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ESTIMATING DOWN TIME OF GLOVE DIPPING MACHINES OPERATION USING EXPONENTIAL AND WEIBULL DISTRIBUTIONS

(*Menganggar Masa Kegagalan Operasi Mesin Sarung Tangan Menggunakan Taburan Eksponen dan Weibull*)

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ABSTRACT

Reliability of any particular component or system is very important as it involves the whole stage of the product life-cycle. It is necessary for a factory to ensure that down time is managed proactively to ensure efficient product manufacturing process. It is common for many manufacturing plants to conduct preventive and corrective maintenance because long machine repair time will cause loss of productivity and revenue. In order to minimize repair time, both inventory and workforce preparation are crucial. This paper uses statistical methods to determine proper time slots to conduct preventive maintenance of a machine. The proposed method determines Mean Time to Failure (MTTF) via Exponential and Weibull distributions using time to failure data of two sets of similar machines from glove manufacturing production line. Results revealed that Weibull distribution offered better MTTF prediction performance compared to Exponential distribution. By pairwise comparison, these two methods do not present significant difference, and hence, both methods could serve as the benchmark in designing potential preventive maintenance strategy.

Keywords: exponential; MTTF; reliability; Weibull

ABSTRAK

Kebolehpercayaan komponen atau sistem tertentu adalah amat penting kerana ia melibatkan keseluruhan kitaran hayat produk. Pihak kilang perlu memastikan masa henti diurus secara proaktif untuk memastikan proses pengeluaran yang efisyen. Kebanyakan kilang tidak asing daripada menjalankan proses penyelenggaraan pencegahan dan pemulihan kerana tempoh membaiki mesin yang masa bakal menjejaskan produktiviti dan pendapatan syarikat. Persediaan inventori dan tenaga kerja adalah penting untuk meminimumkan masa pembaikan. Artikel ini menggunakan kaedah statistik untuk menentukan slot masa yang sesuai untuk melaksanakan menyelenggaraan mesin. Kaedah dicadangkan mengira Min Masa Kegagalan (MTTF) menggunakan taburan Eksponen dan Weibull mengguna pakai data masa kegagalan dua set mesin serupa daripada sebuah syarikat pembuat sarung tangan. Dapatan menunjukkan taburan Weibull memberikan ramalan MTTF yang lebih baik berbanding taburan Eksponen. Secara perbandingan berpasangan, kedua-dua kaedah ini tidak menunjukkan perbezaan ketara, dan kedua-dua kaedah boleh menjadi petanda aras dalam mereka bentuk strategi penyelenggaraan.

Kata kunci: eksponen; MTTF; kebolehpercayaan; Weibull

1. Introduction

The operation of technically complex objects poses numerous challenges and obstacles. Engineers and operators usually face difficulties on how to maintain an acceptable level of reliability, particularly regarding availability and readiness. This issue can cause significant distraction among those involved in the management and operation of production lines. The life cycle phase of every technical operation varies, and it could occupy up to 95% of the equipment's life (Żyluk *et al.* 2023).

Production process is one of the top priorities in manufacturing companies, and many opt for the adoption of a maintenance policy to optimize this process. Development of effective maintenance policy can help such entities to prolong the life span of their machines and overall production efficiency (Zhong *et al.* 2023). For instance, several Malaysian manufacturing companies reported better performance after they adopted total productive maintenance approach (e.g., improved equipment effectiveness and better equipment life span) (Mad Lazim & Ramayah 2010). Crafting the right maintenance policy is important in a manufacturing environment. Once the machine downtime in serial lines exceeds a threshold cost of downtime incidents, the cost will increase linearly as the downtime increases (Liu *et al.* 2012).

