proposed that $\boldsymbol{R_0}$ is "the number of secondary cases one case would produce in a completely susceptible population" (Dietz 1993). Fine stated that "average number of secondary cases" (Fine 1993). Diekmann used the description "expected number of secondary cases". (Diekmann *et al.* 1990). Counting the number of infected cases during an epidemic can be very difficult. As a result, $\boldsymbol{R_0}$ is always estimated retrospectively from seroepidemiologic data or by using theoretical mathematical models (Li & Blakeley 2011). When mathematical models are used, the values of $\boldsymbol{R_0}$ are often estimated by using ordinary differential equations (Keeling & Grenfell 2000; Heesterbeek 2002). A precise definition of $\boldsymbol{R_0}$ was proposed for a general compartmental disease transmission model based on a group of ordinary differential equations (Van den Driessche & Watmough 2002). In this paper, if $\boldsymbol{R_0} < 1$, then the disease free equilibrium is locally asymptotically stable; whereas if $\boldsymbol{R_0} > 1$, then it is unstable. In the past three years, many scholars have used the generation matrix method (Diekmann *et al.* 2010) to obtain $\boldsymbol{R_0}$ and the global stability for COVID-19 spreading (Mwalili *et al.* 2020; Annas *et al.* 2020).

The basic reproduction number for SIR model with demography is $\boldsymbol{R}_0 = \dfrac{\beta}{M+\Gamma}$ (Kousar *et al.* 2016). When vaccination is implemented, $\boldsymbol{R}_V = \boldsymbol{R}_0(1-V) = \dfrac{\beta}{M+\Gamma}(1-V)$ (Chauhan *et al.* 2014). This number indicates whether the infection will spread through the population or not. If $\boldsymbol{R}_0 < 1$, the disease is disappeared and abolished soon. If $\boldsymbol{R}_0 = 1,$, the disease is controlled and exist in certain area with any condition. If $\boldsymbol{R}_0 > 1,$, the disease will become an endemic disease.

2.3. *The Runge-Kutta fourth order (RK4) method*

The Runge-Kutta fourth order method (RK4) is a classic method for solving ordinary differential equations (ODE) models (Kennedy & Carpenter 2019). RK4 is often referred to as a one-step approach from one previous information step used to calculate the approximate next step. Compared to the Euler method, this method has a smaller error value and is also easier to program, has fewer cut errors, and also has fewer rounding errors. In engineering, the RK4 has a high level of precision that can meet most precision requirements, and the computational complexity can also meet the performance requirements of most processors. Therefore, RK4 is also known as the standard Runge-Kutta method.

The RK4 method computes the approximate solution to the initial value problem $\frac{dy}{dt} = f(t, y)$ with initial condition, $y(t_0) = y_0$. Then, we have

$$\begin{cases} y_{i+1} = y_i + \frac{h}{6}(p_1 + 2p_2 + 2p_3 + p_4), \\ p_1 = f(t_i, y_i), \\ p_2 = f(t_i + \frac{1}{2}h, y_i + \frac{1}{2}p_1 h), \\ p_3 = f(t_i + \frac{1}{2}h, y_i + \frac{1}{2}p_2 h), \\ p_4 = f(t_i + h, y_i + p_3 h). \end{cases} \tag{3}$$

3. SIR Hybrid Numerical Model Based on Runge-Kutta

Using the SIR model equation, the Runge-Kutta method is used in this study to numerically simulate the spread of polio and predict the development trend of infectious diseases. The SIR numerical solution using RK4 method is in good agreement with the results obtained using other simulation tools (such as Matlab Simulink) (Kousar et al., 2016). Results obtained by this model are robust with less computation error. The integration of RK 4 method accelerate the convergence rate during the training process. The calculated values for the parameters in RK 4, increased the efficiency of the method.

