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DYNAMIC MODELLING FOR ASSESSING THE IMPACT OF MARINE DEBRIS ON THE POPULATION OF SEA TURTLES

(Pemodelan Dinamik untuk Menilai Kesan Serpihan Marin Terhadap Populasi Penyu Laut)

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ABSTRACT

Marine debris has significant impacts on marine animals including the sea turtles, which are particularly vulnerable to the presence of waste in the marine environment. We propose a novel mathematical model with three compartments to examine this effect: the sea turtle population, the concentration level of pollution inside sea turtles' bodies, and the concentration level of pollution in marine environment. We locate the equilibrium points (also known as equilibria) for the suggested model and perform an analytical check on their stability. We also use the bifurcation analysis to examine how changing a model parameter affects the stability of the model's equilibria. Our findings demonstrated the existence of two equilibria: the sea turtles' survival equilibrium and their extinction equilibrium. The eigenvalues of the Jacobian matrix applied to the proposed model have been used to demonstrate the conditions for stability of these equilibria. The resulting bifurcation diagram demonstrates that both equilibrium points undergo transcritical bifurcations when the values of response intensity of toxicity parameter is varied. The findings of this study can help local or national governments make decisions and educate the public about sea turtle conservation in order to sustain sea turtle populations in the future.

Keywords: dynamics model; sea turtle; marine debris; stability; bifurcation

ABSTRAK

Serpihan marin mempunyai kesan yang besar terhadap haiwan laut termasuk penyu, yang sangat terdedah kepada kehadiran sisa di persekitaran marin. Kami mencadangkan model matematik baru dengan tiga petak untuk mengkaji kesan ini: populasi penyu laut, tahap kepekatan pencemaran di dalam badan penyu laut, dan tahap kepekatan pencemaran dalam persekitaran marin. Kami mencari titik keseimbangan untuk model yang dicadangkan dan melakukan pemeriksaan analisis terhadap kestabilan mereka. Kami juga menggunakan analisis bifurcation untuk mengkaji bagaimana perubahan parameter model mempengaruhi kestabilan keseimbangan model. Penemuan kami menunjukkan kewujudan dua titik keseimbangan: kewujudan penyu laut (terus hidup) dan kepupusan penyu. Nilai eigen yang diperolehi daripada matriks Jacobian telah digunakan untuk menunjukkan syarat-syarat untuk kestabilan titik-titik keseimbangan ini. Rajah dwicabangan yang terhasil menunjukkan bahawa kedua-dua titik keseimbangan menjalani dwicabangan jenis transkritikal apabila nilai parameter keamatan tindak balas divariasikan. Penemuan kajian ini diharapkan dapat membantu kerajaan tempatan atau nasional membuat keputusan dan mendidik orang ramai mengenai pemuliharaan penyu laut bagi mengekalkan populasi penyu laut pada masa akan datang.

Kata kunci: model dinamik; penyu; serpihan marin; kestabilan; dwicabangan

1. Introduction

Researchers predict that global sea turtle populations have reduced by up to two-thirds since the beginning of the Industrial Age in the early twentieth century. Only roughly 6.5 million sea turtles now inhabit the world's subtropical and tropical coastlines. Indeed, sea turtle populations are under threat: The International Union for the Conservation of Nature (IUCN) classifies three of the world's seven sea turtle species as "critically endangered"; green sea turtle populations have declined by 90 percent, while leatherback populations have declined by 40 percent (Iyer 2022). Besides the natural predators that threatened the sea turtles, human activities are also the major contributors which are currently threatening sea turtles at all life stages, both on nesting beaches and at sea (Tomas *et al.* 2002).

Marine debris is mostly caused by litter from ships, fishing and leisure boats, and waste thrown into the water from land-based sources in industrialised and densely populated areas (Derraik 2002). Debris and toxic waste deposited on the coast or at sea pollute the environment and endangers marine life. The debris comprises not only fisheries waste such as lines, plastic ropes, and nets, but also anthropogenic detritus such as plastic bags, six pack rings, tar, styrofoam, glass, and other things that might entangle or be consumed by sea turtles (Bugoni *et al.* 2001; Marn *et al.* 2020; Meaza *et al.* 2021).

