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THERMAL BOUNDARY LAYER OVER A VERTICAL PLATE IN POROUS MEDIUM WITH A CONVECTIVE SURFACE BOUNDARY CONDITION

(Lapisan Sempadan Haba Terhadap Plat Menegak dalam Bahantara Berliang dengan Syarat Sempadan Permukaan Berolak)

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ABSTRACT

This study aims to investigate the thermal boundary layer over a vertical plate in porous medium with a convective surface boundary condition. The governing systems of partial differential equations subject to the boundary conditions are transformed into the system of ordinary differential equations by employing the similarity transformation. The bvp4c method in Matlab software is used to numerically solve the equations. With the use of graphical and tabular data, the velocity and temperature profiles for various parameter values is obtained, analyzed, and discussed. The effects of the governing parameters involved including permeability parameter K and the buoyancy parameter λ are examined and discussed. The local Nusselt number and skin friction coefficient increase as the permeability parameter's K value rises. The results suggest that dual solutions for opposing flow and the solution is unique for assisting flow. The results also shown that by increasing K, the permeability parameter, will increase the range of solutions. Lastly, the permeability parameter, K, has the consequence of expanding the range values of buoyancy parameter, λ for which solutions are discovered.

Keywords: porous medium; vertical plate; surface boundary conditions

ABSTRAK

Tujuan kajian ini adalah untuk menyelidik lapisan sempadan haba terhadap permukaan plat menegak di dalam bahantara berliang dengan syarat sempadan permukaan berolak. Sistem persamaan pembezaan separa menakluk yang tertakluk kepada syarat sempadan dijelmakan ke dalam sistem persamaan pembezaan biasa dengan menggunakan penjelmaan keserupaan. Kaedah bvp4c di dalam perisian Matlab digunakan untuk menyelesaikan persamaan secara berangka. Dengan menggunakan graf dan data berjadual, profil halaju dan profil suhu untuk nilai parameter yang berbeza diperoleh, dianalisis dan dibincangkan. Kesan parameter menakluk yang terlibat termasuk parameter ketelapan, K dan parameter keapungan, λ dikaji dan dibincangkan. Nombor Nusselt setempat dan pekali geseran kulit meningkat apabila nilai parameter ketelapan, K meningkat. Keputusan menunjukkan terdapat penyelesaian dual bagi aliran menentang dan penyelesaian adalah unik bagi aliran membantu. Keputusan juga menunjukkan bahawa dengan meningkatkan parameter ketelapan, K akan meningkatkan julat penyelesaian. Akhir sekali, parameter ketelapan, K, memberi kesan meluaskan julat nilai parameter keapungan, λ di mana penyelesaian wujud.

Kata kunci: bahantara berliang; plat menegak; syarat sempadan permukaan berolak

1. Introduction

Fluid flows with low viscosity and thus very high Reynolds numbers occur in many technical processes. The concept of the boundary layer implies that flows at high Reynolds numbers can be divided up into inviscid outer flow and a very thin boundary layer at the wall. Thermal boundary layer is formed near the wall where the fluid temperature varies between the wall and the free stream temperature. To understand the Earth's thermal boundary layer, we need to split it into two sections with different properties. There are three reasons for this. First, it helps to compare different approaches to studying the boundary layer. Second, under different conditions, heat flows differently through the layers. Third, these layers have different effects on mantle convection and the Earth's cooling. For example, the mantle plumes that rise towards the lithosphere must pass through the convective boundary layer, which can impact their temperature. Therefore, we cannot rely solely on heat flow data to understand the boundary layer, as other factors like crustal heat generation can complicate the analysis, especially in continents. Heat flow data can provide useful models, but we need to combine them with other approaches and consider heat transport systems to fully understand mantle dynamics. Porous media is a substance that has tiny spaces, or pores, that let fluid or gas pass through it. Rocks, soils, and many other natural and man-made materials have porous media. Porous media is applied as filters or for storing liquids or gases. The way fluids or gases move through porous media depends on its porosity, permeability, and surface area. Porous media is important in geology, petroleum engineering, and environmental science, as it helps store and transfer fluids underground. Understanding fluid and heat movement in porous media is also crucial in fields like biomedical research and energy storage.

The convective boundary condition, known as the Newton boundary condition, comes from the balance of energy at the surface. It is important to understand how heat and flow of heat work to meet quality standards. Since most heat transfer on surfaces is exposed to convective environments, this boundary condition is often seen in real life.

Makinde (2005) studied the effects of thermal radiation and mass transfer on free convection flow past a moving vertical porous plate. The study found that as the Prandtl number increases from 0.71 to 7.0, the fluid temperature decreases significantly. Makinde and Olanrewaju (2010) studied the buoyancy effects on thermal boundary layer over a vertical plate with a convective surface boundary condition. The results showed that a similarity solution for the momentum and the thermal boundary layer equation is possible if the convective heat transfer of the fluid heating the plate on its left surface is proportional to $x^{-\frac{1}{2}}$ and the thermal expansion coefficient, β is proportional to x^{-1}.

