Sains Malaysiana 53(3)(2024): 719-731
http://doi.org/10.17576/jsm-2024-5303-18

Mathematical Modelling of a Rumour Spreading with the Attitude of Adjusting Mechanisms

(Pemodelan Matematik bagi Penyebaran Khabar Angin dengan Mekanisme Penyesuaian Sikap)

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Received: 28 September 2023/Accepted: 15 February 2024

ABSTRACT

With the advent of the internet, social media of Facebook and Twitter, as well as the communication technology of WhatsApp and Telegram, the speed and scope of the rumour dissemination has been expanded. Understanding the characterization of rumour dissemination and how it spreads can help in mitigation measures to avoid the spread of the rumour. Therefore, it is crucial to propose a mathematical model, and in particular this paper is concerned with the epidemic model to understand the dissemination of the rumour in social network. The mechanism of rumour propagation is behaving like infectious diseases spread; hence this research adopted the epidemiological model approach. In this network, the compartment is divided into susceptible, ignorant, propagation and stiflers. The basic influence number, the equilibrium points of rumour-free and the endemic equilibrium state were obtained and discussed. For the local stability, the Next Generation Matrix was used. Numerical simulation is performed to understand the dynamics of the spread of rumour in a population or social networks, its impact in a population, and adjusting mechanisms in curbing the spread of rumour.

Keywords: Adjusting mechanism; mathematical model; rumour spreading; stability

ABSTRAK

Dengan kemunculan internet, media sosial seperti Facebook dan Twitter, serta teknologi komunikasi seperti WhatsApp dan Telegram, penyebaran khabar angin tersebar meluas dan berlaku dengan pantas. Memahami ciri penyebaran khabar angin dan bagaimana ia merebak dapat membantu dalam langkah mitigasi untuk mengawal penyebarannya. Oleh itu, penting untuk mencadangkan model matematik dan kajian ini membincangkan model epidemik untuk memahami penyebaran khabar angin dalam rangkaian media sosial. Mekanisme penyebaran khabar angin berperilaku seperti penyebaran penyakit berjangkit; oleh itu, penyelidikan ini mengambil pendekatan model epidemiologi. Dalam rangkaian ini, kompartmen dibahagikan kepada populasi rentan, populasi yang tidak ambil tahu dan populasi penyebar dan populasi penghalang. Nombor pengaruh asas, titik keseimbangan tanpa khabar angin dan keadaan keseimbangan endemik diperoleh dan dibincangkan. Untuk kestabilan tempatan, Matriks Generasi Seterusnya digunakan. Simulasi berangka dijalankan untuk memahami dinamik penyebaran khabar angin dalam populasi atau rangkaian sosial, kesannya dalam populasi dan mekanisme penyesuaian dalam mengekang penyebaran khabar angin.

Kata kunci: Mekanisme perubahan; model matematik; penyebaran khabar angin; stabiliti

INTRODUCTION

A rumour refers to unverified information that is spread within a particular context through various channels (Li et al. 2021; Zhou et al. 2022). In the era of the internet, a plethora of online rumors can rapidly circulate due to technological advancements and the widespread influence of social media platforms such as LiveJournal, Facebook, Twitter, YouTube, and Hi5 among others. With the world evolving into a global community through social media, spreading rumors has become instantaneous with just a click of the share button. The dissemination of rumours poses a significant challenge to governments and can have detrimental effects on societal well-being. In recent times, numerous mathematical models have emerged to explain the dynamic behavior of rumor propagation. These models offer insights into various phenomena, such as information dissemination, viral marketing, and panics triggered by epidemics or emergencies (Daley & Kendall 1964; Maki & Thompson 1973; Zan et al. 2014). Among the references cited, many researchers that modelled the spread of rumour gave credit to the work of Daley and Kendall (1964). They have adopted the epidemiological model in modelling the spread of rumour. The model is known as DK model and categorizes the population into three distinct groups i.e., the ignorant (those initiating rumors), spreaders (those disseminating the rumor), and stiflers (individuals aware of the rumor but choose not to spread it). However, a limitation of the DK model is its assumption that all ignorant individuals invariably become spreaders upon hearing a rumor which does not always hold true. To address this, Maki and Thompson (1973) expanded upon the DK model, introducing the MT model, which offers a more nuanced interaction among the three groups: ignorant, spreaders, and stiflers. Despite these advancements, both the DK and MT models lack consideration for the topological characteristics inherent in social networks, making them less suitable for capturing the complexities of rumor propagation on large-scale social networks (Zan et al. 2014). As rumor propagation occurs within complex networks, Zanette (2001) extended the DK model to better capture this complexity. Several researchers have since incorporated various mechanisms into epidemiological models to mitigate its impact using control mechanisms. Among the references cited therein are modelling the rumours in the presence of the mechanism of incubation (Al-Tuwairqi, Al-Sheikh & Al-Amoudi 2015), the mechanism of forgetfulness (Zhao et al. 2013), the mechanism of hesitation (Xia et al. 2015), the mechanism of punishment and control by government (Li & Ma 2017; Zhao & Wang 2014), the mechanism of memory effect (Zhang & Xu 2015). Li et al. (2021) have extended DK model to its corresponding stochastic counterpart and added two distinct inhibiting and attitude adjusting mechanisms to the model. They analyse a deterministic and stochastic model for the spread of rumours in a homogeneous social network. For simplicity, Li et al. (2021) focus solely on rumors spreading through direct human contact, categorizing individuals into three compartments of newcomer (S), spreader (I) and stifler (R). Zhao et al. (2013) explored the dynamics of a rumor-spreading model with four compartments, taking into account forgetting and remembering mechanisms within inhomogeneous networks. The model by Li et al. (2021) have ignored resisted compartment. In this research, we employed the compartmental model introduced by Zhao et al. (2013) within a homogeneous network, integrating the two adjusting mechanisms highlighted by Li et al. (2021). Our model incorporates the resisted (recovered) compartment, representing individuals who, despite hearing the rumor, resist spreading it or adjust their attitude due to the mechanisms. Therefore, this research introduces an epidemiological framework termed Susceptible-Hesitate-Propagate-Resisted (SHPR) that incorporates two inhibitory and attitude-adjusting mechanisms within homogeneous networks.

This paper is organised as follows. Next section provides the deterministic rumours spreading model of SHPR with two inhibiting and attitude adjusting mechanisms. Qualitative analysis of the model is given in subsequent section which includes the basic influence number, positivity of the solution and the stability at equilibria. In the section that follows, numerical simulations are performed to illustrate the validity of the theoretical results. Concluding remarks are given in the last section.

