

Estimation of Stress Concentration Factor for a plate with hole under Axial Tension Loading

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ABSTRACT

The purpose of this study is to investigate the effects of hole/holes on stress concentrations in a plate using a current industry standard software - Finite Element Analysis (FEA). Plate with hole/holes is a common engineering application such as in automobile, marine, aerospace and mechanical structures. Hole/Holes can be seen in many thin-walled mechanical and automobile structures and components. For example, hole/holes are found in residential/commercial buildings' steel structural studs to allow plumbing installation, web or flange of steel box girders in bridges is furnished with holes to ease inspection works, electrical and heating conduits in the walls or ceilings and ribs attached to the main spar of an aeroplane's wing are frequently come with holes. These hole/holes are one type of discontinuities within the structure (e.g. thin plate) that leads to changes in elastic stiffness and may tend to failures. Proper knowledge of stresses, strains, deflection and stress concentration (SC) are required to design any structures. In the geometry of the plate, under different loading, stress concentration rises from any abrupt change. And due to this, throughout the cross-section, the uniform stress distribution does not occur. At a point of stress concentration, as a results failures like fatigue cracking and plastic deformation commonly occur. Hence, it is very important in any engineering structural design to know about the stress concentration on plates with holes. In the present study stress concentration in a plate with a circular central hole and offset hole subjected to uniaxial loading (axial tension) is calculated using Finite Element of Analysis (FEA). Equations of SCF given by Peterson are dependent on three parameters namely, hole radius (a), distance from the bottom edge to the centre of hole (c) and distance from top edge to centre of hole (e). By varying either ' c ' or ' e ' or the ratio of ' e/c ', an effort is made to study stress concentration factors for the determination of the edge effect by varying the hole to edge distance. The results show that von Mises stresses increased with increasing ' e/c ' or ' a/c ', which in turn increases SCF and finally leads to failure.

Keywords: Axial tension; finite element analysis (FEA); stress concentration (SC); stress concentration factor (SCF); equivalent (von Mises) stress, edge effect

NOTATIONS

K_t	=	Stress concentration factor
K_{tg}	=	Theoretical stress concentration factor
σ_{max}	=	Maximum stress (N/mm ²)
σ_o	=	Nominal stress (N/mm ²)
L	=	Length of the plate (mm)
w	=	Width of the Plate (mm)
t	=	Thickness of the Plate (mm)
d	=	Diameter of the hole across the plate
P	=	Axial tensile load (N)
e	=	Distance from top of the plate to centre of the hole
c	=	Distance from the bottom of the plate to the centre of the hole

ABBREVIATIONS

SC	=	Stress Concentration
SCF	=	Stress Concentration Factor
TSCF	=	Theoretical stress concentration factor
FEA	=	Finite Element Analysis

INTRODUCTION

Machine components are usually provided with geometric irregularities such as keyways, notches, fillets, holes, threads etc. These irregularities are also known as geometrical discontinuities (Norton R. L. 2006). Due to these geometric irregularities or discontinuities in machine structures, the stress distribution around the irregularities is concentrated and failure from the fatigue-cracking happens. Likewise, in so many cases, pre-existing geometrical irregularities are also

major causes of the failures. Due to functional requirements of different geometrical discontinuities on different machine parts (i.e. on plates) are unavoidable. For example, steps are provided to couple the bearings and pulleys, holes for making riveted, bolted and pin joints and a keyway is required to align the shaft. These geometric discontinuities will be the source of significant stress concentration (SC). The most common application of plates with central holes and offset holes are riveted joints, bolted joints, pin joints etc. which are in turn used in pressure vessel application. Due to these irregularities on a plate, stress is concentrated in this localised vicinity of plates. This concentrated stress is measured by a factor which is known as stress concentration factor (SCF). This SCF leads to the failure of the plates. Mathematically SCF is expressed as a ratio of maximum stress to nominal stress (Pilkey W. D. 1997).

$$K_t = \frac{\sigma_{max}}{\sigma_0} \tag{1}$$

Figure 1 shows a plate with a hole under axial tension loading. Significant stress concentrations are caused by these geometric discontinuities. Therefore, the assessment of

stress, strain and deflections at geometries is very important in the designing of components or structures with stress raisers against different types of failure modes (Shigley J. E and Michke C. R. 2008).

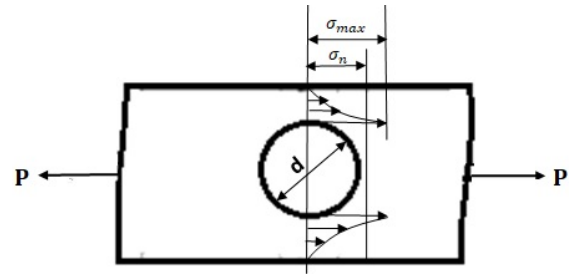


FIGURE 1. Geometry of circular hole (Pilkey W D 1997)

Figure 2 shows curves of stress concentration factors for the tension of a finite-width element having an eccentrically located circular hole (Pilkey W.D. 1997). Plates with hole/holes are the most vital component used in almost every mechanical system and machine.