According to Manes *et al.* (2023), fibropapillomas, a disease that is currently killing many sea turtles, may be connected to pollution in the oceans and near-shore waterways. Pollution contaminates and kills aquatic plant and animal life, which is typically food for sea turtles. Water contamination is caused by oil spills, urban runoff from chemicals, fertilisers and petroleum, and other factors. Because the ocean is so vast, many people mistakenly believe that pollutants will be diluted and spread to acceptable levels. However, the toxins produced by these pollutants become more concentrated as they degrade in size. As a result, numerous links in the food chain, including sea turtles, consume these tiny, more deadly particles.

There are numerous mathematical models for investigating the impactness of pollution on organisms. For example, Maystruk and Abdella (2011) proposed a mathematical model with five state variables: animal species, plant species, concentration of toxicant in individual animal, concentration of toxicant in individual plant and concentration of toxicant in environment. To see the effect of pollution on animals and plants, they varied the values of amount of pollution being input to the system. Their findings showed that when the input is low the coexistence between the organisms and pollution occurred. On the other hand, when the input is much higher, the number of animals and plants decreases, while the level of pollution increases.

Huang *et al.* (2020) used the Lotka-Volterra model to investigate predator-prey population dynamics in terms of toxicological response intensity (strength to population growth rate) to microplastic particles, as well as the negative effects on prey feeding ability and predator performance caused by microplastic particles. They discover four important response patterns of predator and prey dynamics to microplastic particles, which are the predator is more vulnerable than prey to the detrimental impact of microplastic particles, the coexistence of a predator and prey population is dependent on the presence of prey, and prey density is the key factor to ensure the stability of a predator-prey system, and the impact of microplastic particles is reduced when response intensity is low (values less than 1.0).

However, there is a gap in the literature regarding the mathematical modelling on the impact of pollution on sea turtles. Only a few studies have considered this research and these studies have become limited. Therefore, this research aimed to fill this gap by developing a mathematical model for investigating the dynamics of sea turtles with the presence of pollution. In fact, this study is motivated by Huang *et al.* (2020), where we modified their model to the case of sea turtle populations. In particular, the pollution is divided into two compartments: pollution in the sea turtles' bodies and pollution in the marine environment. We will perform stability analysis as well as bifurcation analysis for the proposed model. In this research, we are interested to investigate the effect of response intensity of toxicity by the sea turtles on their survivaliblity.

2. Methodology

In this section, we introduce some mathematical notations including equilibrium point, stability and bifurcation theory.

2.1. *Model development of mathematical model of sea turtle-pollution*

In this study, we consider a simple model which describes the effect of pollution in sea turtles' body and environment on sea turtle population. In our model, we assume that the population of sea turtles, $S(t)$, is increased by the natural growth rate r and may decreased due to natural death rate, d, and also due to response intensity of toxicity, m. Whereas, the concentration of pollution in sea turtles' body $P_S(t)$ is increased by the absorption rate of toxicity into sea turtles' body, b and it may decrease by egestion rate, g. Besides, the concentration of pollution in the environment $P_E(t)$ will increased by toxicity excreted by sea turtle, n and also the input rate of pollution by human activities, U.

From the assumptions made, the dynamics of sea turtles and pollution can be written as follows:

$$\begin{aligned} \frac{dS}{dt} &= rS - dS - mSP_S - nSP_S, \\ \frac{dP_S}{dt} &= bP_E - gP_S + mSP_S, \\ \frac{dP_E}{dt} &= nSP_S + U - bP_E, \end{aligned} \tag{1}$$

with initial data $S(0) > 0$, $P_E(0) > 0$ and $P_S(0) > 0$. All parameters in the above equation are assumed to be nonnegative. In the following section, we prove the positivity of the equilibrium points for model in Eq. (1). This proof is crucial to ensure that the solution is biologically meaningful. Table 1 listed the description of variables and parameters used in this model.

Table 1: Description of parameters for model in Eq. (1)

Symbol	Description
$\frac{dS}{dt}$	the rate of changes of the number of sea turtles
$\frac{dP_S}{dt}$	the rate of changes of concentration level of pollution in sea turtles' body
$\frac{dP_E}{dt}$	the rate of changes of concentration level of pollution in marine environment
r	natural growth rate of sea turtle population
d	natural death rate of sea turtle population
m	response intensity of toxicity
b	absorption rate of toxicity from environment into sea turtles' bodies
g	egestion rate
n	toxicity excreted by the sea turtles
U	input rate of pollution (marine debris)

2.2. *Equilibria and stability theory*

A special class of solutions of an autonomous initial value problem (IVP) are the equilibria (also called singular points, steady solutions or stationary solutions). The solutions which denoted by x^* are constant in time.