The numerical solution of the SIR model is obtained using the RK4 method for three groups over time. That is, the $S(i+1), I(i+1)$ and $R(i+1)$ in the demographics-based SIR model without vaccination and the demographics-based SIR model with vaccination respectively for each iteration i.

$$\begin{cases} S(i+1) = S(i) + \frac{h}{6}(p_1 + 2p_2 + 2p_3 + p_4) \\ I(i+1) = I(i) + \frac{h}{6}(q_1 + 2q_2 + 2q_3 + q_4) \\ R(i+1) = R(i) + \frac{h}{6}(r_1 + 2r_2 + 2r_3 + r_4) \end{cases} \tag{4.1}$$

$$\begin{cases} p_1 = f_s(t_i, I_i, S_i) \\ q_1 = f_I(t_i, I_i, S_i) \\ r_1 = f_R(t_i, I_i, R_i) \end{cases} \tag{4.2}$$

$$\begin{cases} p_2 = f_s(t_i + \frac{h}{2}, I_i + \frac{hq_1}{2}, S_i + \frac{hp_1}{2}) \\ q_2 = f_I(t_i + \frac{h}{2}, I_i + \frac{hq_1}{2}, S_i + \frac{hp_1}{2}) \\ r_2 = f_R(t_i + \frac{h}{2}, I_i + \frac{hq_1}{2}, R_i + \frac{hr_1}{2}) \end{cases} \tag{4.3}$$

$$\begin{cases} p_3 = f_s(t_i + \frac{h}{2}, I_i + \frac{hq_2}{2}, S_i + \frac{hp_2}{2}) \\ q_3 = f_I(t_i + \frac{h}{2}, I_i + \frac{hq_2}{2}, S_i + \frac{hp_2}{2}) \\ r_3 = f_R(t_i + \frac{h}{2}, I_i + \frac{hq_2}{2}, R_i + \frac{hr_2}{2}) \end{cases} \tag{4.4}$$

$$\begin{cases} p_4 = f_s(t_i + h, I_i + hq_3, S_i + hp_3) \\ q_4 = f_I(t_i + h, I_i + hq_3, S_i + hp_3) \\ r_4 = f_R(t_i + h, I_i + hq_3, R_i + hr_3) \end{cases} \tag{4.5}$$

where h is the step size, and

$$\begin{cases} f_s(t, I, S) = \frac{d(S)}{dt} = M - \frac{BSI}{N} - MS \\ f_I(t, I, S) = \frac{d(I)}{dt} = \frac{BSI}{N} - \Gamma I - MI \\ f_R(t, I, R) = \frac{d(R)}{dt} = \Gamma I - MR \end{cases} \tag{4.6}$$

for the demographics-based SIR model without vaccination and

$$\begin{cases} f_s(t, I, S) = \frac{d(S)}{dt} = M(1 - V) - \frac{BSI}{N} - MS \\ f_I(t, I, S) = \frac{d(I)}{dt} = \frac{BSI}{N} - \Gamma I - MI \\ f_R(t, I, R) = \frac{d(R)}{dt} = MV + \Gamma I - MR \end{cases} \tag{4.7}$$

for the demographics-based SIR model with vaccination.