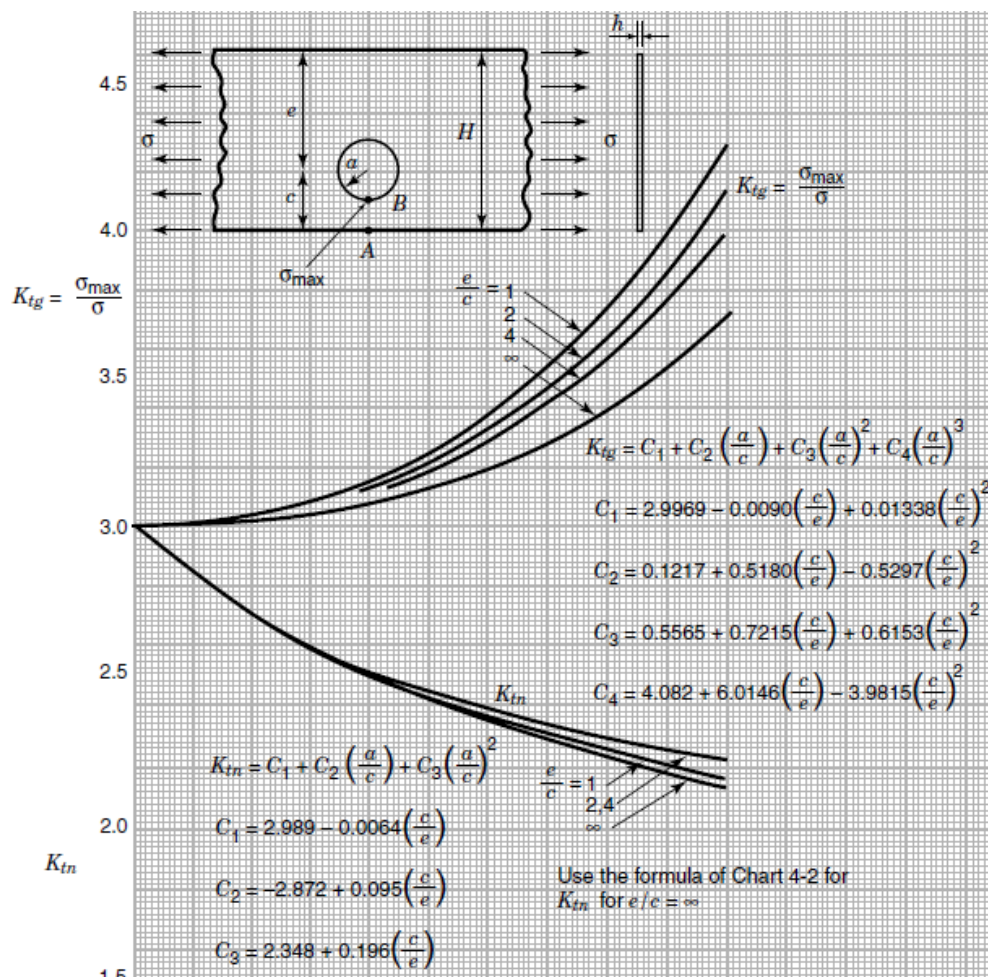


FIGURE 2. Stress concentration factors for the tension of a finite-width element having an eccentrically located circular hole (From Peterson's Handbook) (Pilkey W D, 1997)

