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Derivation of a Rotation Curve Model Based on Time of Events Theory and Its Application on Samples of Several Spiral Galaxies

(Terbitan Model Lengkung Putaran Berdasarkan Teori Masa Peristiwa dan Aplikasinya pada Sampel Beberapa Galaksi Lingkaran)

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ABSTRACT

A new model of rotation curves in spiral galaxies is derived and applied to several spiral galaxies. This model is based on a new theory called the 'time of events' theory, which was published elsewhere. The theory led to derive a multiscale model that applies to the problem of rotation curves in the galaxies as a non-adiabatic system. The equations of the derived multiscale model will be generalized in the current work to apply to the gases inside the galaxy to reach a general semi-empirical model that relates rotational velocity with the galaxy radius. The derived equation showed an excellent agreement with experimental results for five galaxies. The findings showed that the bulge plays a limited role for the galaxies with large extensions like IC 2574 and NGC 3198, where the dark matter in the halo region controls and determines the shape of the diagram of the rotation curves in these galaxies. Furthermore, in three bared galaxies NGC 1068, NGC 1097, and NGC 6503, the bulge has the most major role in controlling the rotation dynamics in these galaxies.

Keywords: Dark matter; rotation curve; spiral galaxies; time of events theory

ABSTRAK

Suatu model baharu lengkungan putaran galaksi berputar diperoleh dan diaplikasikan kepada beberapa galaksi berputar. Model ini adalah didasarkan pada teori baharu yang diperoleh daripada hasil terbitan artikel jurnal lain dan dikenali sebagai 'masa kejadian'. Teori ini dapat menghasilkan model berbilang skala yang diaplikasikan kepada masalah lengkungan putaran galaksi sebagai sistem bukan adiabatik. Persamaan model berbilang skala terbitan akan digeneralisasikan di dalam kertas ini untuk digunakan pada gas di dalam galaksi untuk mencapai model separa empirik umum yang mengaitkan halaju putaran dengan jejari galaksi. Persamaan yang diterbitkan menunjukkan keserasian yang sangat baik dengan keputusan uji kaji untuk lima galaksi yang dipilih. Penemuan menunjukkan bahawa tonjolan pusat galaksi memainkan peranan terhad untuk galaksi yang mempunyai tambahan seperti IC 2574 dan NGC 3198 dengan jirim gelap di kawasan lingkaran mengawal dan menentukan bentuk rajah lengkung putaran dalam semua galaksi tersebut. Sebagai tambahan, dalam tiga galaksi berpalang NGC 1068, NGC 1097 dan NGC 6503, tonjolan pusat galaksi memainkan peranan paling utama dalam mengawal dinamik putaran semua galaksi tersebut.

Kata kunci: Galaksi lingkaran; lengkung putaran; perkara gelap; teori masa kejadian

INTRODUCTION

The rotation curve in galaxies is one of the most active fields in modern astronomy. Understanding the gas dynamics and its relation to gas mass distribution with the presence of dark matter are some of the big challenges in this subject (Kauffmann et al. 2015; McGaugh 2020). In addition, the problem of halo extension and estimating the density profile for the halo which extends outside the optical radius of the galaxy mainly due to the existence of the dark matter and determining its total mass make the problem even harder to solve (Zasov et al. 2017). The high-resolution data served in offering precise information for galaxies masses and their rotation curves (Azeez et al. 2022, 2021, 2015). Dark matter's nature, despite forming about 95% of the mass in galaxies, is still not understood (Freese 2009). The cusp-core problem and the extreme differences in the rotation curve behavior between different galaxies lead to arise of so many models with different assumptions (Del Popolo & Le Delliou 2021). Each model can predict and interpret the rotation curve behavior of certain classes of galaxies, but cannot fit well with all types. Some works like Baes, Dejonghe and Buyle (2005) and Xu (2023) tested a so-called γ- model that assumes the density profile varies according to r^{γ} relation. They have tested different values for γ like 5/2, 3/2, 2/3, 0.5, 1, 2, 3. Other works handled the halo density profile like the quasi-isothermal halo model, which is associated with constant velocity dispersion due to self-gravitating objects in the halo (Mbelek 2004; Su et al. 2017). The Navarro, Frenk and White (NFW) model proved its excellence for many galaxies, it is characterized by a halo density profile given by Navarro, Frenk and White (1997):