Khan *et al.* (2014) investigated triple diffusion with a convective boundary condition along a horizontal plate in a porous medium. Their study found that when there is an aiding flow, the dimensionless surface velocity increases as the buoyancy ratios of the salts increase.

Rosali *et al.* (2015) investigated mixed convection boundary layer flow past a vertical cone in a porous medium with a convective boundary condition. The study found that there are dual solutions in opposing flow, while there is only one solution in assisting flow. Shi and Wang (2015) investigated the combined use of regenerative combustion technology and porous medium combustion technology. Their study found that using these technologies together enables the burner to combust a wider range of low calorific fuels with higher heat loads while ensuring combustion stability and security.

Das *et al.* (2015) carried out a study regarding magnetohydrodynamic mixed convective slip flow over an inclined porous plate with viscous dissipation and Joule heating. They came to a few conclusions including that both fluid velocity and temperature accelerates due to increasing of magnetic field and thermal buoyancy force. A study on a linear stability of horizontal throughflow in a Brinkman porous medium with mixed thermal boundary conditions was done by Dubey and Murthy (2019). They found that if the heat provided at the bottom boundary is more than that supplied at the top boundary, the viscous dissipation stabilizes the flow.

Srihari *et al.* (2022) did research on a implicit Keller box approach for solution of MHD three-dimensional flow through a porous medium. They used implicit Keller box and regular perturbation methods to examine the Soret and Dufour impact on the free-convection flow of a viscous incompressible fluid through a porous material that is confined by an infinite vertical porous plate. The effects of thermal radiation and heat generation/absorption in the mixed convection flow along a vertical permeable shrinking flat plate in was investigated by Naganthran *et al.* (2018). They found that the effect of governing parameters are able to defer the flow separation. A number of studies on thermal boundary layer and convective boundary conditions (Bognár & Hriczó 2011; Olanrewaju *et al.* 2012; Aman & Ishak 2012; Rashidi *et al.* 2014; Etwire *et al.* 2015; Wang *et al.* 2017; Isa *et al.* 2017; Fenuga *et al.* 2018; Jha & Samaila 2020) have been discussed.

These publications explore different fluid types and processes to advance global science and technology. The research builds on previous work to meet modern demands, but porous medium receives less attention than the thermal boundary layer. This article focuses on thermal boundary layer flow over a vertical plate in porous medium with a convective boundary condition using the byp4c Matlab Solver. The paper mainly develops a focus on porous medium cases from Makinde and Olanrewaju's (2010) research on buoyancy effects on the thermal boundary layer over a vertical plate with a convective surface boundary condition.

2. Mathematical Formulation

Consider a two-dimensional steady incompressible fluid flow coupled with heat transfer by convection over a vertical plate. A stream of cold fluid at temperature T_∞ moving over the right surface of the plate with a uniform velocity U_∞ while the left surface of the plate is heated by convection from a hot fluid at temperature T_f, which provides a heat transfer coefficient h_f. The x-axis is taken along the plate and y-axis is normal to the plate. Porosity K_1 is applied to the y-direction. Incorporating the Boussinesq's approximation within the boundary layer, the governing equations of continuity, momentum, and energy equations according to Makinde and Olanrewaju (2010) are respectively given as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\nu}{K_1}(U_\infty-u), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and v are the x and the y components of the velocity respectively; g is the acceleration due to gravity; x, y are the horizontal and vertical dimension respectively, K_1 is the porosity, β is the coefficient of thermal expansion, ν is the Kinematic viscosity, α is the coefficient of thermal conductivity and T is the temperature of the fluid. The velocity boundary conditions can be expressed as (Makinde & Olanrewaju 2010):

$$u(x,0)=v(x,y)=0, \qquad -k\frac{\partial T}{\partial y}(x,0)=h_f\left[T_f-T\right],$$
$$u(x,\infty)=U_\infty,\;\; T(x,\infty)T_\infty \tag{4}$$

In order to solve Eqs. (1)-(3) subject to boundary conditions in Eq. (4), we propose the following similarity transformation.

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}},\ u = axf'(\eta), T = \theta(\eta)(Tf\ - T\infty) + T\infty$$
$$v = \frac{1}{2}\sqrt{\frac{U_\infty}{x}}\left(\eta f'(\eta) - f(\eta)\right), \qquad \theta = \frac{T - T_\infty}{T_f - T_\infty} \tag{5}$$

where a is the thermal diffusivity, η is the similarity variable, $f(\eta)$ is a dimensionless stream function and $\theta(\eta)$ as the temperature and the stream function $\psi(x, y)$.