With this importance of maintenance strategy, researchers have been focusing on statistical approaches to come out with the best model that represent appropriate maintenance policies in manufacturing industries. Failure time of operational systems is one of the useful metrics in reliability analysis. As such, Rifaai *et al.* (2022) proposed an integrated approach using logistic regression for estimating pipe failure prediction in water infrastructure systems. The probability of failure within certain time interval may help in predicting the timing to repair related machine/components. Meanwhile, Jeon and Sohn (2015) investigated failure pattern analysis for warranty purposes using Weibull regression and association rule analysis. Weibull regression identifies factors affecting variation in mean time between failures from the extracted rules, which contribute to quality improvement in manufacturing industry, especially on issues related to warranty. Zyluk *et al.* (2023) utilized Poisson and Weibull distributions to identify mean time to failure in the logistical operational support systems. The finding supports the importance of reliability analysis to maintain high availability of aircraft technical objects.

Overall, reliability analysis is crucial in ensuring safe and efficient production process. By choosing the appropriate operational strategy and combining it with proper maintenance practices, organizations can optimize their operations and minimize risk of equipment failure, which leads to lower expenditure and prevention of accidents (Gackowiec 2019). Hence, this paper attempts to estimate machines down time, focusing on glove dipping machine using two established Exponential and Weibull distributions. Weibull distribution is one of the extreme value distributions and the associated parameters are commonly used in failure analysis. The advantage of the scale and shape parameters in Weibull distribution can navigate the distribution characteristics of the particular component or system (Ikbal *et al.* 2022). Meanwhile, Exponential distribution can represent the lifetime of certain types of item in a simple statistical form (Iskandar 2018). Hence, both distributions were chosen as it widely used in assessing product reliability and life data analysis. Estimating down time can be defined as estimating the mean time to failure, a reliability metric which serves as maintenance strategy.

2. Research Methodology

The overall methodology includes data collection, Exponential and Weibull distributions definition, parameter estimation and mean time to failure prediction.

2.1. *Data collection*

Glove dipping machine is a machine that produces high quality glove using natural rubber plastic. Glove dipping, glove coloring, glove sanitizing, glove finishing and glove curing are the processes that need to be done by the said machine. The data used in this study were the machine's downtime (in minutes) from 1st January 2015 to 31st January 2015 (one month), collected 24 hours daily for two machines, denoted as Machine A and Machine B. These two machines have the same series system, start the operation at the same time and are producing the same product. Descriptive statistics of the machines' downtime are presented in Table 1.

Table 1: Descriptive statistics of the machine's downtime (in minutes)

Machine	Total downtime	Minimum	Maximum	Mean	Median	Standard deviation
Machine A	27	42	6321	1527	515	1942
Machine B	26	5	5220	1455	1405	1357

Summarizing the descriptive analyses, it can be concluded that Machine A is heavily skewed to the right as the value of mean is significantly greater than the value of median, indicating the tendency of longer operational times before it fails, compared to Machine B. During data collection period, both machines recorded quite a number of failures, with a minimum downtime of five minutes and maximum downtime of about four days.

2.2. *Exponential and Weibull distributions definition*

Exponential reliability function is normally used to define the useful life of an electronic and electro technical component or system. The probability density function of Exponential distribution can be defined as Eq. (1).

$$f(t) = \lambda e^{-\lambda t} \tag{1}$$

where λ is the failure rate and t =1, 2, 3, … is the time to failure.

Weibull distribution is one of the most important time dependent failure detection methods used in reliability engineering. The Weibull distribution allows for failure prediction probability at any level of stress providing information about the reliability of a material is given (Bütikofer *et al.* 2015). The probability density function of Weibull distribution can be defined as Eq. (2).

$$f(t) = \left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}} \tag{2}$$

where α is the scale rate, β is the shape rate and t is the time to failure. The shape rate indicates the failure pattern. Shape rate less than one represents infant mortality, rate of one defines random failure and rate more than one indicates wear-out failure (Galar & Kumar 2017). Both of these models are employed in this study as these approaches have been widely used in reliability analysis application.