$$\rho_{NFW}(r) = \frac{\rho_o}{\left(\frac{r}{r_o}\right)\left(1+\frac{r}{r_o}\right)^2} \tag{1}$$

where ρ_o and r_o are constants; r is the radius from the galaxy center. The density profile follows a r^{-1} for small radii; and r^{-3} for large radii (Lilley, Evans & Sanders 2018). Many other models have close behavior to NFW, except in large radii in which they have differences, like Hernquist model has ρ^{-4} profile, while super NFW model has $\rho^{-3.5}$ profile for large radii, respectively (Hernquist 1990; Lilley, Evans & Sanders 2018). Despite many models that handled the halo density profile in addition to bulge and disk density profiles, the interaction among the masses in these three regions seemed to be absent from the majority of models in the literature. In the present work, we will introduce a new rotation curve model that is derived from a multiscale model (Fadhil, Azeez & Hassan 2021), which is constructed from a new theory that is called the time of events theory (Fadhil, Azeez & Whahaeb 2014). The rotation curves for the galaxies are the most important tool to determine mass distribution in the galaxies (Sofue 2017, 2013). Therefore, mass distribution should be considered by rotation curves' models. The significance of the present model appears in its easiness of transferring among different scales without the need for complicated mathematical processes. The rotation curves in galaxies can be assumed to be a problem of a system of many particles consisting mainly of baryonic and dark matter. The rotation curve model will be derived by taking into account the interaction between different regions in the galaxy.

DERIVATION OF THE THEORETICAL MODEL

To handle the problem of the galaxy rotation curve and interpret its results in different galaxies, we need to understand the galaxy structure versus the dynamics of each structure. Considering the galaxy as an adiabatic system as it is usually the dominant approximation in most classical models may oversimplify the problem and hide many of its details (Binney & McMillan 2011). The problem with assuming the galaxy as an adiabatic system lies in missing the knowledge of its main and dominant structure which is the dark matter. The dark matter structure and its extension remain unknown, despite the huge advance in the technology of observation instruments (Frenk & White 2012). All these reasons suggest that the galaxy rotation curves should be manipulated as a non-adiabatic problem. The non-adiabatic problem even in classical mechanics may involve the violation of the total energy conservation in certain cases (Takatsuka 2018). Despite being unaware of the composition and extent of dark matter, the rotation curve clearly shows the gravitational effects of the dark matter. These effects are affected by the mass distribution of the dark matter around the galaxy. However, in all cases, the response to Hubble's expansion of dark matter should be the same (or close) to the galaxy baryonic matter in the

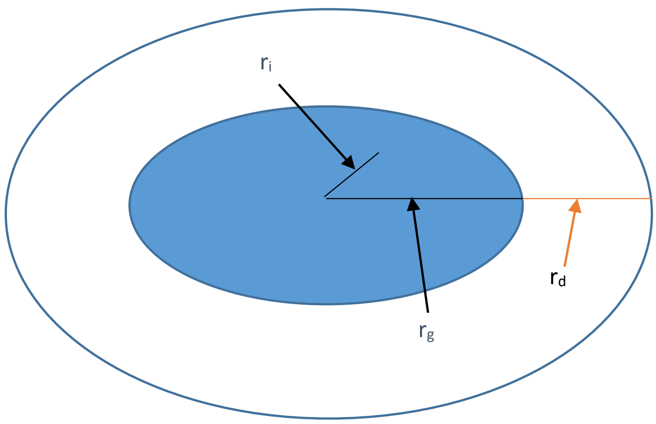

FIGURE 1. Schematic section diagram for a galaxy showing an assumed arrangement for the galaxy with radius rg surrounded by dark matter region with radius rd according to the present model

halo region, so that the galaxy structures are kept together versus time. Therefore, if the galaxy is assumed as a lever in an equilibrium state then for all baryonic and dark matter the summation of the ratio of the kinetic energy over associated mass and potential energy over associated mass should equal the ratio of the total energy over associated mass; as a new multiscale model (Fadhil, Azeez & Hassan 2021) derived from time of events theory proposed (Fadhil, Azeez & Whahaeb 2014). Figure (1) shows a section diagram for a galaxy that the current model is intended to apply to.