Momentum Eq. (2) and Energy Eq. (3) are transformed into a set of coupled non-linear ordinary differential equation as

$$f'''(\eta) + \frac{1}{2}f(\eta)f''(\eta) + \lambda\theta - K(1 - f') = 0 \tag{6}$$
$$\theta''(\eta) + \frac{1}{2}f(\eta)\theta'(\eta)Pr = 0 \tag{7}$$

with boundary conditions Eq. (4) are derived to become

$$f(0) = f'(0) = 0,\ f'(\eta) = 0 \text{ as } \eta \to \infty$$
$$\theta'(0) = -Bi\,[1 - \theta(x,0)],\quad \theta(\infty) = 0 \tag{8}$$

where Gr is the Grashof number $Pr = \frac{\nu}{a}$ is the Prandtl number, $\lambda = \frac{Gr}{Re^2}$ is the buoyancy parameter and $Bi = -\frac{h_f}{-k}\left(\frac{a}{\nu}\right)^{\frac{1}{2}}$ is the Biot number.

The physical quantities of interest are the skin friction coefficient,

$$C_f = \frac{\tau_w}{\rho {U_w}^2/2} = 2(Re)^{\frac{1}{2}}f''(0)$$

and the local Nusselt number,

$$Nu_x = \frac{q_w x}{U(T_w - T_\infty)} = -(Re)^{-\frac{1}{2}}\,\theta(0)$$

where,

$$\tau_w = \mu\frac{\partial u}{\partial y}\bigg|_{y=0.} \quad \text{and} \quad q_w = -U\frac{\partial T}{\partial y}\bigg|_{y=0.}$$

The Matlab bvp4c function is utilized to solve the systems of differential equations in Eqs. (6) and (7) along with the corresponding boundary conditions in Eq. (8) in a numerical manner. The solving process starts with an initial guess at the first mesh point and adapts the step size to reach the desired accuracy. The bvp4c approach introduces the new variables for the first time.

3. Results and Discussion

The study utilizes similarity transformations to convert the system of governing Eqs. (6) and (7) under boundary condition in Eq. (8) into an ordinary differential equation (ODE) system. The ODE system is solved using the bvp4c technique in Matlab R2021a software. The study presents an analysis of the effect of the governing parameters on various profiles, such as velocity profiles $f'(\eta)$, temperature profiles $\theta(\eta)$, skin friction coefficient $f''(0)$, and local Nusselt number $-\theta'(0)$. The results are compared with those of Makinde and Olanrewaju (2010) and shown to be in good agreement. The profiles also satisfy the boundary conditions, indicating the effectiveness of the numerical approach.

Table 1: Comparison study with Makinde & Olanrewaju (2010) for *Gr* = 0, *Pr* = 0.72, *Bi* = 0 and $\boldsymbol{\lambda}$ = 0

Bi_x	$-\theta'(0)$ Makinde & Olanrewaju (2010)	$-\theta'(0)$ Presents
0.05	0.0428	0.0428
0.1	0.0747	0.0746
0.2	0.1193	0.1193
0.4	0.1700	0.1700
0.6	0.1981	0.1980
0.8	0.2159	0.2159
1	0.2282	0.2283
5	0.2791	0.2791
10	0.2871	0.2872
20	0.2913	0.2913

The skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ values have been determined. The changes in the local Nusselt number $-\theta'(0)$ have been presented in Table 1 for comparison with Makinde and Olanrewaju's (2010) study, while Table 2 displays the point of inflection that was identified for K values of 1, 1.5, and 2, at *Pr* =0.72 and *Bi* =1. These inflection points are clearly visible in Figures 1 and 2.

Table 2: Variations critical value of buoyancy parameter $\boldsymbol{\lambda_c}$ with $\boldsymbol{K=1, K=1.5}$ and $\boldsymbol{K=2}$

K	λ_c
1	-1.88016
1.5	-2.64714
2	-3.48016

The skin friction coefficient and the local Nusselt number increase with increasing permeability parameter, K. The study revealed the existence of a dual solution when the buoyancy parameter, λ decreases, particularly when it becomes negative or opposing flow. Figures 1 and 2 show that increasing K leads to a wider range of solutions, with each K value having a unique set of solutions that expands as K increases. Thus, K has the effect of increasing the range of λ solution values. Notably, the calculations were stopped when the solution did not converge.

In this research, we also can obtain the velocity profiles $f'(\eta)$ and temperature profiles $\theta(\eta)$ from the variations of the Biot number, buoyancy parameter, Prandtl number and permeability parameter.