MATHEMATICAL MODEL

In this section, we describe the formulation of susceptible-hesitate-propagate-resisted (SHPR) rumour spreading model with adjusting mechanisms. Let $N(t)$ denote the total population at time t. As indicated in Figure 1, the population is divided into four compartments, the susceptible class (S), the ignorant class (I), the propagating class (P) and the hesitates class (H). Let denote the number of susceptible, propagates, ignorant and hesitated at time t as $S(t)$, $P(t)$, $I(t)$ and $H(t)$, respectively. Then, the total population at time t is

$$N(t) = S(t) + P(t) + I(t) + H(t) \tag{1}$$

Figure 1 shows the structure of the SHPR model with adjusting mechanism, U for the rumour spreading.

As illustrated in Figure 1, the following assumptions are made for this model: 1. Any person added to the social network population is susceptible. 2. The rumour spreads in a population with constant immigration and emigration rate. The recruitments into susceptible class, S occurs at a positive constant rate Γ, while each compartment is characterized by a constant leaving rate of μ. 3. Upon hearing a rumour, the susceptible, S may subsequently display two different attitudes. The first attitude is denoted by I that is referred to the individuals who have heard the rumours but hesitate to spread it (ignorance). Meanwhile the second attitude is denoted by P nodes is those who believe the rumours and spread it actively. 4. Those who are ignorance will move from S to I nodes with the probability, (1-θ) and adjustment function of $f(U)$, resulting from the adjustment mechanism of governmental work. Ignorant population may change their attitudes and become spreaders with the constant rate of τ. 5. Those who actively spread the rumours will move from S to P nodes with the probability, θ and adjustment function of $f(U)$, resulting from the adjustment mechanism of governmental control. The newcomer from I nodes who have changed their attitudes and actively spread the rumours will be recruited to P nodes with the constant rate of τ. 6. The newcomer from P and I nodes will enter the H (hesitated/ resisted) node with the function rate of $g(U)$ and constant rate of β, respectively. H nodes refer to the recover individuals. It is also known as stiflers. 7. The constant r is the government control rate for adjusting and inhibiting mechanisms and e is represents a decay rate mechanism. The functions $f(U)$ and $g(U)$ characterize the impact of rumour control mechanisms, which influence the attitudes of the spreaders within propagation nodes. The recruitment of the individuals from S to I and P nodes is subjected to the adjustment function of $f(U)$. The constant δ and K quantify the utilization of the inhibitor, with δ representing the maximum uptake rate of P and K is a half-saturation parameter. 8. The movement of individuals from one compartment to another is unidirectional, i.e., the flow shown in Figure 1 is irreversible.

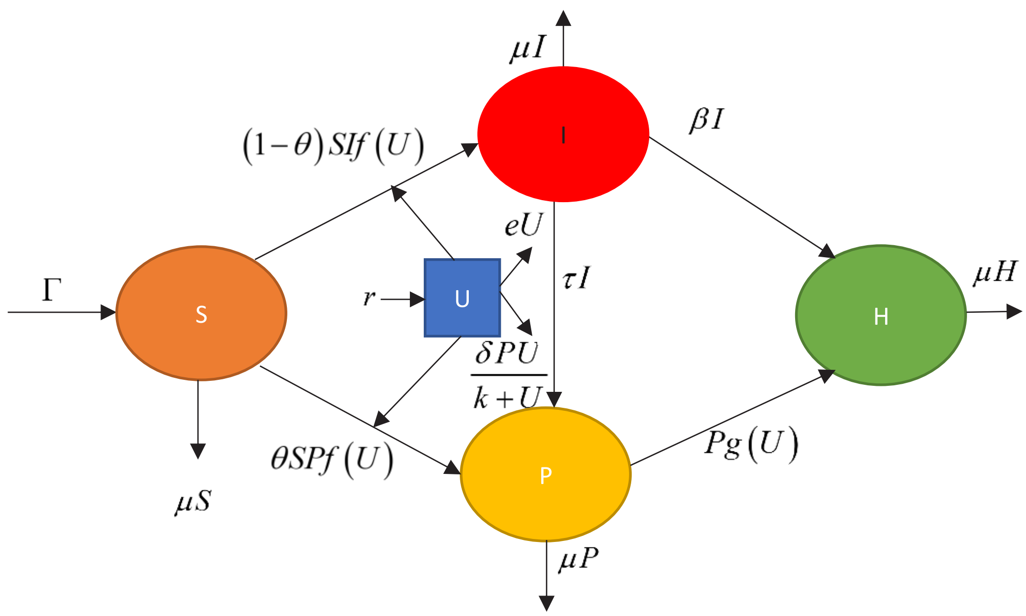

FIGURE 1. Schematic diagram of the flow of rumour spreading with two adjusting mechanisms

TABLE 1. Description of the variables and parameters

Variables/ Parameters	Description
S	Susceptible class
I	The latent (ignorance) class i.e., the class belong to the individuals who have heard the rumours but ignore it
P	The propagating class i.e., the class belong to the individual who believe the rumours and spread it actively
U	Adjustment mechanism
H	The resisted (recovered) class which is the class for those who are from Ignorance class have heard the rumour but resist and do not spread it/ those who have changed the attitude due to the adjustment mechanism
Γ	The rate of the recruitments into susceptible class
μ	Leaving rate for each compartment
θ	The rate of those who actively spread the rumours
r	The rate for adjusting and inhibiting mechanism
e	A decay rate mechanism
δ	The maximum uptake rate of
K	Half-saturation parameter
τ	The constant rate of the hesitate population who change their attitudes and become spreaders
β	The rate of newcomer from I node to the H node

Based on the schematic diagram in Figure 1, we obtain the following system of differential equations:

$$\begin{aligned}
\frac{dS}{dt} &= \Gamma - \left(\left(1-\theta\right)I + \theta P\right)Sf\left(U\right) - \mu S \\
\frac{dP}{dt} &= \theta SPf\left(U\right) + \tau I - \left(\mu + g\left(U\right)\right)P \\
\frac{dI}{dt} &= \left(1-\theta\right)SIf\left(U\right) - \left(\mu + \tau + \beta\right)I \\
\frac{dU}{dt} &= r - eU - \frac{\delta PU}{K+U} \\
\frac{dH}{dt} &= \beta I + Pg\left(U\right) - \mu H
\end{aligned} \tag{2}$$

We assume that:

A1: The function $f:[0,\infty)\to R$ satisfies

1. $f(U) \geq 0, f(0) = 1$

2. f is non-increasing on $[0,\infty)$.