2.3. *Parameter estimations*

The next step is to estimate the parameter. Method of Maximum Likelihood Estimation (MLE) will be used. The idea of MLE is to find the most probability values of distribution parameters for a set of data by maximizing the likelihood function in Eq. (3).

$$L = \prod_{i=1}^{n} f(t) = \prod_{i=1}^{n} \lambda e^{-\lambda t} \quad (3)$$

$$\ln L = \ln \prod_{i=1}^{n} \lambda e^{-\lambda t} = n \ln \lambda - \lambda \sum_{i=1}^{n} t_i \quad (4)$$

Based on the log-likelihood obtained in Eq. (4), the value of λ can be obtained by setting the associated derivative with respect to zero. Hence, the estimated parameter or also known as failure rate can be expressed as in Eq. (5).

$$\lambda = \frac{n}{\sum_{i=1}^{n} t_i} \quad (5)$$

where *n* is the total observations and *t* is the time to failure. Failure rate is the frequency of the system or component will fail to function and is expressed in failures per unit of time.

Similarly, the parameter estimation for Weibull distribution could be obtained. The associated parameters for Weibull distribution are known as scale rate and shape rate. By using MLE method, the shape rate and scale rate are defined as Eq. (6).

$$\alpha \frac{d}{d\alpha}\left(\frac{1}{\alpha^{\beta}} \sum_{i=1}^{n} t_n^{\beta}\right) = -n^{\beta}$$

$$\frac{n}{\beta} + \frac{d}{d\beta}\left(\ln t_n^{\beta-1}\right) - \frac{d}{d\beta}\left(\frac{1}{\alpha^{\beta}} \sum_{i=1}^{n} t_n^{\beta}\right) = n \ln \alpha \quad (6)$$

where *n* is the total observations and $t = 1, 2, 3, \dots$ is the time to failure.

2.4. *Mean time to failure predictions*

After failure rate have been estimated, the next step is to calculate Mean Time to Failure (MTTF), a reliability metric that estimates the average time that a system or a component will function correctly before it fails (Manglik & Ram 2019). MTTF is calculated by dividing the total time of operation by the number of failures during that time. This metric is often used for products with a limited lifespan, such as electronic devices or mechanical systems, to estimate how long they will last before requiring maintenance or replacement.

The MTTF of a component is given by the integral of time weighted with the probability density function, $f(t)$. In the case of Exponential distribution, we have

$$MTTF = \int_{0}^{\infty} t\ f(t)\, dt = \int_{0}^{\infty} t\ \lambda e^{=\lambda t}\ dt = \lambda \int_{0}^{\infty} t\ e^{=\lambda t}\ dt = \frac{1}{\lambda} \quad (7)$$

The integral part of Eq. (7) is solved using integration by part technique. This MTTF is the reciprocal of failure rate, λ. Hence, the final MTTF can be defined as the following Eq. (8).

$$MTTF = \frac{1}{\lambda} = \frac{1}{n / \sum_{i=1}^{n} t_i} = \frac{\sum_{i=1}^{n} t_i}{n} \tag{8}$$

For the case of Weibull distribution, we have

$$MTTF = \int_0^{\infty} t\ f(t)\, dt\ = \int_0^{\infty} t \left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}}\ dt\ = \frac{\beta}{\alpha^{\beta}} \int_0^{\infty} t^{\beta}\ e^{-\left(\frac{t}{\alpha}\right)^{\beta}}\ dt \tag{9}$$

From Eq. (8) and parameter estimation from the previous section, MTTF for Weibull distribution can be defined as the following Eq. (10).

$$MTTF = \alpha \Gamma\left(1 + \frac{1}{\beta}\right) \tag{10}$$

where $\Gamma(t) = \int_0^{\infty} e^{-t} t^{n-1} dt,\ \Gamma(t)$ is the gamma function and t is the time to failure.