According to the present theory, the equation of energy conservation needs to be written as follows for a non-adiabatic system in the case of equilibrium (Fadhil, Azeez & Hassan 2021):

$$\sum_{i=1}^{n} \frac{K_i}{m_i} + \sum_{i=1}^{n} \frac{1}{m_i} \sum_{j=1}^{p} V_{ij} = \sum_{i=1}^{n} \frac{E_i}{m_i} \quad (2)$$

where K_i and V_{ij} are the kinetic and potential energy respectively; E_i is the total energy; m_i is the mass of the i^{th} particle. The kinetic and potential energy are given by:

$$K_i = \frac{1}{2} m_i v_i^2 \quad (3)$$

$$V_{ij} = G \frac{m_i m_j}{r_{ij}} \quad (4)$$

where v_i is the velocity of the i^{th} particle; and r_{ij} is the distance between the i^{th} and the j^{th} particle.
Substituting Equations (3) and (4) in Equation (2) yields after simplification:

$$\sum_{i=1}^{n} \frac{v_i^2}{2} + G \sum_{i=1}^{n} \frac{1}{m_i} \sum_{j=1}^{p} \frac{m_i m_j}{r_{ij}} = \sum_{i=1}^{n} \frac{E_i}{m_i} \quad (5)$$

For any system under equilibrium with no changes with time, the total energy should be constant versus time, i.e., $\frac{dE_i}{dt} = 0$, therefore Equation (5) becomes:

$$\sum_{i=1}^{n} v_i \frac{dv_i}{dt} - G \sum_{i=1}^{n} \frac{1}{m_i} \sum_{j=1}^{p} \frac{m_i m_j}{r_{ij}^2} \frac{dr_{ij}}{dt} = 0 \quad (6)$$

If the distance between the particles i and j is constant then only the rotational component of velocity will survive, therefore, the term $\frac{dr_{ij}}{dt}$ can be expressed as: $\frac{dr_{ij}}{dt} = r \frac{d\theta_{ij}}{dt} \approx v_i$. Also, the acceleration will be a centripetal acceleration given as $\frac{dv_i}{dt} = \frac{v_i^2}{r_i}$, where r_i is the distance from the center of the galaxy to the particle under consideration. Substituting these assumptions in Equation (6) yields:

$$\sum_{i=1}^{n} v_i^2 = G \sum_{i=1}^{n} \frac{1}{m_i} \sum_{j=1}^{p} \frac{m_i m_j}{r_{ij}^2} r_i \quad (7)$$

To find a relation between v and r from Equation (7), we will study several conditions for the equation. First, for the transition region between the central and the halo regions, if this region is divided into partitions each partition is associated with a certain density ρ_i, i.e., for m_i α $r_i^3 \rho_i$ and m_j α $r_j^3 \rho_j$. As a special case, the term m_i in the outer summation of i (which represents the gas mass for the sample under study) can be assumed constant versus r if the partitions are taken with equal masses and due to the balance between inner (central region) and outer (halo) region the outer summation term can be considered independent on r. Considering these assumptions then the right side of Equation (7) becomes:

$$\frac{\rho_j {r_j}^3 {r_i}^3 r_i}{{r_i}^2} = \frac{\rho_j {r_i}^7}{{r_i}^2} = \rho_j {r_i}^5 \quad (8)$$

where r_j is assumed close to r_i. ρ_j is the density of the region under study. Usually, this region appeared as the end of a peak in the rotation curve for certain galaxies or the start of the plateau curve for others.