4. Simulation and Analysis

4.1. *Data selection and processing*

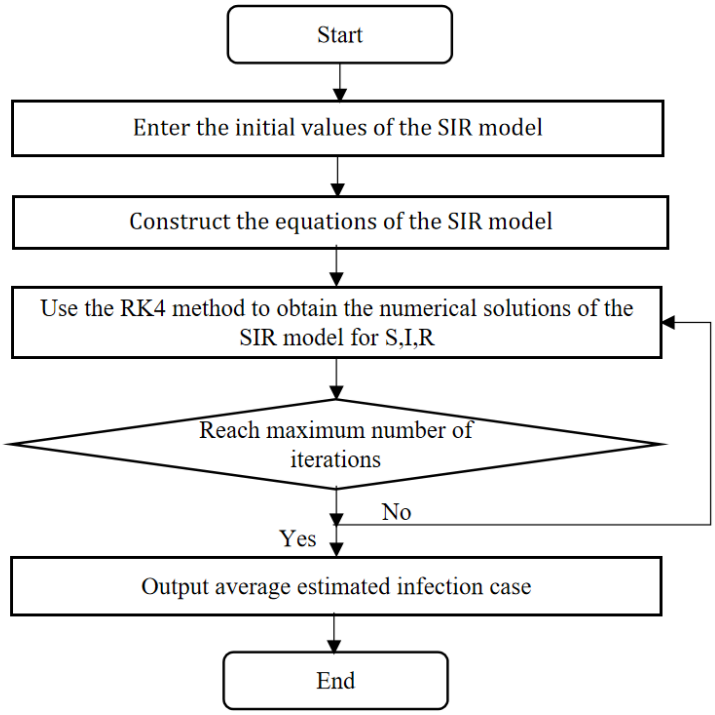


Figure 2: Flow chart of hybrid model of numerical SIR model using Runge-Kutta

MATLAB R12 software was used in Windows 7 with a 3.40 GHz processor and 16GB RAM to conduct simulations to evaluate the impact of vaccine on polio transmission in this section.

This experiment simulates the transmission of poliomyelitis within 30 days. There are two demographics-based SIR model: non-vaccination and vaccination. These two models are solved by RK4 method, using worldwide data on polio disease in 2021 (Saloni *et al.* 2022), $S(1) = 1873,\ I(1) = 649,\ R(1) = 197,\ \beta = 0.14286, M = 0.02.$

In the demographics-based SIR model without vaccination, $\Gamma = 0.16667$. In the demographics-based SIR model with vaccination, $\Gamma = 0.25,\ V = 0.95$. It should be noted that, the number of susceptible individuals in the year 2021 was taken from the number of diseased individuals in the year 2020. Here we are assuming that the number of people exposed in year 2021 equals the number of infected from 2020 but of the 1873 exposed, only 649 was infected.

4.2. *Simulation results and analysis*

4.2.1. *Hybrid model of numerical sir for polio transmission without and with vaccination over 30 days*

As shown in Table 1, we can obtain the following results.

Table 1: Simulation results of SIR model for polio transmission over 30 days

Days	Without Vaccinated			With Vaccinated		
	S	I	R	S	I	R
1	1873.00	649.00	197.00	1873.00	649.00	197.00
2.0357	1773.80	590.68	298.81	1776.20	541.86	345.17
3.0714	1685.10	534.86	388.78	1693.60	450.21	464.88
4.1071	1605.50	482.10	467.62	1622.30	372.51	560.47
5.1429	1534.00	432.77	536.12	1560.00	307.1	635.8
25.857	851.40	30.64	772.23	951.19	4.2355	698.84
26.893	832.67	26.44	761.26	931.50	3.3706	685.5
27.929	814.53	22.79	749.85	912.26	2.6794	672.24
28.964	796.94	19.66	738.10	893.44	2.1278	659.09
30	779.84	16.88	726.08	875.03	1.6881	646.08
Average estimated infection case $\boldsymbol{I}$		193			122	
Reproduction number $\boldsymbol{R_0}$		0.77			0.03	

One obvious observation is the reduction in predicted cases following the introduction of vaccines, which illustrates the effectiveness of vaccination in preventing the spread of Poliomyelitis. This trend can be expected from the data we have used.

From Fig. 3(a) and 3(b), we see that there is not much difference between the evolution of the number of susceptible individuals for both cases. What is more noteworthy is the pattern of change in both the proportion of infected and the proportion of recovered. With vaccines, the corresponding graphs show a sharper rate of change in particular, rate of decline, hence ending up with low values at the end of 30 days.