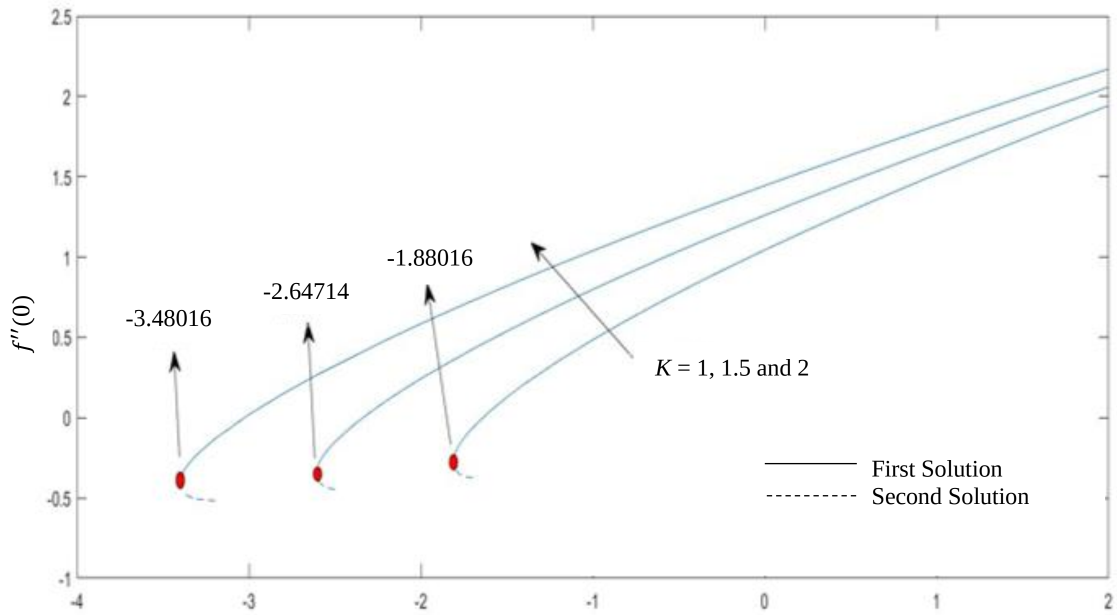

Figure 1: Skin friction coefficient $\boldsymbol{f''(0)}$ as a function of $\boldsymbol{\lambda}$ when $\boldsymbol{K=1, K=1.5}$ and $\boldsymbol{K=2}$ where $\boldsymbol{Pr=0.72}$ and $\boldsymbol{Bi=1}$

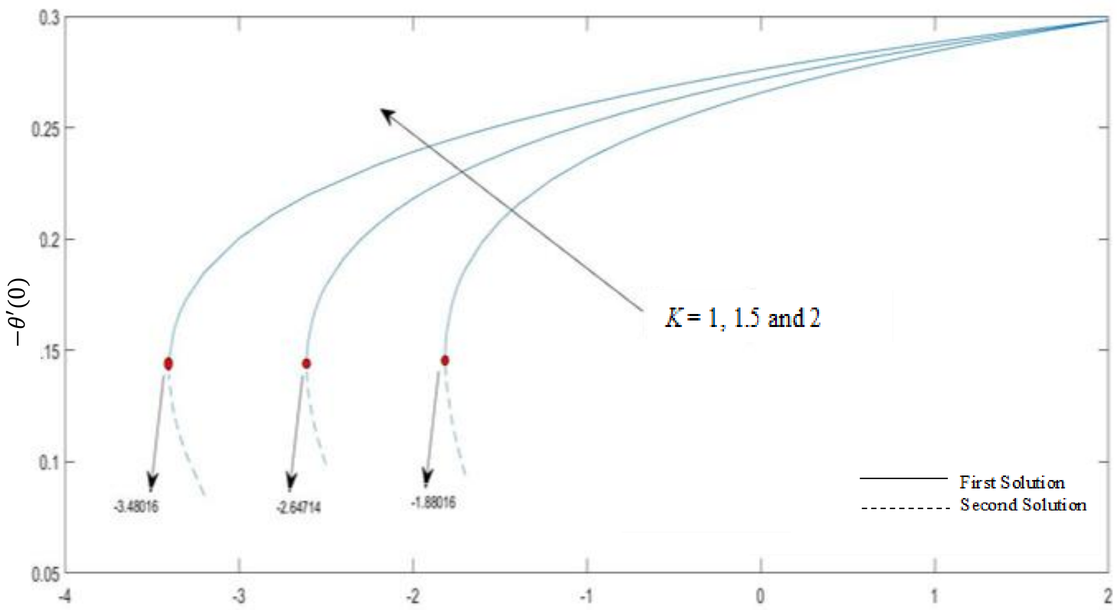

Figure 2: Value of local Nusselt number $\boldsymbol{-\theta'(0)}$ as a function of $\boldsymbol{\lambda}$ when $\boldsymbol{K=1, K=1.5}$ and $\boldsymbol{K=2}$ where $\boldsymbol{Pr=0.72}$ and $\boldsymbol{Bi=1}$