For the central region; all the mass terms within the summation are important, therefore, we cannot neglect the term $\sum_{i=1}^{n} \frac{1}{m_i}$. Assuming the same radii for all the masses terms within the summation, also near the center $r_{ij} \sim r_i$, and both radii are close to the radius of the assumed masses' zones, substituting these assumptions in Equation (7) yields:

$$\sum_{i=1}^{n} \sum_{j=1}^{p} \frac{{r_i}^3 \rho_i {r_j}^3 \rho_j}{{r_i}^3 \rho_i {r_i}^2} r_i \rightarrow \frac{\rho_j {r_i}^7}{{r_i}^5} \approx \rho_j {r_i}^2 \quad (9)$$

This condition is more applicable near the center of the galaxy as isotropic models proposed (Baes & Dejonghe 2004; Baes, Dejonghe & Buyle 2005). Assuming average ρ_j for every specific zone makes it semi-constant within the studied zone for a certain r in the galaxy, therefore, the terms of the summation approach r_i^2 as written above for the zone under consideration. Near the edge of the galaxy, the effect of dark matter attraction will arise in addition to the attraction from the masses within the galaxy, therefore, r_{ij} should be replaced by r where r is given by r_i-r', where r' is the distance between a mass affecting the mass under study by gravity and the center of the galaxy. In the case of dark matter, the term r_i-r' will be negative, because $r' > r_i$, so the negative terms will be in equilibrium with the positive terms near the edge. At the same time, the term r_i is close to r for the masses near the edge, accordingly, the term $\sum_{i=1}^{n} \sum_{j=1}^{p} \frac{m_i m_j}{m_i r^2} r \rightarrow \frac{1}{r}$ near the edge of the galaxy.

Therefore, the summation will yield three main terms according to different regions in each galaxy, which are r_i^5 which is in the transition region between the central and halo regions. The second is the central region term which is proportional with r_i^2. The third is the near edge term is proportional with $1/r_i$. This term should interpret the semi-plateau behavior near the galaxy edge. This leads to

$$v_i^2 \propto (r_i^5, r_i^2, \frac{1}{r_i}) \quad (10)$$

These three terms can be considered as independent components and, therefore, can be written in the matrix form as:

$$v^2 = vv' = \begin{bmatrix} \alpha_1 r^{5/2} & 0 & 0 \\ 0 & \alpha_2 r & 0 \\ 0 & 0 & \alpha_3 \frac{1}{r^{1/2}} \end{bmatrix} \begin{bmatrix} \alpha_1 r^{5/2} & 0 & 0 \\ 0 & \alpha_2 r & 0 \\ 0 & 0 & \alpha_3 \frac{1}{r^{1/2}} \end{bmatrix} \quad (11)$$

where α_1, α_2, and α_3 are constants found from the fitting of the experimental data, assuming independent components which is an ideal case. In reality, there can be more than one compon1.0ent found to act at the same time. For a general case, all the probabilities will be considered, i.e., assuming the first term and the second term act together will yield another term in the diagonal of the matrix and will increase its dimension to 4×4. Multiplying the first term with9 the third, the third with the second, and the three terms together will make the matrix has a dimension of 7×7, so v can be given

$$v= \begin{bmatrix} \alpha_{11} r^{\frac{5}{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_{22} r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_{33} \frac{1}{r^{\frac{1}{2}}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_{44} r^{\frac{7}{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_{55} r^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_{66} r^{1/2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{77} r^3 \end{bmatrix} \quad (12)$$

where α_{11}, α_{22}, α_{33}, α_{44}, α_{55}, α_{66} and α_{77}, in Equation (12), are respectively, the corresponding fitting constants for the first term, second term, third term, multiplying the first and third, second and third, and multiplying the first three terms. These terms implicitly contain the density and are proportional to it in each region, however, handling density and its variation versus r will be left to another work.

Equation (12) can be re-written in the scaler form as:

$$v = \alpha_{11} r^{\frac{5}{2}} + \alpha_{22} r + \alpha_{33} \frac{1}{r^{\frac{1}{2}}} + \alpha_{44} r^{\frac{7}{2}} + \alpha_{55} r^2 + \alpha_{66} r^{1/2} + \alpha_{77} r^3 \quad (13)$$

RESULTS AND DISCUSSION

The present work involves applying Equation (13) to several samples of spiral galaxies. After performing the fitting process using MATLAB software the fitting parameters are shown in Table (1). The fitting curves with the error bars are shown in Figures (2) to (6).