A2: The function $g:[0,\infty)\to R$ satisfies

1. $g(U) \geq 0$

2. g is non-decreasing on $[0,\infty)$.

QUALITATIVE ANALYSIS

The initial conditions are assumed to be non-negative because the model portrays human population dynamics concerning the propagation of rumours. Now, it suffices to show that the solutions of the model are also positive and bounded.

Theorem 1 Let $\coloneqq \{(S, I, P, U, H) \in \mathbb{R}^5 : S_0 > 0, I_0 > 0, P_0 > 0, U_0 > 0, H_0 > 0\}$ then the solution of the model (2) for is positive invariant and bounded for all $t \geq 0$.

Proof From equation (1), we have

$$\frac{dN(t)}{dt}=\frac{dS(t)}{dt}+\frac{dP(t)}{dt}+\frac{dI(t)}{dt}+\frac{dH(t)}{dt}=\Gamma-\mu N(t)$$

which implies that

$$N(t)=\left(N(0)-\frac{\Gamma}{\mu}\right)e^{-\mu t}+\frac{\Gamma}{\mu}$$

for all $t \geq 0$. We obtain $0 \leq S(t)+P(t)+I(t)+H(t) \leq \frac{\Gamma}{\mu}$.
This ensures the boundedness of $S(t)$, $P(t)$, $I(t)$ and $H(t)$.

Please note that $\left.\frac{dU(t)}{dt}\right|_{U(t)=\frac{r}{e}}=-\frac{\delta P\left(\frac{r}{e}\right)}{K+\frac{r}{e}}\leq 0$ in which prevents the function of from growing indefinitely. This implies that the function $U(t)$ is decreasing over time and approaching the constant value of $\frac{r}{e}$. Hence, the function of $U(t)$ is bounded.

The state variables *S, P, I, U* and *H* are remains within specific region of a positive invariant set of

$$\Omega \coloneqq$$

$$\left\{(S,P,I,U,H)\in\mathbb{R}^5: 0\leq S+P+I+H\leq\frac{\pi}{\mu}\ and\ 0\leq U\leq\frac{r}{e}\right\}.$$

This ensures that the solution of *S, P, I, U* and *H* remains positive for all $t \geq 0$.

The Basic Influence Number

The system (2) has a rumour-free equilibrium, E_0 of

$$E_0=\left(\frac{\Gamma}{\mu},0,0,\frac{r}{e},0\right)$$

The dynamic of the model is investigated using the next generation method. Let consider the infected-like compartment of *P*, *I*, and the uninfected-like compartment is given by *S,U* and *H*. Then we apply the next generation method to find the basic influence number, R_0. Let denotes

$$X=(P,I)^T \text{ and } Y=(S,U,H)^T.$$

Then, the system (2) can be written as

$$\frac{dX}{dt}=F(X,Y)-V(X,Y)$$
$$\frac{dY}{dt}=G(X,Y)$$

This yield

$$\frac{dX}{dt}=\begin{pmatrix}\theta SPf(U)+\tau I\\ 0\end{pmatrix}-\begin{pmatrix}(\mu+g(U))P\\ -(1-\theta)SIf(U)+(\mu+\tau+\beta)I\end{pmatrix}$$

in which

$$F(X,Y)=\begin{pmatrix}\theta SPf(U)+\tau I\\ 0\end{pmatrix},$$

$$V(X,Y)=\begin{pmatrix}(\mu+g(U))P\\ -(1-\theta)SIf(U)+(\mu+\tau+\beta)I\end{pmatrix}$$

Then at rumour-free equilibrium we have

$$DF|_{E_0}=\left.\begin{pmatrix}\frac{\delta F_1}{\delta P} & \frac{\delta F_1}{\delta I}\\ \frac{\delta F_2}{\delta P} & \frac{\delta F_2}{\delta I}\end{pmatrix}\right|_{E_0}=\begin{pmatrix}\frac{\theta\Gamma f\left(\frac{r}{e}\right)}{\mu}+\tau & 0\\ 0 & 0\end{pmatrix}$$

$$DV|_{E_0}=\left.\begin{pmatrix}\frac{\delta V_1}{\delta P} & \frac{\delta V_1}{\delta I}\\ \frac{\delta V_2}{\delta P} & \frac{\delta V_2}{\delta I}\end{pmatrix}\right|_{E_0}$$

$$=\begin{pmatrix}\mu+g\left(\frac{r}{e}\right) & -\tau\\ 0 & \frac{-(1-\theta)\Gamma f\left(\frac{r}{e}\right)}{\mu}+(\mu+\tau+\beta)\end{pmatrix}.$$

The basic influence number that is a threshold parameter for the stability of the system (2) can be obtained by determined the spectral radius of matrix FV^{-1} given by

$$R_0=\frac{\theta\Gamma f\left(\frac{r}{e}\right)+\tau\mu}{\mu\left(g\left(\frac{r}{e}\right)+\mu\right)}.$$

R_0 provides insight into how the value r influences the potential of rumour spreading. The parameter r corresponds to factors such as mitigation factors or allotted budget by government to control the spread of the rumour. Under assumption A1, the numerator is non-increasing function of r i.e., the function $f\left(\frac{r}{e}\right)$ decreases or remain constant as r increases. Assumption A2 imply that the denominator is an increasing function of r i.e., the function $g\left(\frac{r}{e}\right)$ increases as r increases. This imply the basic influence number R_0 is non-increasing function of r. It can be concluded that as more budget is allocated for government control mechanism will lead to the less influence of the spreader in the system, hence able to control the spreading of the rumours.
Next, we investigate the dynamic of the system (2) at a positive equilibrium point.