Definition 2.1. (Glendinning 1994) We say x^* is an *equilibrium point* of the IVP $\dot{x} = f(x)$ if any of the equivalent conditions hold.

(1) x^* is constant
(2) x^* solves the IVP for some $x(0)$ where $f\big(x(0)\big) = 0$
(3) $\dot{x} = 0$ for all t.

Hence, the equilibria are given simply by finding solutions x^* of the system of equations

$$f(x) = 0.$$

Now, we discuss the stability of equilibria. Consider $\dot{x} = Ax$ where $x \in \mathbb{R}^d$ and the matrix coefficient $A_{d\times d}$. Suppose that λ_i are eigenvalues of A, then we can classify x^* as follows:

Theorem 2.2. *(Glendinning 1994) Suppose that $Re(\lambda_i) < 0$ for all $i = 1, \dots, d$. Then x^* is stable.*

Theorem 2.3. *(Glendinning 1994) Suppose that $Re(\lambda_i) > 0$ for all $i = 1, \dots, d$. Then x^* is unstable.*

Theorem 2.4. (Glendinning 1994) *Suppose that at least one of the eigenvalues is positive. Then x^* is said to be saddle, where saddle implies unstable.*

Theorem 2.5. (Lynch 2014). *Suppose that $\lambda = \alpha \pm \beta i$, where λ is the complex eigenvalue. The conditions of stability of x^* is given as follows:*
(1) If $\alpha > 0$, then x^* is unstable spiral.
(2) If $\alpha < 0$, then x^* is stable spiral.
(3) If $\alpha = 0$, then x^* is a center.

We now define the term hyperbolic system.

Definition 2.6. (Glendinning 1994) A nonlinear system $\dot{x} = f(x)$ with equilibrium x^* is *hyperbolic* if and only if the Jacobian $Df(x^*)$ has no eigenvalues with zero real part.

In the result's section, we will prove that the model in Eq. (1) can also exhibit non-hyperbolic phenomenon in which there exists eigenvalue with zero real part.

2.3. Bifurcation theory

In this paper, we also investigate the stability of the equilibria as a system parameter is varied. This is crucial to study the impact of certain parameters on the system studied. Consider the following system:

$$\dot{x} = f(x, \mu), \qquad x \in \mathbb{R}^d, \mu \in \mathbb{R}^d,$$

where μ is a bifurcation parameter (i.e. the parameter where its values is varied).

Definition 2.7. (Glendinning 1994) Suppose that for some $\mu = \mu^*$ there is an equilibrium x^* that is not hyperbolic, i.e., $f(x^*, \mu^*) = 0$ and Jacobian $Df(x^*, \mu^*)$ has at least one eigenvalue with zero real part. The we say that (x^*, μ^*) is a *bifurcation point* for the system.

Definition 2.8. The plot of the state variable x versus parameter μ is called as a *bifurcation diagram*.

Classically, there are four common types of bifurcation for ODE system: saddle-node, transcritical, pitchfork and Hopf bifurcation. In this study, we consider the transcritical bifurcation for model in Eq. (1).

3. Analytical Results

In this section, we explain analytical results for equilibria and their positiveness, conditions of stability of equilibria as well as proving the transcritical bifurcation for model in Eq. (1).

3.1. *Equilibrium analysis*

To find the equilibrium points for model (1), we take the equations equal to 0 and solve the variables simultaneously for $S(t)$, $P_E(t)$ and $P_S(t)$. There are two possible equilibrium points for this model in the form of $E = \left(S^*(t), P_S^*(t),\ P_E^*(t)\right)$:

$$E_1 = \left(0, \frac{U}{b}, \frac{U}{g}\right), \tag{2}$$

$$E_2 = \left(\frac{U(m+n)-g(r-d)}{(m+n)(d-r)}, \frac{Um(m+n)-gn(d-r)}{b(m+n)^2}, -\frac{d-r}{m+n}\right). \tag{3}$$

From the above, it is obvious to see that the solution in E_1 is positive. However, we have to prove the positivity of equilibrium point E_2 to ensure that the solution is biologically meaningful which is concluded in Lemma 3.1.