Researchers have contributed a lot to understanding the stress analysis, optimization and failure analysis of a plate with any discontinuities on it. Ozkan et al. 2019, used five different methods namely, empirical, regression, ANN, FEA and analytical to determine the stress concentration factor (K_t) in a rectangular plate with a hole under tensile stress. In this study, the empirical (Peterson's) stress concentration factor K_t was compared with the results of regression analysis (REGA), finite element analysis (FEA), analytical model and the artificial neural network (ANN) model and they found the best results using the ANN model. S.G. Sarganachari et al. 2019, used stress reduction techniques such as an experimental, analytical and numerical method to assess stress intensity in a uniaxially loaded plate with a hole. E Devaraj et al. 2019, do numerical and analytical analysis for a rectangular plate with holes as discontinuities under static load to investigate the SCF. They determined the von Mises stress for a rectangular plate with a hole of a different radius by analytical and numerical methods. They used Solidwork for modelling different geometries and ANSYS for analysis. Sohanur Rahman 2018, used FEM to do a stress analysis of a finite steel plate with a rectangular hole under uniaxial stress. The author investigated the effect of von-Mises stress on various types of reinforcements and from this analysis they observed that a single doubler plate gives the most suitable reinforced model. Also, the author performed the convergence test for the most suitable reinforced model and observe the effect of variation in reinforcement. Olesya Maksymovych et al. 2017, used the integral equation method for stress calculation and optimization in composite plates with holes. They developed a numerical method for solving integral equations by the mechanical quadrature method for systems of holes. For a large number of holes with controlled accuracy, they calculate the stress. Soni Kumari et al. 2017, does a stress analysis of an infinite plate with circular holes. The authors studied stress concentration in an infinite isotropic plate around a circular hole subjected to transverse, longitudinal and biaxial loading using an analytical approach. They used complex variable and bipolar coordinate methods for calculating stress concentration around two holes. Kolachala et al. 2019, carried out a stress concentration analysis of rectangular plates with an elliptical hole as a discontinuity by using composite (silicon carbide). They observed that when an elliptical hole is introduced in a rectangular composite plate, discontinuity arises which leads to increased stress at the edge of the hole which will lead to failure of the plate. FEA has been used to analyse and determine the optimal size and position of mitigation curves. The results are obtained with the help of tables, graphs and Figures which are done theoretically, experimentally and also with ANSYS.

The methodology for this research work consisted of four phases:

1. Calculation of theoretical stress concentration factor (TSCF) as per Peterson handbook.
2. Software simulations for determination of equivalent von Mises stress in a plate with a circular central hole and offset hole.
The steps required to carry out the simulations are as follows:
 - a. Prepare geometries as per the size of the plate with a hole in Solidworks 16.0.
 - b. Import solid model in ANSYS software to perform FEA.
 - c. Do meshing of the imported model/geometries and refine the meshing process by conducting a convergence test.
 - d. Define load (magnitude and direction) and the supports that give the static equilibrium.
 - e. Perform the simulation and obtain the stress distribution (von Mises stress) present on the plate.
 - f. Calculate the SCF from the von Mises stresses.
3. Tabulate the results obtained from calculations of TSCF and SCF from FEA simulation.
4. Validation of TSCF and simulation results obtained by the use of ANSYS software.

PROCEDURE TO DETERMINE OF TSCF

From Figure 2, parameters are identified which are useful for calculating stress concentration factors. These parameters are hole radius (a), distance from the bottom edge to the centre of hole (c) and distance from top edge to centre of hole (e). In the Peterson handbook, equations to determine the theoretical stress concentration factor (TSCF) for a plate with a hole were given. Those equations are used to determine the TSCF (Pilkey W D 1997). These equations are:

$$K_{tg} = C_1 + C_2 \left(\frac{a}{c}\right) + C_3 \left(\frac{a}{c}\right)^2 + C_4 \left(\frac{a}{c}\right)^3 \quad (2)$$

$$\left. \begin{aligned} C_1 &= 2.9969 - 0.0090 \left(\frac{c}{e}\right) + 0.01338 \left(\frac{c}{e}\right)^2 \\ C_2 &= 0.1217 + 0.5180 \left(\frac{c}{e}\right) - 0.5297 \left(\frac{c}{e}\right)^2 \\ C_3 &= 0.5565 + 0.7215 \left(\frac{c}{e}\right) + 0.6153 \left(\frac{c}{e}\right)^2 \\ C_4 &= 4.082 + 6.0146 \left(\frac{c}{e}\right) - 3.9815 \left(\frac{c}{e}\right)^2 \end{aligned} \right\}$$

By varying the center of the hole to edge distance (either ' c ' or ' e '), the attempt is made to study TSCF, to determine the edge effect on a plate with a circular hole.

PROCEDURE TO CONDUCT FEA

Computational analysis is carried out for a plate with circular holes subjected to axial tension using an approach of Finite Element Analysis (FEA). For FEA a well-known and widely

used industrial software ANSYS is used. Eighteen different geometries of 3D models of a plate with holes are modelled in Solid works 16.0. EN31 material was modelled as a linear elastic material in ANSYS. Table 1 shows the material properties of EN31 (ANSYS Workbench 19.0).

TABLE 1. Material properties of EN31

Property	Value	Unit
Density	7600	Kg/m ³
Isotropic properties		
Young's modulus	207	MPa
Poisson's ratio	0.3	--
Bulk modulus	172.5	GPa
Shear modulus	80	GPa
Tensile Yield strength	460	MPa
Tensile Ultimate strength	560	MPa

The plate with a hole has meshed with rectangle elements. The fine mesh is selected to get more accurate results, which in turn results in 1186 nodes and 148 elements. Solid Works model and ANSYS meshed model of a plate with a centre hole with dimensions of $L = 120$ mm, $B = 40$ mm, $t = 6$ mm and hole diameter of 6 mm is shown in Figure 3.