The Existence of a Positive Equilibrium
The positive equilibrium, $E^* = \left(S^*, P^*, I^*, U^*, H^*\right)$ let the system (2) take the form of

$$
\begin{aligned}
&\Gamma - \left((1-\theta) I + \theta P\right) Sf(U) - \mu S = 0 \\
&\theta SPf(U) + \tau I - \left(\mu + g(U)\right) P = 0 \\
&(1-\theta) SIf(U) - (\mu + \tau + \beta) I = 0 \\
&r - eU - \frac{\delta PU}{K+U} = 0 \\
&\beta I + Pg(U) - \mu H = 0
\end{aligned}
\tag{3}
$$

By algebraic manipulation, we obtain

$$
\begin{aligned}
S^* &= \frac{\Gamma}{\mu} - \frac{\left((1-\theta) I^* + \theta P^*\right)(\mu+\tau+\beta)}{(1-\theta)\mu} \\
P^* &= \frac{\left(r - eU^*\right)\left(k+U^*\right)}{\delta U^*} \\
I^* &= \frac{-\theta S^* P^* f\left(U^*\right) + \left(\mu + g\left(U^*\right)\right) P^*}{\tau} \\
H^* &= \frac{\beta I^* + P^* g\left(U^*\right)}{\mu}
\end{aligned}
\tag{4}
$$

The Jacobian matrix at $J(E^*)$ is given by

$$
J\left(E^*\right) =
\begin{bmatrix}
-\mu - f\left(U^*\right)\left(\theta P^* + (1-\theta) I^*\right) & -\theta S^* f\left(U^*\right) & -(1-\theta) S^* f\left(U^*\right) & 0 & 0 \\
\theta P^* f\left(U^*\right) & \theta S^* f\left(U^*\right) - (\mu + g) & \tau & 0 & 0 \\
(1-\theta) I^* f\left(U^*\right) & 0 & (1-\theta) S^* f\left(U^*\right) - (\mu+\tau+\beta) & 0 & 0 \\
0 & -\frac{\delta U^*}{k+U^*} & 0 & \frac{\delta P^* U^*}{\left(k+U^*\right)^2} - \frac{\delta P^*}{k+U^*} - e & 0 \\
0 & g\left(U^*\right) & \beta & 0 & -\mu
\end{bmatrix}
$$

The Stability at Equilibria

Theorem 2 The rumour-free equilibrium $E_0 = \left(\frac{\pi}{\mu}, 0, 0, \frac{r}{e}, 0\right)$ is locally asymptotically stable if

$$
\Lambda < R_0 < 1, \text{ where } \Lambda = \frac{\left(\beta - \mu - \frac{\Gamma}{\mu} f\left(\frac{r}{e}\right)\right)}{\mu\left(\mu + g\left(\frac{r}{e}\right)\right)}.
$$

Proof The Jacobian matrix of system (2) at $E_0 = \left(\frac{\Gamma}{\mu}, 0, 0, \frac{r}{e}, 0\right)$ is

$$
J\left(E_0\right) =
\begin{bmatrix}
-\mu & -\theta \frac{\Gamma}{\mu} f\left(\frac{r}{e}\right) & -(1-\theta)\frac{\Gamma}{\mu} f\left(\frac{r}{e}\right) & 0 & 0 \\
0 & \theta \frac{\Gamma}{\mu} f\left(\frac{r}{e}\right) - \left(g\left(\frac{r}{e}\right) + \mu\right) & \tau & 0 & 0 \\
0 & 0 & (1-\theta)\frac{\Gamma}{\mu} f\left(\frac{r}{e}\right) - (\mu+\tau+\beta) & 0 & 0 \\
0 & \frac{-\delta r}{e\left(k + \frac{r}{e}\right)} & 0 & -e & 0 \\
0 & g\left(\frac{r}{e}\right) & \beta & 0 & -\mu
\end{bmatrix}
$$

The characteristic equation is

$$
(\lambda+\mu)^2 (e+\lambda)\left(-\theta \Gamma f\left(\frac{r}{e}\right) + g\left(\frac{r}{e}\right)\mu + \mu^2 + \lambda\mu\right)
$$

$$
\left(\theta \Gamma f\left(\frac{r}{e}\right) - \Gamma f\left(\frac{r}{e}\right) + \beta\mu + \mu^2 + \tau\mu + \lambda\mu\right) = 0
$$

The eigen values of $J(E_0)$ are given by

$$
\begin{aligned}
\lambda_{1,2} &= -\mu, \\
\lambda_3 &= -e, \\
\lambda_4 &= \theta \frac{\Gamma}{\mu} f\left(\frac{r}{e}\right) - \left(\mu + g\left(\frac{r}{e}\right)\right), \\
\lambda_5 &= \left(\left(\frac{\beta - \mu - \frac{\Gamma f\left(\frac{r}{e}\right)}{\mu}}{\mu\left(g\left(\frac{r}{e}\right) + \mu\right)}\right) - R_0\right)\left(g\left(\frac{r}{e}\right) + \mu\right).
\end{aligned}
$$

We observe that $\lambda_1, \lambda_2, \lambda_3 < 0,$ the remaining λ_4 and λ_5 are investigated as follows. The eigen value of λ_4 can be written as

$$\lambda_4 = \left(\theta \frac{\Gamma}{\mu} f\left(\frac{r}{e}\right) - \left(\mu + g\left(\frac{r}{e}\right)\right)\right)\left(\frac{\mu + g\left(\frac{r}{e}\right)}{\mu + g\left(\frac{r}{e}\right)}\right)$$

$$= \left(\frac{\theta \frac{\Gamma}{\mu} f\left(\frac{r}{e}\right)}{\mu + g\left(\frac{r}{e}\right)} - \frac{\left(\mu + g\left(\frac{r}{e}\right)\right)}{\mu + g\left(\frac{r}{e}\right)}\right)\left(\mu + g\left(\frac{r}{e}\right)\right)$$

$$= \left(R_0 - 1\right)\left(\mu + g\left(\frac{r}{e}\right)\right)$$

For $R_0 < 1,$ then $\lambda_4 < 0$ which imply $E_0 = \left(\frac{\Gamma}{\mu}, 0, 0, \frac{r}{e}, 0\right)$ is locally asymptotically stable. Note that E_0 is unstable if $R_0 > 1$.

The remaining λ_5 can be written as follows:

$$\lambda_5 = -\left(\frac{\theta \Gamma f\left(\frac{r}{e}\right) + \tau\mu}{\mu}\right) - \left(\beta + \mu - \frac{\Gamma}{\mu} f\left(\frac{r}{e}\right)\right)$$

$$= \left(\Lambda - R_0\right)\left(g\left(\frac{r}{e}\right) + \mu\right)$$

where $\Lambda = \dfrac{\left(\beta - \mu - \frac{\Gamma}{\mu} f\left(\frac{r}{e}\right)\right)}{\mu\left(\mu + g\left(\frac{r}{e}\right)\right)}$. This imply if then is locally asymptotically stable.