Lemma 3.1. *Consider E_2 in Eq. (3) and rewrite it as $E_2 = \left(S^*(t), P_S^*(t),\ P_E^*(t)\right)$. $S^*(t) > 0, P_S^*(t) > 0,$ and $P_E^*(t) > 0$ if and only if $r > d$.*

Proof. For P_S to be positive, let

$$P_S^* = -\frac{d-r}{m+n} > 0.$$

Thus, it implies $r > d$. Then, for S to be positive, let

$$S^* = \frac{U(m+n)-g(r-d)}{(m+n)(d-r)} > 0.$$

Thus, it implies that $U(m+n) > g(r-d)$, which is true if $r > d$. Then, for P_E to be positive, let

$$P_E^* = \frac{Um(m+n) - gn(d-r)}{b(m+n)^2} > 0.$$

Thus, it implies that $Um(m+n) > gn(d-r)$ which is true if $r > d$. □

3.2. Stability analysis

The stability analysis for model in Eq. (1) involves evaluating the eigenvalues of the Jacobian matrix of partial derivatives with respect to the variables involved in the proposed model. The general formula of Jacobian matrix for proposed system is

$$J = \begin{pmatrix} \frac{\partial \dot{S}}{\partial S} & \frac{\partial \dot{S}}{\partial P_S} & \frac{\partial \dot{S}}{\partial P_E} \\ \frac{\partial \dot{P_S}}{\partial S} & \frac{\partial \dot{P_S}}{\partial P_S} & \frac{\partial \dot{P_S}}{\partial P_E} \\ \frac{\partial \dot{P_E}}{\partial S} & \frac{\partial \dot{P_E}}{\partial P_S} & \frac{\partial \dot{P_E}}{\partial P_E} \end{pmatrix},$$

where all the listed partial derivatives are with respect to S, P_S and P_E, respectively. Therefore, the Jacobian matrix for model (1) can be written as:

$$J = \begin{pmatrix} -(m+n)P_s - d + r & -S(m+n) & 0 \\ mP_s & mS - g & b \\ nP_s & nS & -b \end{pmatrix}. \tag{4}$$

Therefore, we have the following corollaries:

Corollary 3.2. *The equilibrium E_1 is said to be*

(1) *asymptotically stable if and only if $U(m+n) > -g(d-r)$,*
(2) *saddle if and only if $U(m+n) < -g(d-r)$, where saddle implies unstable.*

Proof. By substituting E_1 into Eq. (4) and substituting $\det(J_{E_1} - \lambda I) = 0$, we obtain the following eigenvalues:

$$\begin{aligned} \lambda_1 &= -g, \\ \lambda_2 &= -b, \\ \lambda_3 &= -\frac{U(m+n) + g(d-r)}{g}, \end{aligned} \tag{5}$$

where λ denotes the eigenvalue and I is the identity matrix. Obviously, both λ_1 and λ_2 are negative while λ_3 can be either positive or negative. To make E_1 asymptotically stable, λ_3 must be negative, Thus,

$$\frac{U(m+n) + g(d-r)}{g} > 0.$$

Then $U(m+n) + g(d-r) > 0$. Rearrange to get

$$U(m+n) > -g(d-r).$$

Vice-versa, to get that E_1 is saddle or unstable,

$$\frac{U(m+n)+g(d-r)}{g} < 0.$$

Then $U(m+n)+g(d-r) < 0$. Rearrange to get

$$U(m+n) < -g(d-r). \quad \square$$

From Corollary 3.2, if $d-r<0$, then E_1 is unstable, and if $d-r \geq 0$, then E_1 is stable. In order to check the stability for E_2, we use Routh-Hurwitz criterion by using $\det(J_{E_2} - \lambda I) = 0$ to obtain the following characteristic polynomial:

$$P(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0, \qquad (6)$$

where

$$a_0 = -\frac{b}{8}\left(U(m+n)+g(d-r)\right),$$

$$a_1 = -\frac{1}{4}\frac{U(b+d-r)m^2+\left(2n\left(b+\frac{d}{2}-\frac{r}{2}\right)U+g(d-r)^2\right)m+Ubn^2}{(m+n)(d-r)},$$

$$a_2 = -\frac{1}{2}\frac{\left(Um \quad (d-r)(b+g)\right)n+m\left(Um-b(d-r)\right)}{(m+n)(d-r)}.$$

Corollary 3.3. *The equilibrium E_2 is said to be:*

(1) locally asymptotically stable if and only if $a_0, a_1, a_2 > 0$ and $a_2a_1 > a_0$,

(2) unstable if otherwise.