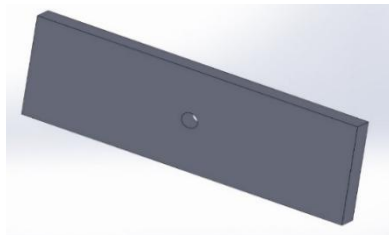


FIGURE 3. Solid Works model

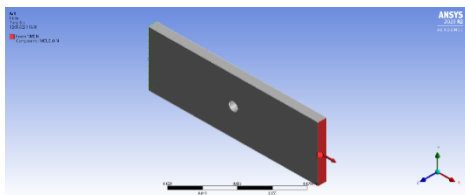


FIGURE 4. Loading and boundary conditions

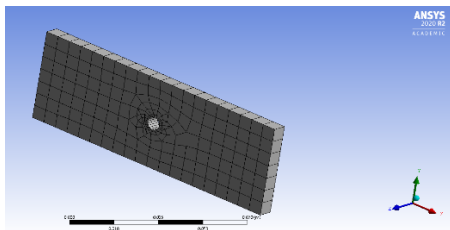


FIGURE 5. Meshed model of a plate with a centre hole

As shown in Figure 4, for axial tension loading, the load was applied to one face of the model and the other face was taken as fixed support. Figure 5 shows the meshed ANSYS model. For this problem, Figure 6 shows a finite element (FE) mesh sensitivity analysis. The fine mesh had more than twenty times more nodes and elements than the coarse mesh. Initially, coarse mesh results show 1186 nodes and 148 elements in mesh analysis. Up to 17134 elements and 26829 nodes were used in the study.

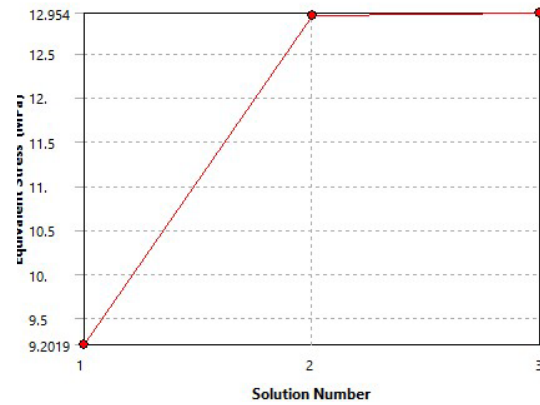


FIGURE 6. Results convergence study case-1

Table 2 shows the convergence test results for the rectangle plate with a circular hole at the centre.

TABLE 2. Results of convergence test for case-1

	Equivalent Stress (MPa)	Change (%)	Nodes	Elements
1	9.2019		1186	148
2	12.93	33.688	8663	5061
3	12.954	0.18438	26829	17193

Figure 7 shows the results of FE mesh, if we further refined the mesh, the straight horizontal line form is no longer and the uneven shape is formed.

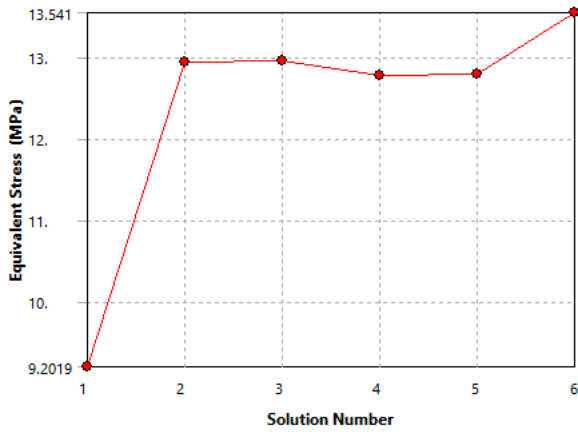


FIGURE 7. Results convergence study case-2

TABLE 3. Results of convergence test for case-2

	Equivalent Stress (MPa)	Change (%)	Nodes	Elements
1	9.2019		1186	148
2	12.93	33.688	8663	5061
3	12.954	0.18438	26829	17193
4	12.775	-1.3864	58251	38954
5	12.782	5.7102e-002	178040	124688
6	13.541	5.7624	573785	412089

The results of the finite element analysis using 3D geometries are presented. A total of eighteen different models are prepared and analyzed. Figures 8 to 25 show various FEA results for all models under axial tension loading (ANSYS Workbench 19.0). Here for all the variations of geometries load value is fixed as 1000 N. This value of the load is selected based on the feasible load range applied through an experimental setup.