Next, we investigate the stability of the system (2) at positive equilibrium using the similar way as for the stability at rumour-free equilibria. For simplicity, the following notations are introduced.

$$\omega_1 = -\left(\mu + f\left(U^*\right)\left(\theta P^* + (1-\theta) I^*\right)\right),\ \omega_2 = -\theta S^* f\left(U^*\right),$$

$$\omega_2 = -\theta S^* f\left(U^*\right),\ \omega_3 = -(1-\theta) S^* f\left(U^*\right)$$

$$\eta_1 = \theta P^* f\left(U^*\right),\ \eta_2 = \theta S^* f\left(U^*\right) - (\mu + g), \eta_3 = \tau$$

$$\nu_1 = (1-\theta) I^* f\left(U^*\right),\ \nu_3 = (1-\theta) S^* f\left(U^*\right) - (\mu + \tau + \beta)$$

$$\psi_1 = -\frac{\delta U^*}{k + U^*},\ \psi_2 = \frac{\delta P^* U^*}{\left(k + U^*\right)^2} - \frac{\delta P^*}{k + U^*} - e$$

$$\zeta_1 = g\left(U^*\right),\ \zeta_2 = \beta, \zeta_3 = -\mu$$

The characteristic equation is

$$\lambda^5 + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0 \qquad (3)$$

where

$$a_0 = -\psi_2\left(-\eta_2\nu_1\omega_3 + \nu_3\left(\omega_1\eta_2 - \omega_2\eta_1\right)\right)\xi_2 > 0$$

$$a_1 = -\eta_2\psi_2\left(-\eta_2\nu_1\omega_3 + \nu_3\left(\omega_1\eta_1 - \omega_2\eta_2\right)\right) > 0$$

$$\begin{aligned} a_2 = \psi_2\Big(&\left(\left(\omega_1 + \nu_3\right)\eta_2 + \omega_1\nu_3 - \omega_2\eta_1 - \omega_3\nu_1\right)\xi_2 \\ &+ \eta_2\left(\omega_1\nu_3 - \omega_3\nu_1\right) - \eta_1\omega_2\nu_3\Big) \\ &+ \xi_2\left(\eta_2\left(\omega_1\nu_3 - \omega_3\nu_1\right) - \eta_1\omega_2\nu_3\right) > 0 \end{aligned}$$

$$\begin{aligned} a_3 = {}&\psi_1\left(-\left(\omega_1 + \eta_1 + \nu_3\right)\xi_2 - \left(\omega_1 + \nu_3\right)\eta_2 - \omega_1\nu_3 + \omega_2\eta_1 + \omega_3\nu_1\right) \\ &+ \psi_2\left(-\left(\omega_1 + \nu_3\right)\eta_2 - \omega_1\nu_3 + \omega_2\eta_1 + \omega_3\nu_1\right) > 0 \\ &+ \eta_2\left(\omega_3\nu_1 - \omega_1\nu_3\right) + \eta_1\omega_2\nu_3 \end{aligned}$$

$$a_4 = -\left(\omega_1 + \eta_2 + \nu_3 + \psi_2 + \xi_3\right) > 0$$

By Routh-Hurwitz criterion, a tabular method is applied to determine the stability of the polynomial (3). The system (2) is stable if the Routh-Hurwitz coefficients

$$a_4, a_3, a_2, a_1, a_0 > 0,$$

$$a_2 < a_3 a_4,$$

$$a_0 < a_1 a_4, a_4 + a_0 a_4 - a_1 a_4^2 - a_2^2 + a_2 a_3 a_4 > 0,$$

$$a_0^2 + a_0 a_3^2 a_4 + 2a_0 + a_1^2 a_4^2 + a_1 a_2^2 + a_3^2 a_4 > a_1 a_2 a_3 a_4 \cdot$$
$$+ 2a_1 a_4 + a_2 a_3 + 2a_0 a_1 a_4 + a_0 a_2 a_3.$$

NUMERICAL SIMULATION

The numerical simulation is performed to support the qualitative analysis of system (1). The initial values and parameter values are indicated in Table 2. The Markov Chain Monte Carlo (MCMC) method, as studied by Fahmi, Norhayati and Noryanti (2021), is employed to estimate the parameter values of θ, τ and β, while other parameters are sourced from Chen (2019) and Li et al. (2021).

The simulated results of evolution of the newcomer in susceptible class, those who are ignorant, the spreader, inhibitor (government mechanism) and the hesitated class is depicted in Figures 2 and 3. According to the theoretical finding, the rumour propagation will fade out when the basic influence number, $\Lambda < R_0 < 1$. For the parameter values given in Table 1, the values of $R_0 = 0.86$ and $\Lambda = -3.6$. The evolution density of the spreaders is influenced by the government control mechanism given by the state of $U(t)$ (Figure 2). In Figure 3, the density of the spreaders, $P(t)$ declines to zero, hence indicate that the propagation of the rumour will disappear finally over time. The density of newcomers to the state of $S(t)$ is sharply decrease for the time interval [0, 5]. As time increases it reach the respective equilibrium states. The latent and hesitating states also dies out indicate that the spread of the rumour is controllable.

TABLE 2. Initial values and parameter values

Variables/ Parameters	Values	Source
$S(0)$	3.0	Li et al. (2021)
$H(0)$	1.0	Li et al. (2021)
$P(0)$	1.0	Li et al. (2021)
$U(0)$	1.0	Li et al. (2021)
$R(0)$	1.0	Li et al. (2021)
$f(U)$	$0.5(1 + e^{-U})$	Li et al. (2021)
$g(U)$	$0.2\left(\frac{U}{1+U}\right)$	Li et al. (2021)
U	[0,100]	Li et al. (2021)
Γ	(2,6)	Chen (2019)
μ	0.7	Chen (2019)
θ	0.8	MCMC
r	1.0	Fixed
e	0.5	Fixed
δ	0.5	Li et al. (2021)
k	1.0	Li et al. (2021)
τ	0.02	MCMC
β	0.05	MCMC