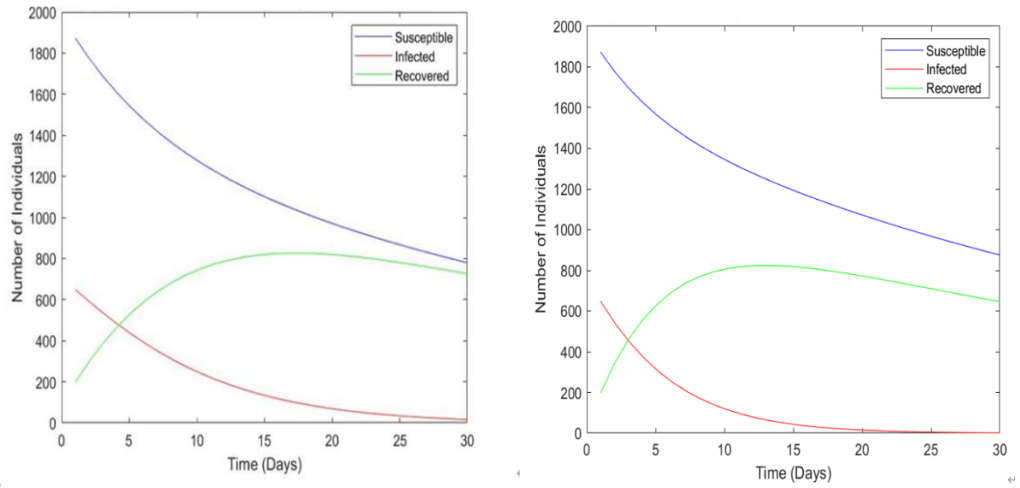


Figure 3: (a) Demographics-based SIR model without vaccination; (b) Demographics-based SIR model with vaccination

Poliomyelitis recovery rates without vaccines are 0.16667, while recovery rates with vaccines are 0.25. This indicates that more people are recovering from Poliomyelitis infection in a given period of time.

Because $\boldsymbol{R_0} = 0.77 < 0$, $\boldsymbol{R_V} = 0.03 < 0$, both SIR models predict that the disease will be eradicated soon. But note that significantly large difference between with and without vaccines. Considering that the input data in the demographics-based SIR model without vaccination, is just an estimate as of current, polio vaccination is widely mandated throughout the world, the actual numbers without vaccines are expected to be higher, and thus $\boldsymbol{R_0}$ would be expected to exceed 1. In light of this, the difference in both cases would in reality be larger, highlighting the effectiveness of vaccines.

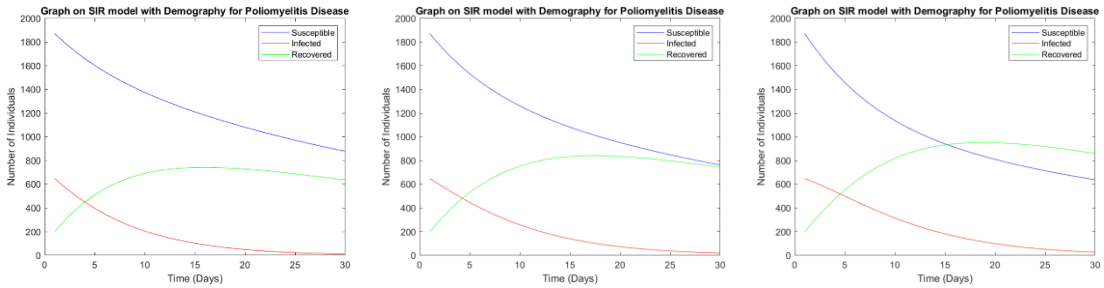
Finally, the average estimated infection cases between the two SIR models are compared, 193 and 122. The average estimated infection cases have been decreased by 36.79%.

4.2.2. *The performance of vaccination on the polio within 30 days under different infection rates*

In recent years, when trying to prevent and control COVID-19, people have found that measures such as vaccination, social distance, wearing masks, isolation, and so on can effectively reduce the infection rate, thereby reducing the spread of infectious diseases. Therefore, in the following section, we will examine the impact and performance of vaccination on polio within 30 days at different infection rates.