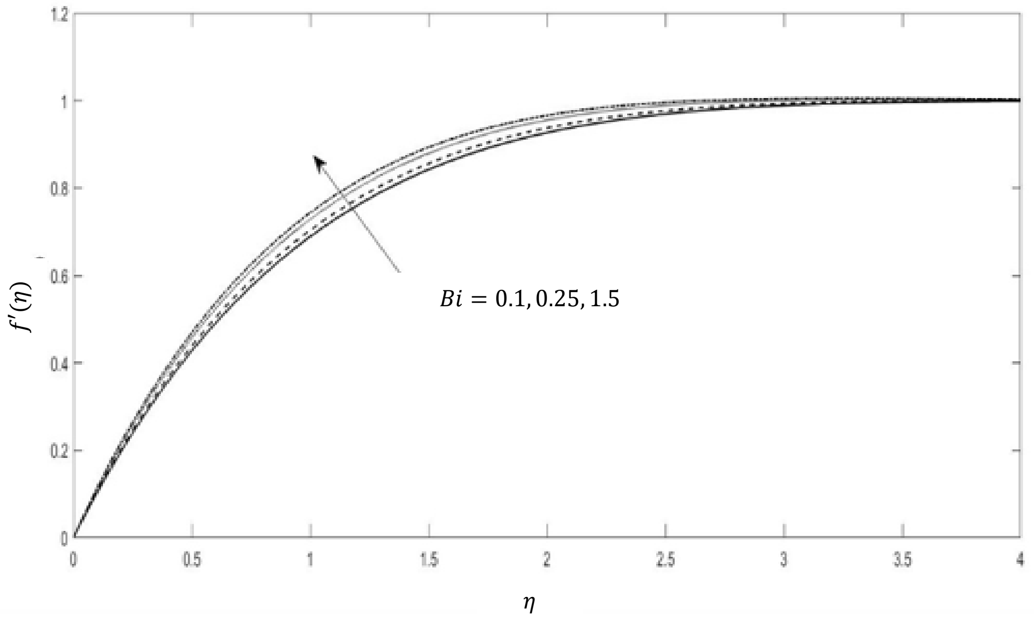

Figure 3: Velocity profiles $\boldsymbol{f'(\eta)}$ for different values of $\boldsymbol{Bi}$

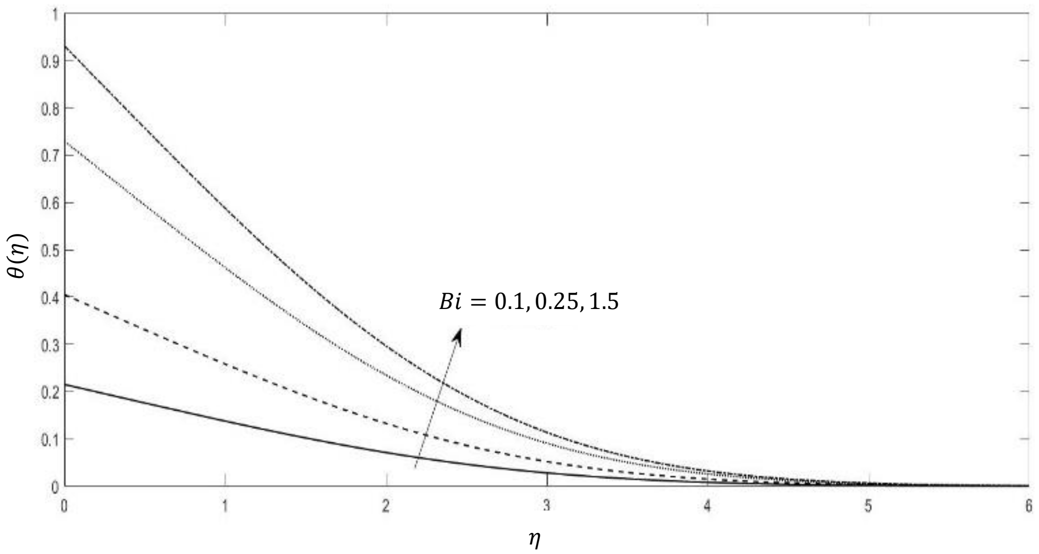

Figure 4: Temperature profiles *θ(η)* for different values of $\boldsymbol{Bi}$

Figures 3 and 4 show the velocity profiles and temperature profiles for different Bi. From the figures, it can be observed that unique solutions exist for Biot number of 0.1, 0.25, 1 and 5. It is also observed that the velocity and temperature profiles increase as the Biot number increases. The effect of buoyancy parameter can be seen in Figures 5 and 6. It has a significant effect on the velocity and temperature profiles. The velocity profile $f'(\eta)$ decreases as we decrease the value of buoyancy parameter, λ whereas the temperature profile increases.