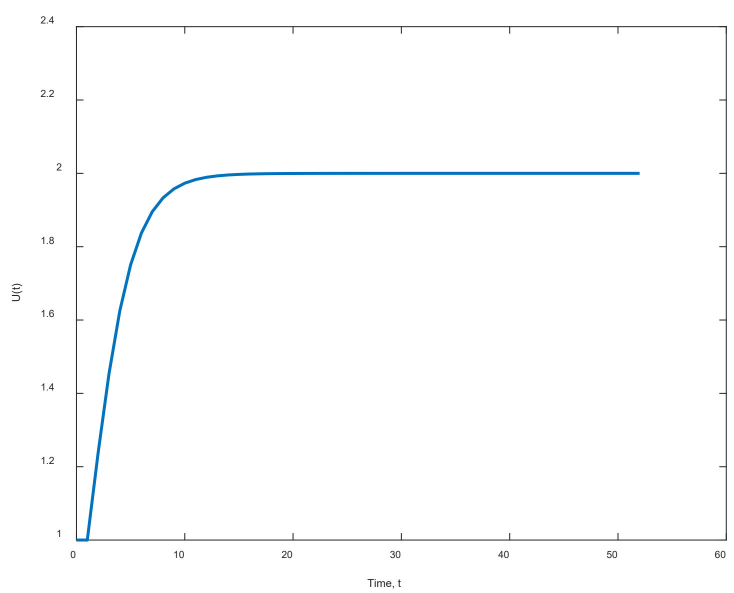

FIGURE 2. Inhibitor mechanism, $U(t)$

Next, we investigate the stability of the solution at positive equilibria points. In Figures 4 and 5, the control mechanism at initial time, $U(0) = 3$, the control mechanism rate, $r = 3$ and the basic influence number is $R_0 = 1.995>1$. In this case the coefficients of the characteristic equations at rumour positive equilibrium is $a_0 = 0.00396$, $a_1 = 0.31620$, $a_3 = 0.015549$, and $a_4 = 4.013801$. Based on the theoretical finding, the system of Equation (2) is locally asymptotically stable. In Figure 4, the government control mechanism $U(t)$ increase sharply for the time interval [0, 10] and then reach the equilibrium state of 5.9. The population of propagating individual will maintain at a positive constant, which means that the rumour propagation will be permanent but at very low density (Figure 5). Latent population density increases at earlier time, then decreases and maintain at a positive equilibrium state. It shows that as more effort the government put to control the spread of the rumours (allotted budget), the spread of the rumours will eventually die out. More individual will move to latent compartment due to the governmental control mechanism and then reach the respective equilibrium states.

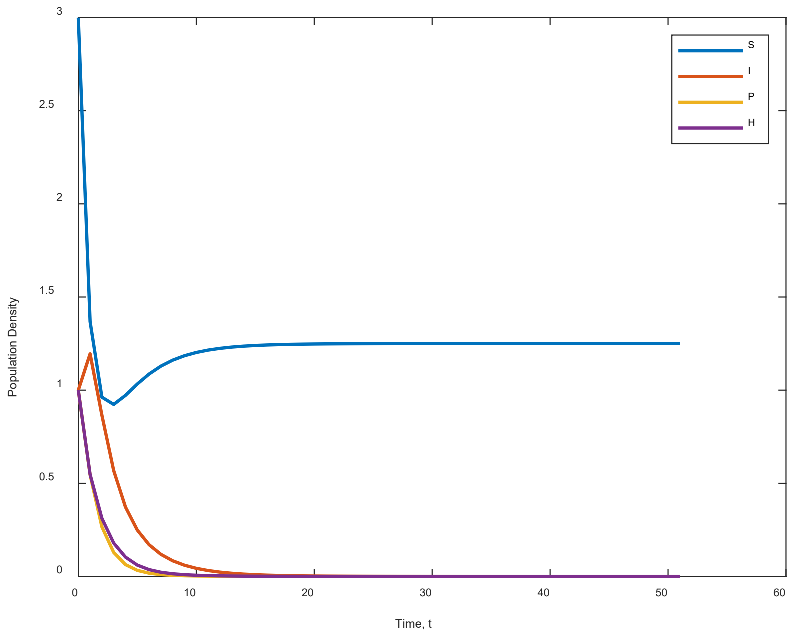

FIGURE 3. The population density of the newcomer in susceptible class, latent class, propagating class and the resisted class