Proof. From the Routh-Hurwitz criterion, the equilibrium E_2 is locally asymptotically stable if both criterion of $a_0, a_1, a_2 > 0$ and $a_2a_1 > a_0$ are satisfied. In particular, $a_0 > 0$ if

$$d-r > -\frac{(m+n)U}{g}.$$

Then, $a_1 > 0$ if

$$d-r > \pm\frac{1}{2}\frac{\left(\mp Um+\sqrt{U^2m^2-Ubgm}\right)(m+n)}{gm}.$$

Now, $a_2 > 0$ if

$$d-r > \frac{Um(m+n)}{b(m+n)+gn}.$$

If any of the above conditions are unsatisfied, then E_2 is unstable. □

3.3. Bifurcation analysis

In this section, we proved that the model in Eq. (1) undergoes a transcritical bifurcation. For model in Eq. (1), we choose parameter of response intensity of toxicity (m) as our bifurcation parameter. The result is proved in the following theorem.

Theorem 3.4. *The proposed model in Eq. (1) undergoes a transcritical bifurcation at* $m = m_c$*, where*

$$m_c = \frac{g(r-d)-Un}{U}.$$

Proof. For model in Eq. (1), we found that there are two equilibria, one at E_1 and another at E_2. So for $m < m_c$ and $m > m_c$ there are two equilibria and if $m = m_c$, there is only one equilibrium point (at $\left(S(t), P_E(t), P_S(t)\right) = (S^*, {P_E}^*, {P_S}^*)$). When $m = m_c$ and following from Definition 2.6, we need to prove that one of the eigenvalues from the Jacobian matrix is equal to zero, so that the equilibria are not hyperbolic. Thus, we equate the last eigenvalue from Eq. (5) to zero:

$$\frac{U(m+n)-g(r-d)}{g} = 0.$$

By simple calculation, we solve the above equation with respect to m and obtain the following:

$$m_c = \frac{g(r-d)-Un}{U}.$$

By substituting the above m, we obtained that $\lambda_3 = 0$. We expect to get a transcritical bifurcation for model in Eq. (1) in which the two equilibria E_1 and E_2 will exchange their stability at this bifurcation point m_c. □

4. Numerical Simulation Results

In this section, we discuss the numerical results for stability analysis, time series and phase portrait as well as for the bifurcation analysis, for deeper understanding about the behaviour of solution in model in Eq. (1) for long-term period.

4.1. *Stability results*

Here, we will discuss the behavior of equilibrium points obtained from the stability analysis of model in Eq. (1), mainly with three components: S as prey, P_S as middle predator and P_E as top predator. The critical point and eigenvalues were calculated with the help of the Maple software using the parameter values in Table 2. The stability investigation's findings are summarized in Table 2. The results of the types of stability based on eigenvalues are summarized in Table 3.

Table 2: Values of parameters for model in Eq. (1)

Parameter	Value	Source
r	0.2174	Mohd Roslan et al. (2019)
d	0.03	Assumed
m	[0,0.2]	Varied
n	0.03	Assumed
b	0.5	Assumed
g	1	Maystruck & Abdella (2011)
U	1	Maystruck & Abdella (2011)

The results are varying with different values of response intensity of toxicity, m. By considering $m = 0$ the equilibrium E_1 has one positive eigenvalue which indicate that E_1 is unstable by Theorem 2.4. On the other hands, E_2 has complex conjugate eigenvalues with negative signs of the real parts. Therefore by Theorem 2.5, E_2 denotes a stable spiral node, inferring that the system maintains a stable equilibrium and that even small minor perturbation will cause oscillations.

When $m = 0.1$, the equilibrium E_1 has different signs of eigenvalues. This E_1 indicates that the system is unstable and may display significant variations. Contrast with equilibrium E_2, the system is stable because all the eigenvalues have negative signs, which is in a good agreement with Theorem 2.2.

Then, by considering $m = 0.2$, equilibrium E_2 is omitted as it is not biologically significant. Negative equilibrium point signifies to the situations which one or more populations have negative values, which is not a biological meaningful scenario. Rather, the only equilibrium left is E_1, where this E_1 is stable. Overall, from Table 3 we can also see that E_1 from unstable to stable, whilst E_2 moves from stable to unstable, as m increases. We will show the critical value of m for which these equilibria exchange their stability in the section of bifurcation. The next section simulates the time series and phase portrait plot for the same values of m.