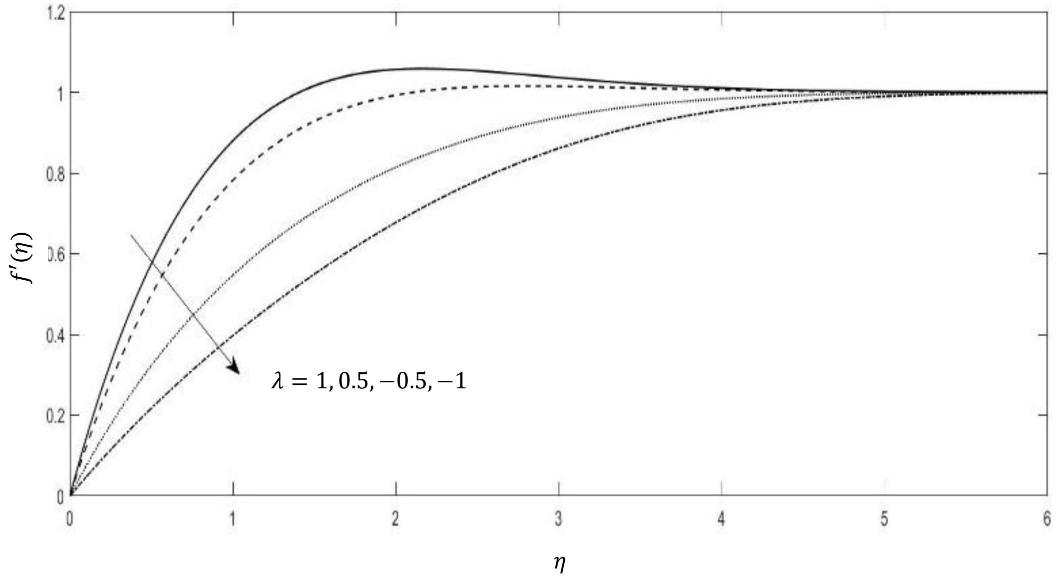

Figure 5: Velocity profiles $\boldsymbol{f'(\eta)}$ for different values of $\boldsymbol{\lambda}$

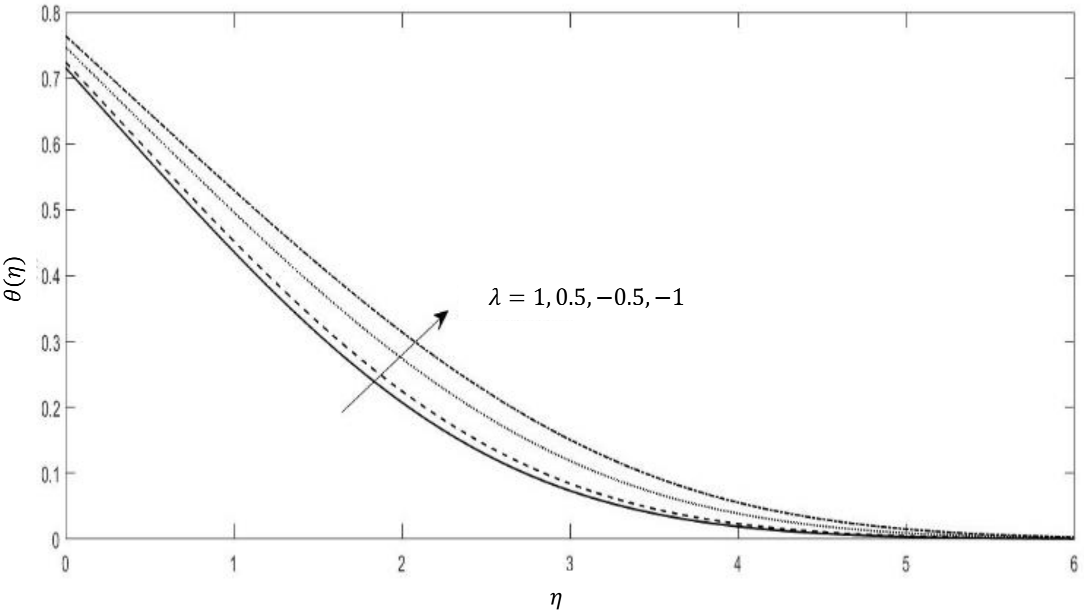

Figure 6: Temperature Profiles *θ(η)* for different values of $\boldsymbol{\lambda}$

Figures 7 and 8 display the temperature and velocity profiles for different values of Prandtl number, Pr. From the figures, it is seen that the velocity profiles decrease as the Prandtl number increases. However, different trend is observed for temperature profiles.