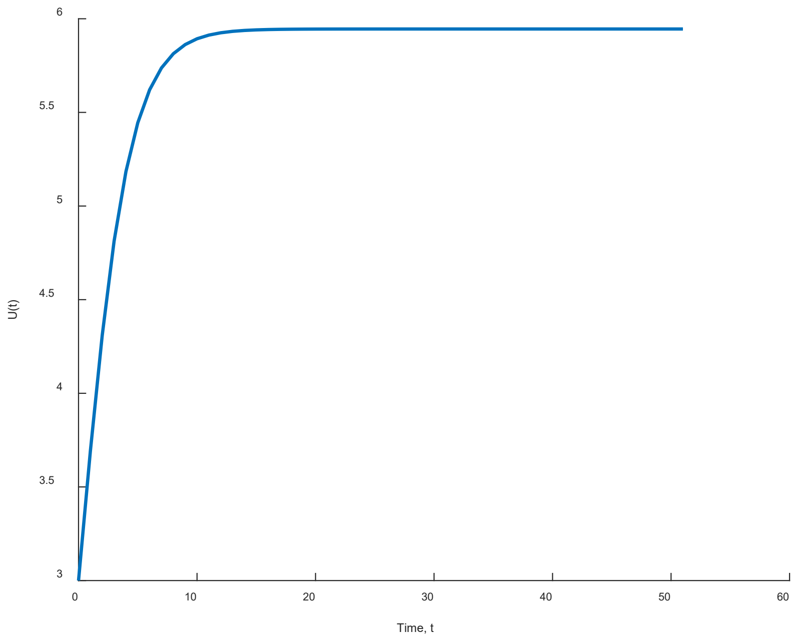

FIGURE 4. Inhibitor mechanism, $U(t)$

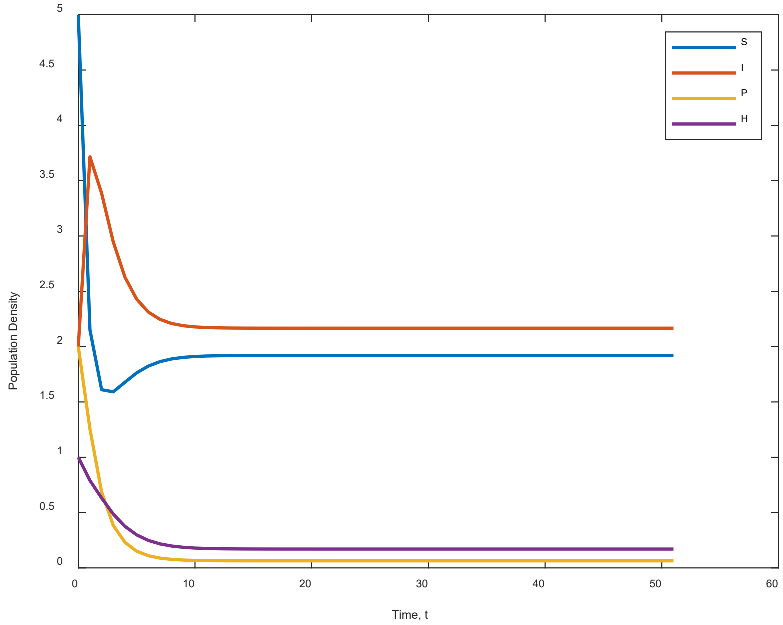

FIGURE 5. The population density of the newcomer in susceptible class, latent class, propagating class and the resisted class for $U(0) = 3$ and $r = 3$, $R_0 = 1.995$

Next, the rate of τ is varies as depicted in Figure 6(a), Figure 6(b) and Figure 6(c).

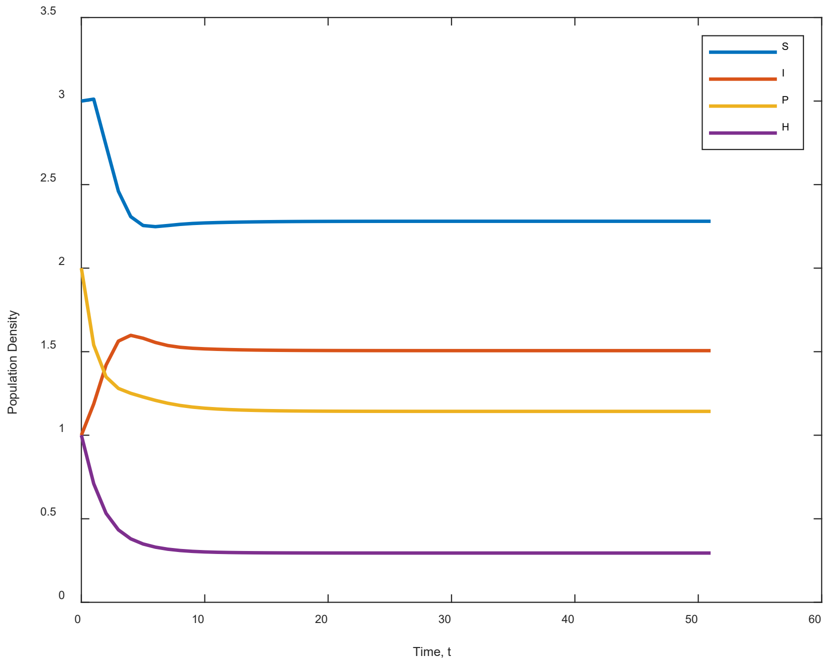

FIGURE 6a. $U(0) = 3$ and $\tau = 0.4$, $R_0 = 2.8155$

Based on Figure 6(a) - Figure 6(c), for the value of $\tau < 1.0$, the population of propagating individual will maintain at a positive constant, which means that the rumour propagation will be permanent at the respective equilibrium states. Next, the parameter rate of τ is varies such that $\tau > 1.0$ as depicted in Figure 7(a) and Figure 7(b).

In Figure 7(a), the population of propagating individuals sharply increases, indicating a rising trend in rumour spreading for $\tau = 1.0$. Conversely, for $\tau = 1.5$, the basic influence number is high, denoted as $R_0 = 4.1355$. The simulated results in this scenario show an instability of the solution at the equilibrium point, as illustrated in Figure 7(b). This leads to the conclusion that when the rate of the hesitant population transitioning to spreaders surpasses 1.0, the simulated results exhibit an increasing trend in propagation nodes and instability in the solution.