Table 3 shows the convergence test results for the rectangle plate with a circular hole at the centre for case 2.

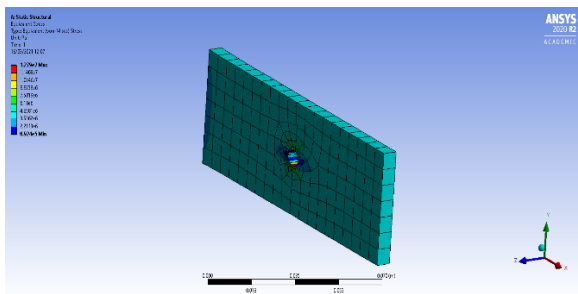


FIGURE 8. Equivalent (von Mises) stress for $e/c = 1$ ($e = 20$ mm and $c = 20$ mm)

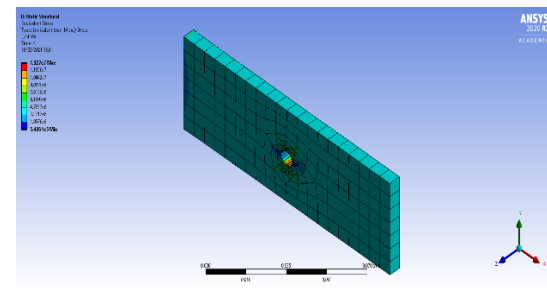


FIGURE 9. Equivalent (von Mises) stress for $e/c = 1.11$ ($e = 21$ mm and $c = 19$ mm)

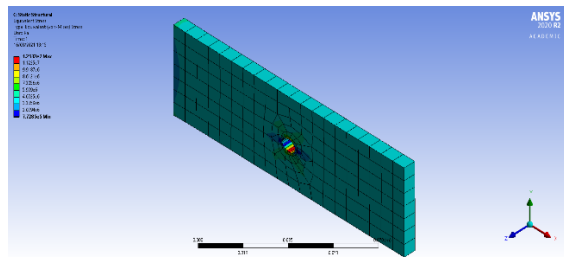


FIGURE 10. Equivalent (von Mises) stress for $e/c = 1.22$ ($e = 22$ mm and $c = 18$ mm)

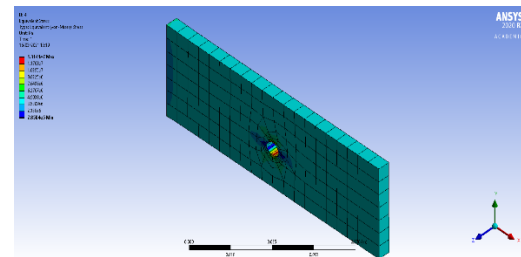


FIGURE 11. Equivalent (von Mises) stress for $e/c = 1.22$ ($e = 23$ mm and $c = 17$ mm)

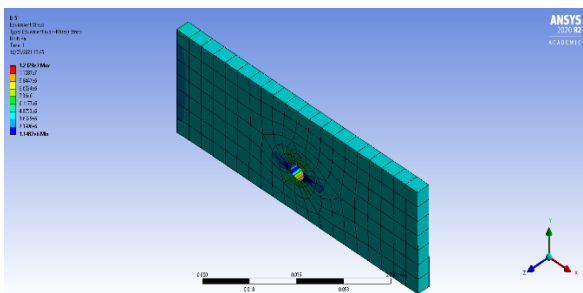


FIGURE 12. Equivalent (von Mises) stress for $e/c = 1.35$ ($e = 24$ mm and $c = 16$ mm)

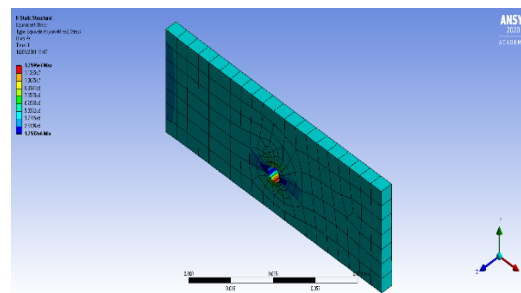


FIGURE 13. Equivalent (von Mises) stress for $e/c = 1.50$ ($e = 25$ mm and $c = 15$ mm)

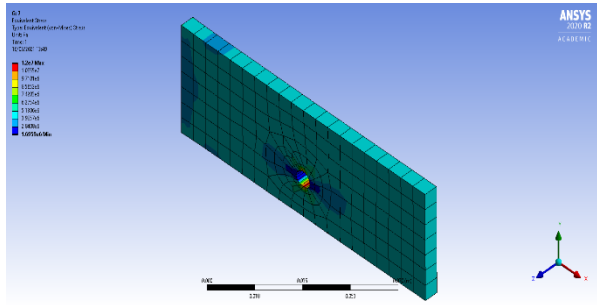


FIGURE 14. Equivalent (von Mises) stress for $e/c = 1.86$ ($e = 26$ mm and $c = 14$ mm)