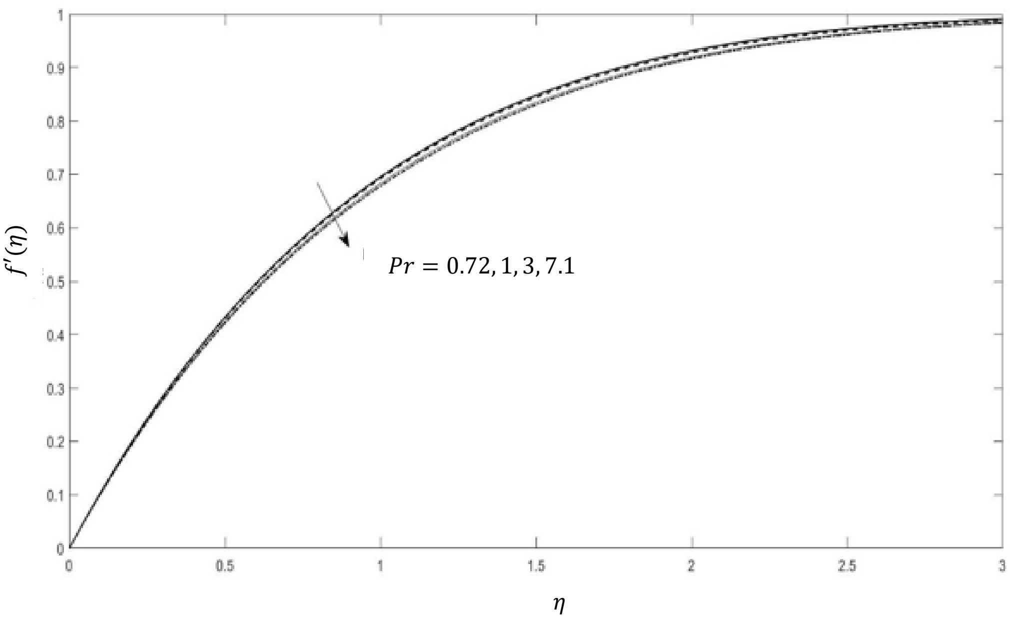

Figure 7: Velocity Profiles $\boldsymbol{f'(\eta)}$ for different values of $\boldsymbol{Pr}$

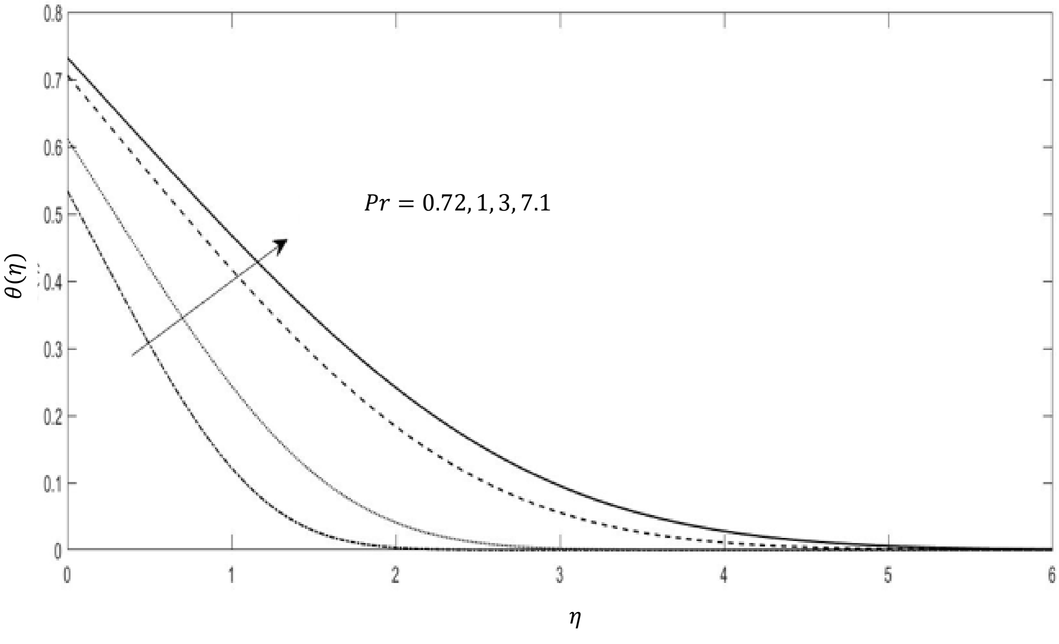

Figure 8: Temperature profiles $\boldsymbol{\theta(\eta)}$ for different values of $\boldsymbol{Pr}$

The effect of permeability parameter, K on the velocity and temperature profiles can be seen in Figures 9 and 10. The velocity profile decrease as the permeability parameter increases. This is due to the resistance on the flow that caused the velocity, whereas, the temperature profiles increase with the permeability parameter K.