2.5. *Model selection criterion*

To evaluate how well both Exponential and Weibull distributions fit the data, Akaike information criterion (AIC) and Bayesian information criterion (BIC) measures will be used (Ikbal *et al*. 2022). For relatively small sample, the AIC is given by Eq. (11) while the BIC is given by Eq. (12).

$$AIC = -2\log L(\theta) + 2k + \frac{(2k+1)}{(n-k-1)} \tag{11}$$

$$BIC = -2\log L(\theta) + k\log(n) \tag{12}$$

where $L(\theta)$ is the likelihood function of the data and k is the number of estimated parameter. The difference between these two measures is the greater penalty imposed for the number of parameters. These two measures were among the most often used for model selection.

3. Results and Discussion

The data was analyzed using Minitab software, and results of parameter estimations are shown in Table 2. For Exponential distribution, failure rate for both machines are 0.000655 and 0.000687, respectively. The failure rate of the first machine is lower than the second one, indicating smaller probability of the first machine will fail in a short period of time.

Table 2: The parameter estimations of each machine using Exponential and Weibull distributions

Machine	**Exponential Distribution**	**Weibull Distribution**	
	Failure Rate, λ	**Scale Rate,** α	**Shape Rate,** β
Machine A	6.55×10^{-4}	1257.40	0.740999
Machine B	6.87×10^{-4}	1337.09	0.821121

Meanwhile, Weibull distribution parameter provides the scale rate or expected life for both machines. There is 63.2% chance for the first and second machines to fail to operate by 1257 and 1337 minutes, respectively. The Weibull shape parameters for both machines indicate the machines have decreasing failure rate, which means that the failure rates are highest when the machines are first started and become lower as time goes on. This shape parameter could provide a clue to engineers or maintenance workers whether scheduled inspection or overhaul is required. For this case, since both of the shape parameters are less than one, overhauls are not cost effective.

Table 3: The MTTF of each machine

Machine	**Exponential Distribution**	**Weibull Distribution**
Machine A	1526.96	1512.247
Machine B	1455.19	1487.686

MTTF value (in minutes) for each machine is shown in Table 3. Higher MTTF indicates that the machine can operate without any failure for a long period of time. Based on Table 3, MTTF value for machine A is greater than machine B for both distributions. The first machine is estimated to be running smoothly about 25 hours before failing. On the contrary, the second machine is estimated to be running smoothly for about 24 days.

Table 4: Model selection criteria

Machine	**Exponential Distribution**		**Weibull Distribution**	
	AIC	**BIC**	**AIC**	**BIC**
Machine A	451.9959	451.3073	449.4433	448.0977
Machine B	432.8355	432.1254	433.4213	432.0339

Subsequently, to test the Exponential and Weibull distributions model fit, two model selection criteria were used, AIC and BIC, where the method with smaller values of AIC and BIC has better fit. It can be observed that the values of AIC and BIC of Weibull distribution are constantly smaller in almost cases, except for AIC in Machine B. These findings further justify the outstanding performance of Weibull distribution in reliability analysis (Méndez-González *et al*. 2017). Nevertheless, by pairwise comparison, the overall performance for both methods do not present significant difference – both distributions can be considered sufficient in estimating the MTTF of the glove dipping machines.

Notwithstanding, a different conclusion can be made if a company needs to design for preventive maintenance policy. The lowest MTTF values should be taken as a benchmark for this purpose because in order to design a warranty policy, for example, the warranty must be based on the probability and time period before a certain product fails to operate certain functions.

4. Conclusion

This article presents the analyses of MTTF reliability metrics for glove dipping machines using Exponential and Weibull distributions which is important from the viewpoint of minimizing losses due to inability of the machines to operate. Planning the optimum scheduled maintenance could be part of the long-term strategy. From the overall analyses, Weibull distribution offers more accuracy and better describes the glove dipping machine's performance and lifetime durability. This possibly due to the characteristics of most electronic systems or components that follow bathtub curve failure model which can be explained by shape parameter. Comparing the two machines, the first machine has higher reliability as it can operate much longer than the second machine. The findings could help to estimate total number of products that can be produced, and approximate the delivery process to the end-users. For future works, similar analyses could be done for other components using larger data. The use of advanced data collection and analysis techniques can also help to identify potential problems before they occur and help to maintain a high level of reliability.

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