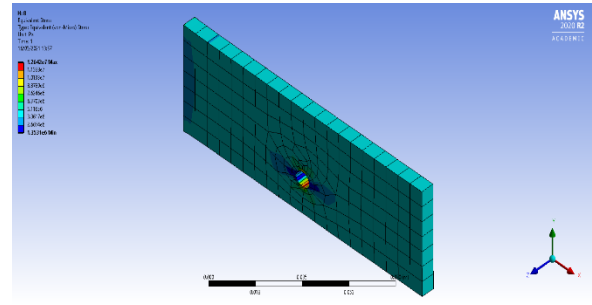


FIGURE 15. Equivalent (von Mises) stress for $e/c = 2.08$ ($e = 27$ mm and $c = 13$ mm)

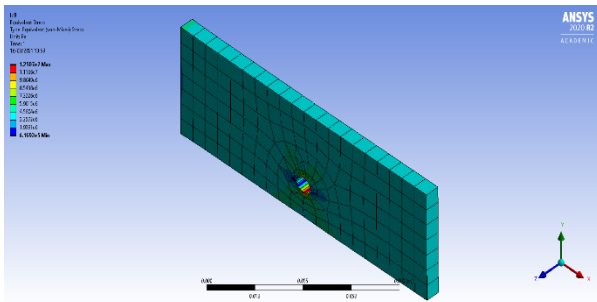


FIGURE 16. Equivalent (von Mises) stress for $e/c = 2.33$ ($e = 28$ mm and $c = 12$ mm)

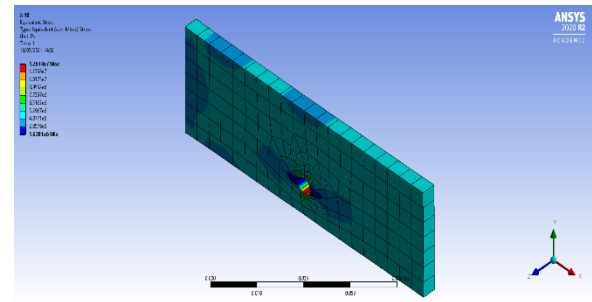


FIGURE 17. Equivalent (von Mises) stress for $e/c = 2.64$ ($e = 29$ mm and $c = 11$ mm)

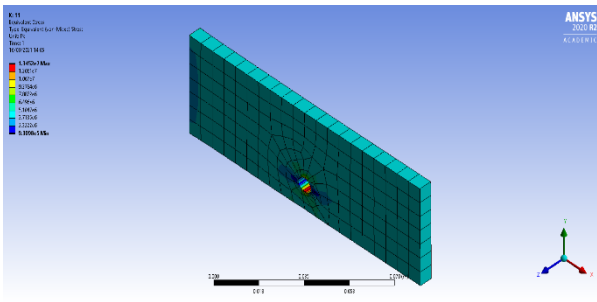


FIGURE 18. Equivalent (von Mises) stress for $e/c = 3.0$ ($e = 30$ mm and $c = 10$ mm)

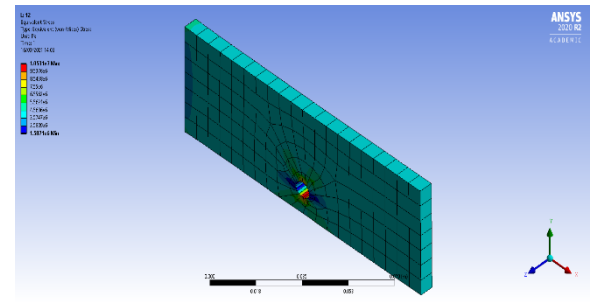


FIGURE 19. Equivalent (von Mises) stress for $e/c = 3.44$ ($e = 31$ mm and $c = 9$ mm)

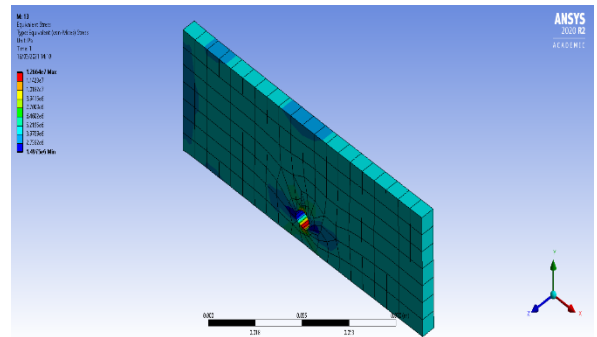


FIGURE 20. Equivalent (von Mises) stress for $e/c = 4.0$ ($e = 32$ mm and $c = 8$ mm)