From Figures (2) and (3) with Table (1), the rotation curves for the first two galaxies IC 2574 and NGC 3198 are mostly affected by the term α_{66}, which represents the intermediate region between the central and the halo region. This factor is associated with a drop in density equal to 0.5. The term α_{22} itself has the highest value among the first three terms, indicating that the high mass central region in both galaxies is the main factor that affects the rotation curves in these galaxies. This conclusion is supported by noticing that all the factors that the factor α_{22} participates in have the highest values for fitting parameters which includes mainly α_{66} and α_{55}. The transition region near the bulge has a relatively small coefficient which indicates a slow variation in gas density, this may be due to the relatively large extent of both galaxies that is represented by their large radius in addition to probably a large percentage of dark matter in both galaxies that largely affected the gas dynamics (Karukes et al. 2015; Oman et al. 2015).

The last three galaxies; NGC 1068, NGC 1097, and NGC 6503 have the largest values mainly in the parameters α_{11}, α_{77}, and α_{55} which indicates the large effects of the transition region in these galaxies. Compared to the first two galaxies, the last three ones have a narrow transition region between the central and halo regions, which may lead to large interactions in the central region between the central and halo regions. Therefore, we note large values for the terms that are related to transition regions in the last three galaxies.

The factors α_{66} and α_{33} have the lowest values in these galaxies indicating the minor effects of dark matter in these galaxies on rotation curves. NGC 1068 was shown in Schartmann et al. (2010) to have an effective interaction between the disk and its surrounding gas mass. Therefore, the transition region term has strong interactions with other terms in the surrounding regions leading to high values for the α_{77}, and α_{55} terms.

TABLE 1. Fitting parameters for samples of spiral galaxies

Galaxy name	α_{11}	α_{22}	α_{33}	α_{44}	α_{55}	α_{66}	α_{77}	α_{88}	Adjusted R-square
IC 2574	84.78 (36.97, 132.6)	387.2 (145.9, 628.4)	-58.91 (-102.5, -15.29)	1.625 (0.7224, 2.527)	-150.1 (-237, -63.22)	-528.5 (-889.4, -167.7)	-19.34 (-30.09, -8.587)	297 (85.29, 508.8)	0.9981*
NGC 3198	-4.856 (-14.09, 4.38)	-102.9 (-302.2, 96.46)	7.059 (-160.5, 174.6)	-0.04121 (-0.1083, 0.02588)	13.35 (-13.78, 40.49)	286.9 (-203.9, 777.7)	0.7335 (-0.5543, 2.021)	-127.6 (-607.7, 352.4)	0.9963
NGC 1068	2.423e+06 (1.652e+06, 3.194e+06)	1.342e+05 (8.933e+04, 1.79e+05)	-271.7 (-376.2, -167.1)	9.105e+05 (6.175e+05, 1.203e+06)	-9.829e+05 (-1.297e+06, -6.684e+05)	-4.408e+04 (-5.954e+04, -2.862e+04)	-2.435e+06 (-3.211e+06, -1.658e+06)	5783 (3660, 7907)	0.959
NGC 1097	-2.732e+04 (-4.865e+04, -5996)	-9267 (-1.546e+04, -3077)	-1.583 (-24.75, 21.58)	-2434 (-4567, -299.6)	2.175e+04 (5805, 3.77e+04)	4219 (1258, 7179)	1.351e+04 (2309, 2.472e+04)	-213.2 (-684, 257.6)	0.8602
NGC 6503	-2.46e+05 (-4.678e+05, -2.416e+04)	-2.081e+04 (-3.439e+04, -7230)	50.19 (15.9, 84.47)	-7.094e+04 (-1.532e+05, 1.137e+04)	1.145e+05 (2.278e+04, 2.063e+05)	7851 (3061, 1.264e+04)	2.161e+05 (-4469, 4.368e+05)	-1002 (-1678, -326.4)	0.977