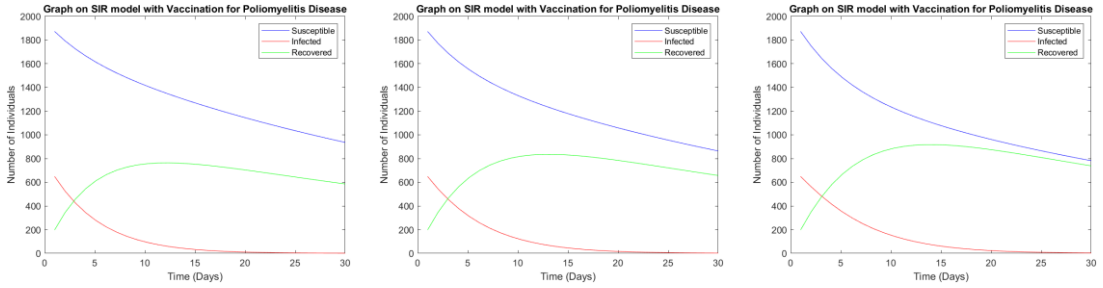
Table 2: Performance of vaccination on polio transmission under different infection rates

	Without Vaccinated			Vaccinated		
Infection rate β	0.10	0.15	0.20	0.10	0.15	0.20
Average estimated infection case I	168	197	229	110	124	140
Reproduction number $\boldsymbol{R_0}$	0.54	0.80	1.07	0.02	0.03	0.04
Vaccine Performance（%）				34.52	37.06	38.86



(a) $\beta = 0.10, I$=168; (b) $\beta = 0.15, I$=197; (c) $\beta = 0.20, I$=229.

Figure 4: The impact of differing transmission rates on infection cases without vaccination



(a) $\beta = 0.10, I$=110; (b) $\beta = 0.15, I$=124; (c) $\beta = 0.20, I$=140.

Figure 5: The effect of different rate of transmission on the infection cases with vaccination

The Table 2, Fig. 4 and Fig. 5 illustrate that the average estimated infection cases of the vaccinated were smaller than those of the unvaccinated at different infection rates.

In the absence of vaccination, when the infection rate is 0.10 and 0.15, the reproduction number is 0.54 and 0.80, respectively, and the disease will gradually disappear. But when the infection rate is 0.20, its reproduction number is 1.07, and the disease will become endemic. This verifies that in the absence of vaccination, the size of the infection rate can affect the size of the reproduction number of Polio. Conversely, if people were vaccinated, the reproduction number would be much less than 1, regardless of the infection rate β of 0.10, 0.15, and 0.20. This suggests that the reproductive numbers of polio are relatively small in the presence of vaccination. As infection rates increased, so did the performance of the vaccine. Regardless of how different the infection rates were, the number of infected people fell by about 35%.

5. Conclusions

Using global polio data for 2021, this paper simulates the 30-day transmission process of polio using the SIR models. As a result of the study, the vaccinated model had better performance, such as fewer estimated infections on average, than the unvaccinated model. In addition, as long as people are vaccinated, the reproduction number is much lower than 1.

At this stage, we can ensure that if there are local cases of polio infection in areas where people are vaccinated, the disease will not become endemic. The spread of polio will slowly decline over time even without additional measures to control infection rates (exposure rates). And if there is a localized polio infection in an unvaccinated area, considering that in the absence of vaccination, the infection rate will play a decisive role in the spread of the disease.

Therefore, after the initial detection of cases, we need to implement effective strategies (i.e., increased social distance, isolation, etc.) to reduce the infection rate, thereby reducing the reproduction number of the disease, in addition to the routine polio vaccine emergency vaccination program. In spite of extreme conditions such as shortages of vaccines, people can still take various measures to reduce the infection rate to $\boldsymbol{R}_0$<1, which prevents the disease from spreading endemically. As a result, the disease gradually decreases and disappears. Of course, this will require more time and resources.

Due to the current lack of a cure for polio, and the fact that vaccines can prevent the spread of the disease, it is imperative that we encourage each other to get vaccinated, especially in high-risk groups such as children under the age of five. The World Health Organization (WHO) recommends that all children receive the polio vaccine. Vaccination is currently the most effective and cheapest means of preventing and treating many diseases. Despite the challenges and difficulties encountered during the COVID-19 pandemic in recent years, vaccination should be implemented with resolute determination. In order to get people to support vaccination, health departments or relevant agencies should also play a more active role in promoting vaccine knowledge and vaccination efforts. It is our hope that with the continuous improvement of medical services in the future, more diseases will be prevented and controlled through vaccination.

Acknowledgments

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