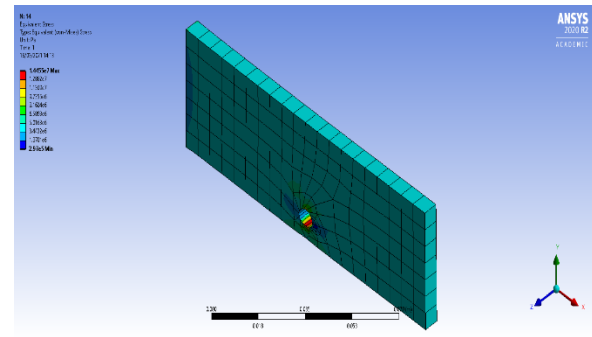


FIGURE 21. Equivalent (von Mises) stress for $e/c = 4.71$ ($e = 33$ mm and $c = 7$ mm)

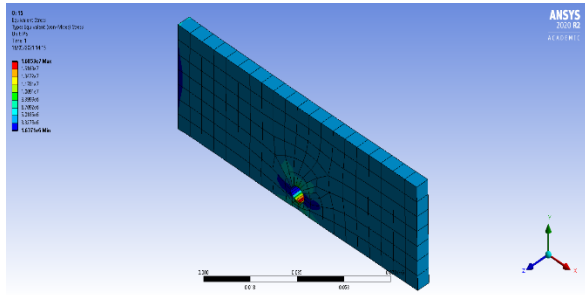


FIGURE 22. Equivalent (von Mises) stress for $e/c = 5.67$ ($e = 34$ mm and $c = 6$ mm)

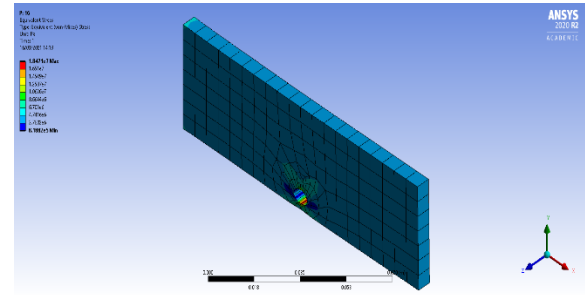


FIGURE 23. Equivalent (von Mises) stress for $e/c = 7$ ($e = 35$ mm and $c = 5$ mm)

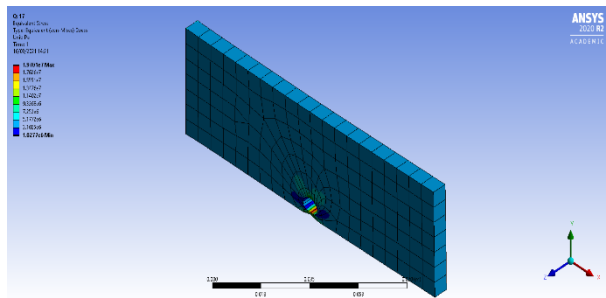


FIGURE 24. Equivalent (von Mises) stress for $e/c = 9$ ($e = 36$ mm and $c = 4$ mm)

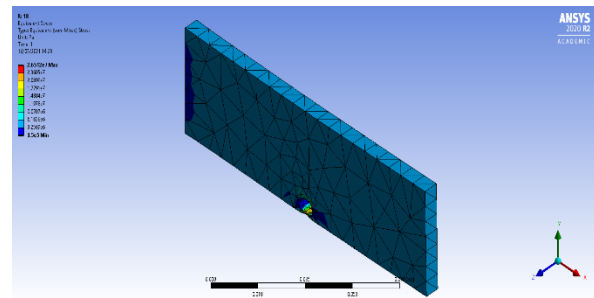


FIGURE 25. Equivalent (von Mises) stress for $e/c = 12.33$ ($e = 37$ mm and $c = 3$ mm)

From the above FEA results, equivalent von Mises stress of plate with a hole is identified and used for calculation of stress concentration factor (SCF). Equivalent

von Mises stresses based on FEA results and TSCF is presented in Table 4.