Table 3: Eigenvalues and types of stability for equilibria in model in Eq. (1) for varies value of response intensity of toxicity, m

Response intensity of toxicity, m	Equilibrium point	Eigenvalues	Types of stability
0	$E_1 = (0,2.0,1)$	$\lambda_1 = 0.1574$ $\lambda_2 = -0.5000$ $\lambda_3 = -1.0000$	Unstable
	$E_2 = (27.10,12.49,6.25)$	$\lambda_1 = -0.0091 + 0.2302i$ $\lambda_2 = -1.4818$ $\lambda_3 = -0.0091 - 0.2303i$	Stable spiral node
0.1	$E_1 = (0,2.0,1)$	$\lambda_1 = 0.0574$ $\lambda_2 = -0.5000$ $\lambda_3 = -1.0000$	Unstable
	$E_2 = (2.356,2.204,1.442)$	$\lambda_1 = -0.1077$ $\lambda_2 = -0.3174$ $\lambda_3 = -0.8392$	Stable
0.2	$E_1 = (0,2.0,1)$	$\lambda_1 = -0.0426$ $\lambda_2 = -0.5000$ $\lambda_3 = -1.0000$	Stable
	$E_2 = (-0.99,1.95,0.82)$	$\lambda_1 = 0.0336$ $\lambda_2 = -0.5271$ $\lambda_3 = -1.2041$	Unstable

4.2. *Time series and phase portrait*

In this section, the time series and three-dimensional phase portrait are plotted for different values of the response intensity m, as shown in Figure 1-3. The time series plotted in these figures used the same initial condition $\left(S(0), P_E(0), P_S(0)\right) = (2.0,0.5,1.0)$. Maple software is used to run the plots.When there is no response intensity of toxicity by the sea turtles ($m = 0$), the population of sea turtle increases and oscillate, as shown in Figure 1. At the same time, the levels of concentration for both inside the sea turtles' bodies and in the marine environment also show the same pattern as sea turtles. Although in the absence of response intensity, the pollution in the environment still absorbed into sea turtles' bodies. And this indicates that pollution in the water always exists since there is an inflow rate of pollution which mainly comes from the marine debris. The phase portrait on the right of Figure 1 shows that the equilibrium point $E_2 = (27.10,12.49,6.25)$ exhibits a stable spiral pattern, indicating that the system is stable, and the population of sea turtle would be sustained in the future.

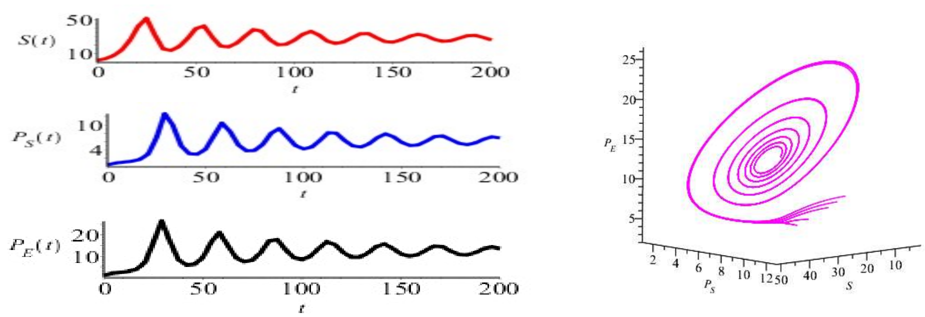

Figure 1: Time series and phase portrait for $m = 0$

When the response intensity exists and occurs with low rate ($m = 0.1$), the number of sea turtles shows decreases pattern and stabilize to an equilibrium point (see Figure 2). During this time, the amount of pollution inside the bodies is approximately the same as the amount of pollution inside the water. In this case, from the phase portrait plot, the stability of the equilibrium point E_2 also shows a stable spiral pattern. The initial conditions move towards the equilibrium point, indicating that the system is stable, and the sea turtle population, concentration level in both bodies and environment remain in balance.

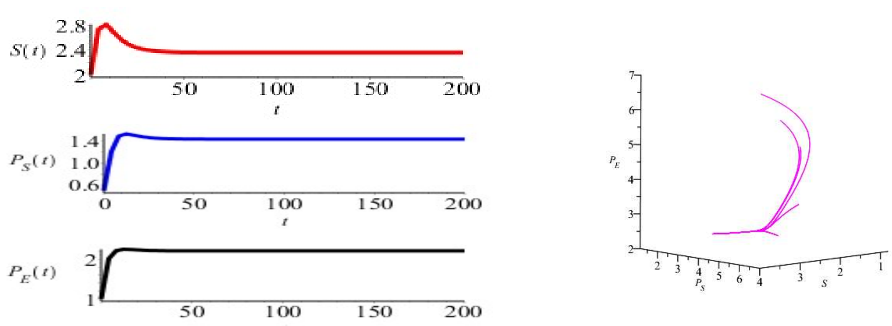

Figure 2: Time series and phase portrait for $m = 0.1$

However, for high intensity of response to toxicity ($m = 0.2$), the sea turtles are expected to extinct approximately in the next 50 years (see Figure 3). This is because increasing in response to toxicity leads to the extinction of sea turtle population. On the other hand, the pollution inside the sea turtles' bodies and in the marine environment remains in balance.