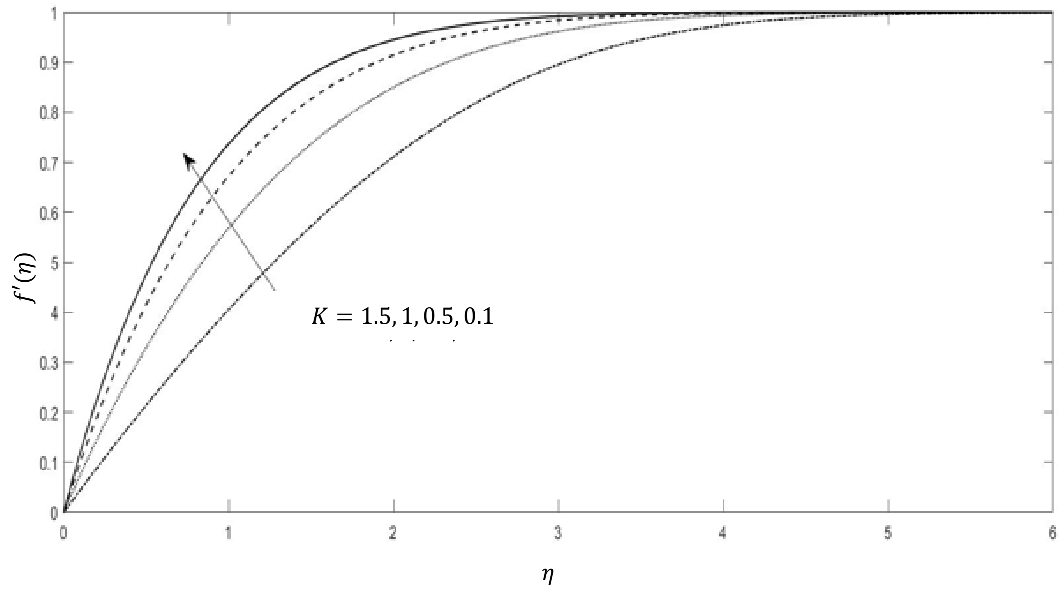

Figure 9: Velocity profiles $\boldsymbol{f'(\eta)}$ for different values of $\boldsymbol{K}$

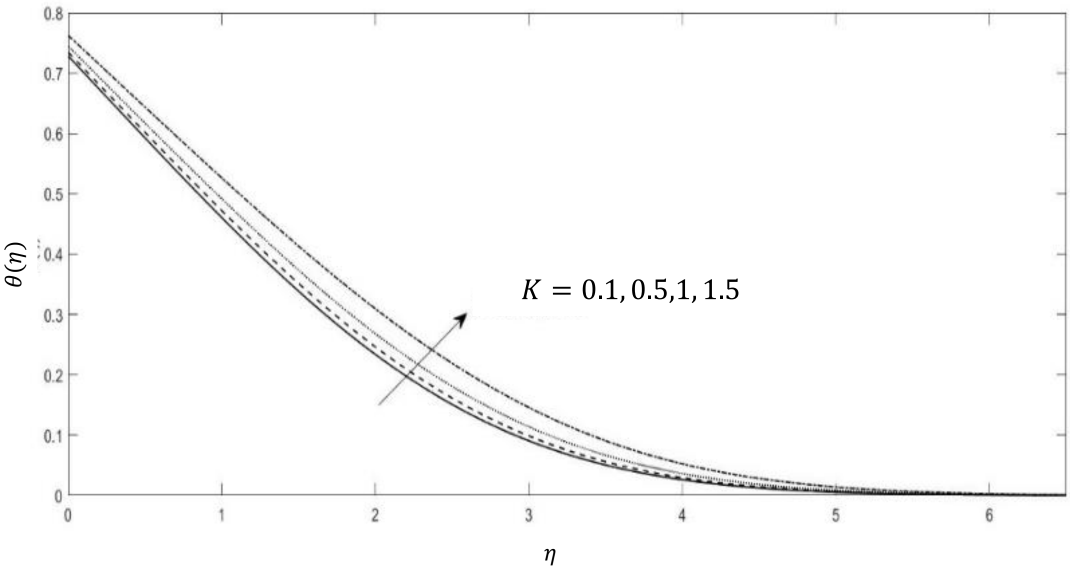

Figure 10: Temperature Profiles $\theta(\eta)$ for different values of K

Certain graphical outcomes, namely Figures 1, 2, and 8, show qualitative similarity with the findings of Makinde and Olanrewaju (2010), who studied the impact of buoyancy on thermal boundary layers over a vertical plate with convective surface boundary conditions. Additionally, these graphical outcomes satisfy the boundary conditions in Eq. (8).

4. Conclusion

This study investigates the behavior of thermal boundary layer and heat transfer over a vertical plate in a porous medium with a convective surface boundary condition. We use the similarity transformation approach to convert the governing partial differential equation (PDE) into an ordinary differential equation (ODE), which is solved using the bvp4c approach in Matlab R2021a. Based on our numerical outcomes, we find that a unique solution exists for both skin friction coefficient $f''(0)$ and local Nusselt number $-\theta'(0)$ when $\lambda > 0$, and dual solutions are present when $\lambda < 0$ approaching the critical value. Increasing permeability parameter K broadens the range of solutions for both $f''(0)$ and $-\theta'(0)$. We also consider various values of Bi, λ, Pr, and K to observe temperature and velocity profiles, which agree with the boundary conditions.

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