TABLE 4. SCF obtained from FEA under axial tension loading for different geometries

L (mm)	B (mm)	d (mm)	e (mm)	c (mm)	a/c	c/e	e/c	Load P (N)	Area A (mm ²)	Nominal stress (MPa)	Equivalent (von Mises) stress (MPa)	FEA based SCF (K_t)	TSCF (K_{tg})
120	40	6	20	20	0.15	1	1	1000	204	5	12.79	2.609	3.081
120	40	6	21	19	0.16	0.90	1.11	1000	204	5	13.27	2.707	3.095
120	40	6	22	18	0.17	0.82	1.22	1000	204	5	12.53	2.556	3.107
120	40	6	23	17	0.18	0.74	1.35	1000	204	5	13.14	2.681	3.120
120	40	6	24	16	0.19	0.67	1.50	1000	204	5	12.32	2.513	3.132
120	40	6	25	15	0.20	0.60	1.67	1000	204	5	12.59	2.568	3.143
120	40	6	26	14	0.21	0.54	1.86	1000	204	5	12	2.448	3.155
120	40	6	27	13	0.23	0.48	2.08	1000	204	5	12.64	2.579	3.182
120	40	6	28	12	0.25	0.43	2.33	1000	204	5	12.5	2.550	3.211
120	40	6	29	11	0.27	0.38	2.64	1000	204	5	12.61	2.572	3.242
120	40	6	30	10	0.30	0.33	3.00	1000	204	5	13.45	2.744	3.296
120	40	6	31	9	0.33	0.29	3.44	1000	204	5	10.53	2.148	3.357
120	40	6	32	8	0.38	0.25	4.00	1000	204	5	12.66	2.583	3.483
120	40	6	33	7	0.43	0.21	4.71	1000	204	5	14.45	2.948	3.632
120	40	6	34	6	0.50	0.18	5.67	1000	204	5	16.85	3.437	3.901
120	40	6	35	5	0.60	0.14	7.00	1000	204	5	18.47	3.768	4.394
120	40	6	36	4	0.75	0.11	9.00	1000	204	5	19.7	4.019	5.468
120	40	6	37	3	1	0.08	12.3	1000	204	5	26.51	5.408	8.312

RESULTS AND DISCUSSION

In the present paper, a study on the stress concentration factor for a plate with a circular central hole and offset hole subjected to uniaxial loading (axial tension) is carried out. By varying both 'c' and 'e', eighteen different geometries are prepared and FEA simulation is carried out. Von Mises stresses from the FEA result are used to calculate the SCF. In

Table 4 SCF based on FEA and TSCF obtained by theoretical calculation is presented and it shows the comparison of TSCF and SCF obtained from the FEA simulation. From Table 4 and the graph in Figure 26, it is observed that both TSCF and SCF from FEA simulations are gradually increased with increasing 'e/c' and 'a/c' ratios. Also one can say that all results are nearer and have a good agreement between TSCF and SCF from FEA simulation.

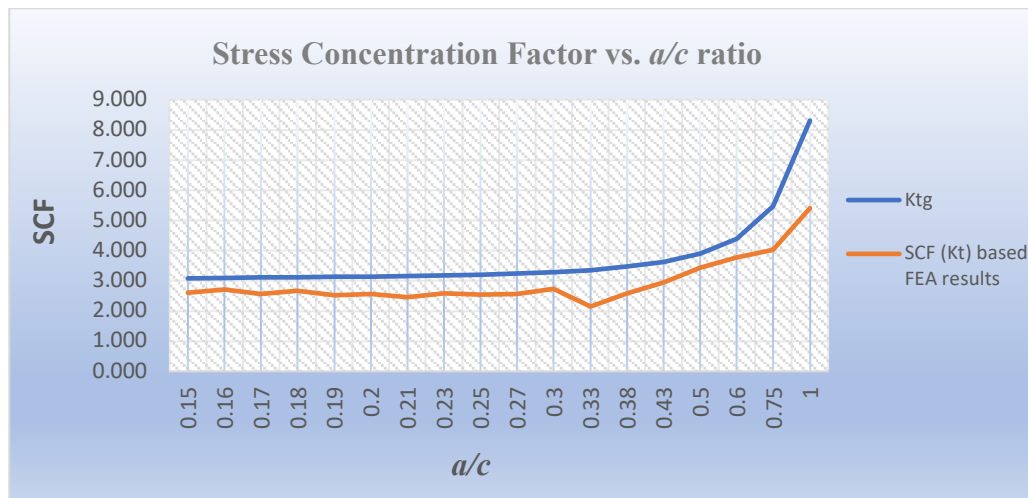


FIGURE 26. SCF vs. a/c

CONCLUSION

The presented work is concluded based on the comparison between the TSCF and SCF obtained from FEA simulation results for a plate with a central hole and offset hole under uniaxial tension loading. The finite element (FE) software (ANSYS) can be used as a tool to determine graphical SCFs for flat plates with a hole as discontinuity. The minimum value of TSCF is 3.081 and the maximum is 8.312 while in FEA simulation results minimum value of SCF is 2.148 and the maximum is 5.408. From Table 4 and Figure 26 it is concluded that von Mises stresses increased with increasing 'e/c' or 'a/c', which results in increasing SCF.

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DECLARATION OF COMPETING INTEREST

None

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