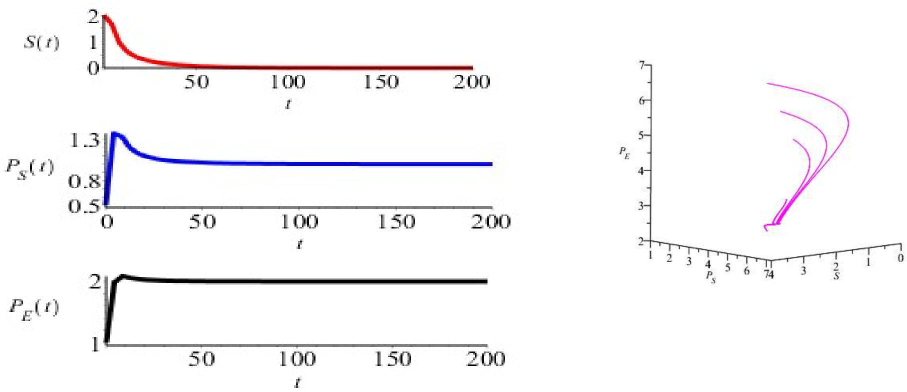

Figure 3: Time series and phase portrait for $m = 0.2$

These results demonstrate the critical role of response intensity of toxicity in model in Eq. (1). Thus, understanding the dynamics of model in Eq. (1) under different levels of this response is crucial for effective ecosystem management and conservation.

4.3. *Bifurcation results on the impact of response intensity of toxicity on sea turtles for model in Eq. (1)*

The dynamical behaviour of sea turtles, concentration level of pollution in sea turtles' body and concentration level of pollution in the environment towards the response intensity of toxicity (m) are discussed in this section. The mathematical software used for this purpose is XPPAUT. The parameter m was varied while the other parameters remained constant, as shown in Table 2. By employing XPPAUT software, one-parameter bifurcation analysis was performed, which resulted to one type of bifurcation phenomena called the transcritical bifurcation. Red lines in Figure 4-6 represents stable equilibrium point while black lines represent the unstable equilibrium point.

Figure 4 showed the bifurcation diagram for sea turtle population toward the response intensity of toxicity m. The straight line represents the equilibrium point E_1 (the extinction of sea turtles) while the curve shows the equilibrium point E_2 (the survival of sea turtles). At the transcritical bifurcation point, the two equilibria exchange their stability. In particular, E_1 changes from being unstable to stable while E_2 changes from being stable to unstable. Based on the formula in Theorem 3.4 and the values of parameters in Table 2, the transcritical bifurcation point is obtained as $m_c = 0.1574$.

In region I ($0 \leq m \leq m_c$), it can be observed that E_2 is stable. This indicate that the sea turtle population survived but shows the decrease in numbers as the response intensity on the toxicity increased, and finally the population hits zero as $m = m_c$. However, in region II, E_1 is now stable, which implies that the sea turtles are expected to extinct due to higher response to the toxicity ($m > m_c$).

On the other hand, Figure 5 and 6 shows the pattern of concentration level of pollution inside sea turtles' bodies (P_S) and concentration level of pollution in the marine environment (P_E) respectively, for response intensity $m \in [0,0.2]$. The findings showed that both patterns are similar. The concentration level for both P_S and P_E decreases in Region I and then in Region II the level remained at positive constant. In particular, the environmental pollution level decreases due to absorption into sea turtles' bodies.