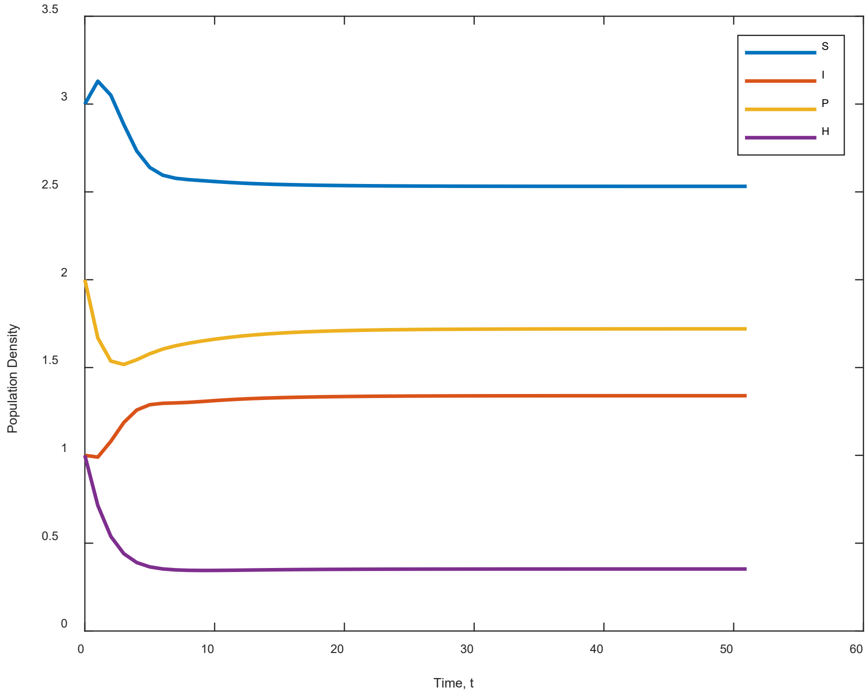

FIGURE 6b. $U(0) = 3$ and $\tau = 0.6$, $R_0 = 3.0555$

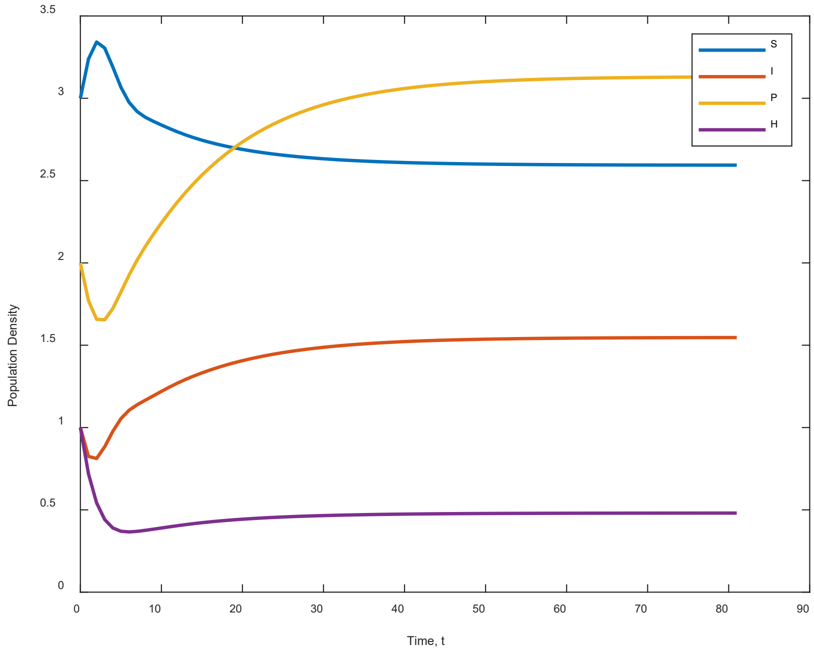

FIGURE 6c. $U(0) = 3$ and $\tau = 0.6$, $R_0 = 3.2955$

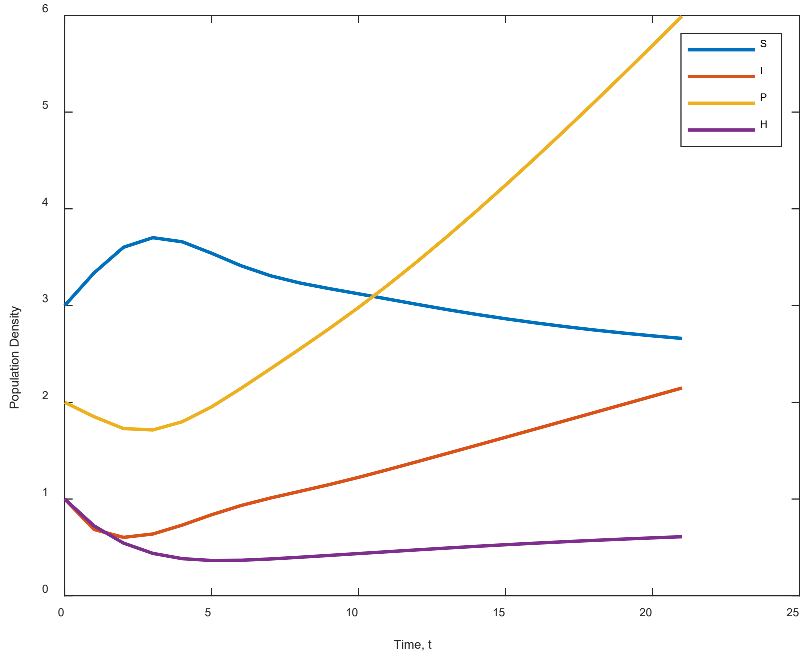

FIGURE 7a. $U(0) = 3$ and $\tau = 1.0$, $R_0 = 3.5355$

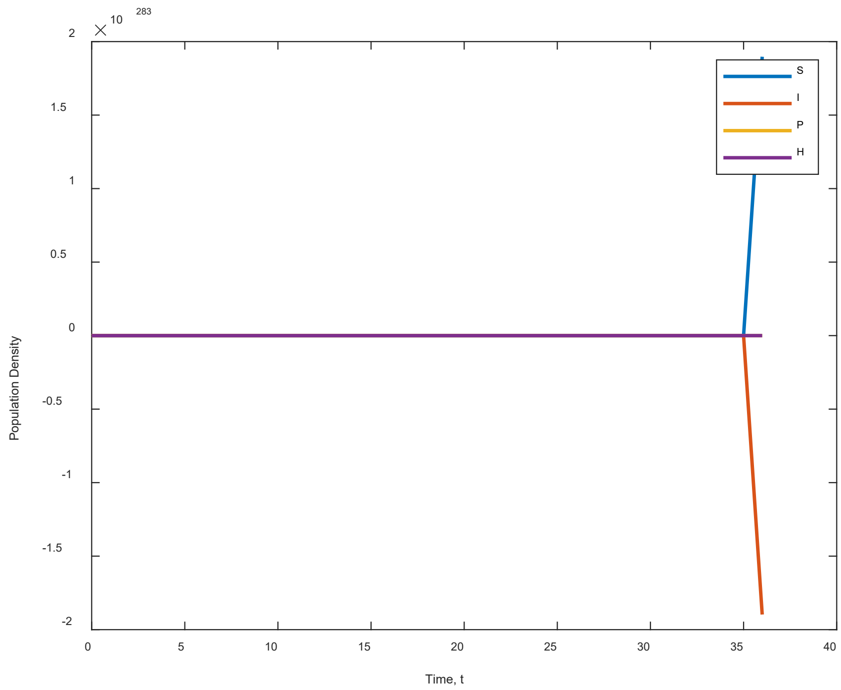

FIGURE 7b. $U(0) = 3$ *and* $\tau = 1.5$, $R_0 = 4.1355$

Conclusion

This study proposed a mathematical model of rumour spreading by extending the traditional Daley and Kendall (1964) model with the effects of the attitude control mechanisms. The explicit expression of the threshold parameter, the basic influence number is calculated. Then, the stability solution at disease free equilibrium and at rumour-prevailing equilibria are computed. R_0 provides insight into how the value r influences the potential of rumour spreading. This imply the basic influence number R_0 is non-increasing function of r. The parameter r mitigation factor to control the spread of the rumour. It can be concluded that as more budget is allocated for government control mechanism will lead to the less influence of the spreader in the system, hence able to control the spreading of the rumours. Furthermore, the spread of the rumours also being influenced by the constant parameter τ. For the value of $\tau > 1.0$, the simulated results exhibit an increasing trend in propagation nodes and instability in the solution.

Acknowledgements

The authors are grateful to the referees for their careful reading and well-thought suggestions which helped us improve this paper significantly. We acknowledge the financial support of the International Matching Grant UIC221517 (RDU222704), Universiti Malaysia Pahang Al-Sultan Abdullah for financial support.

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