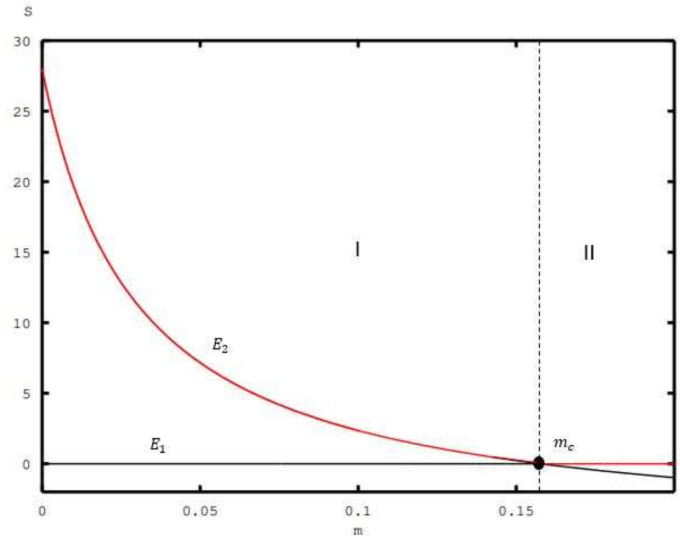

Figure 4: One-parameter bifurcation diagram for sea turtle population (S) versus response intensity of toxicity for $0 \leq m \leq 0.2$

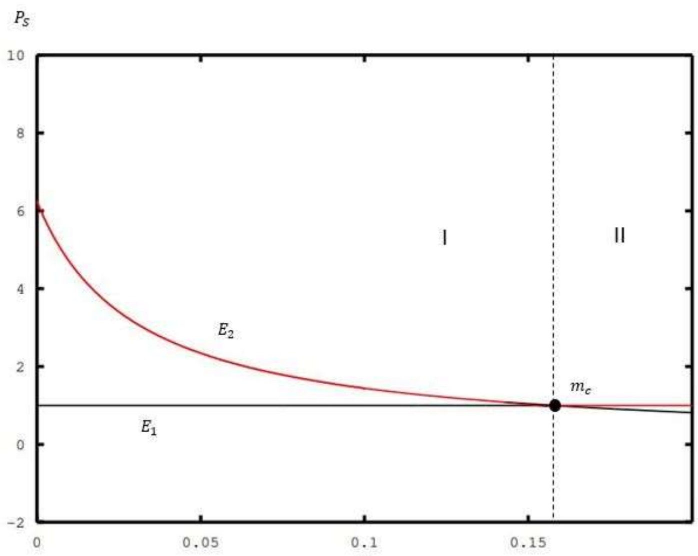

Figure 5: One-parameter bifurcation diagram for sea turtle population (P_S) versus response intensity of toxicity for $0 \leq m \leq 0.2$

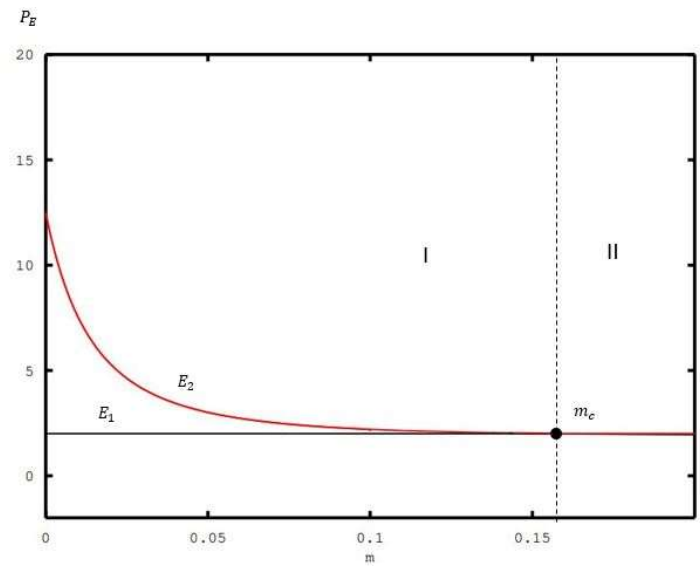

Figure 6: One-parameter bifurcation diagram for sea turtle population (P_E) versus response intensity of toxicity for $0 \leq m \leq 0.2$

5. Conclusion

In conclusion, we have studied the impact of marine debris on the sea turtle population in the context of response intensity by the sea turtles towards the toxic substance. We have done the analysis for the model proposed both analytically and by simulation. Our results demonstrated that the sea turtle population will be sustained even with a poor sensitivity to toxicity, albeit with a declining number of populations. This shows that the degree of reaction intensity has a significant impact on sea turtle survival. Therefore, understanding the model's dynamics at various levels of this response is therefore essential for efficient ecosystem management and conservation.

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