*The NFW adjusted R-square for this galaxy is 0.9526

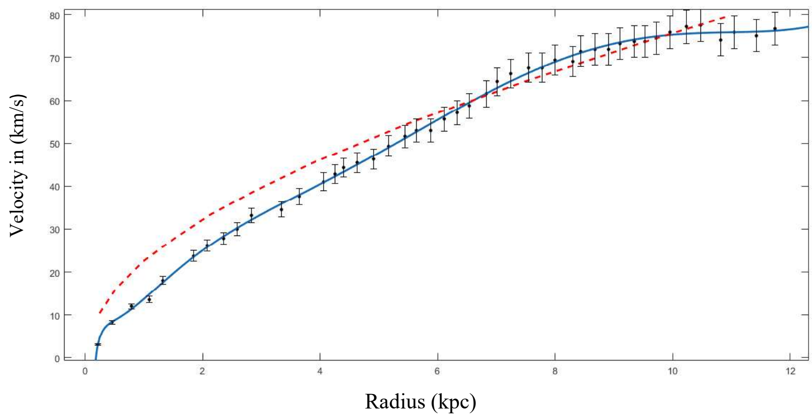

FIGURE 2. The rotation curve for the galaxy IC 2574. The experimental data are represented in dots. The solid line represents the fitting from Equation (13) with the error bars shown. The red dashed line represents the fitting from the NFW model

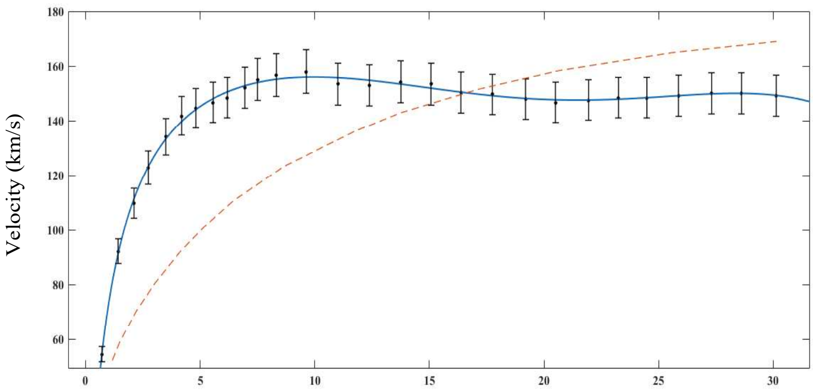

FIGURE 3. The rotation curve for the galaxy NGC 3198. The experimental data are represented in dots. The solid line represents the fitting from Equation (13) with the error bars shown. The red dashed line represents the fitting from the NFW model

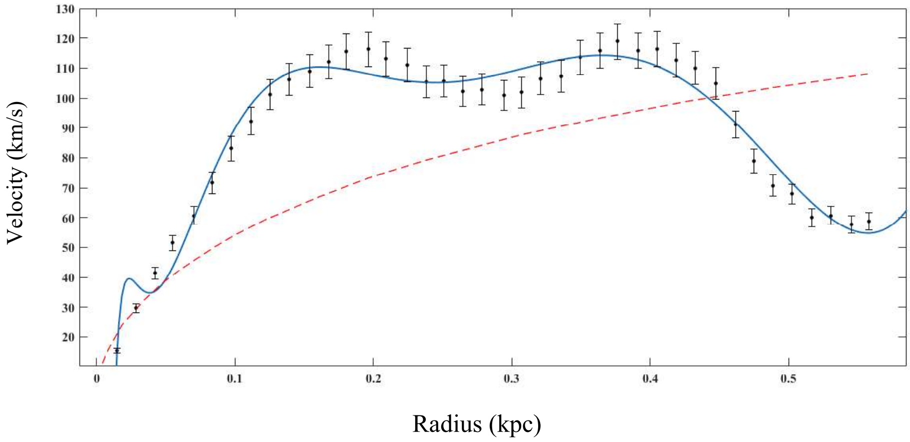

FIGURE 4. The rotation curve for the galaxy NGC 1068. The experimental data are represented in dots. The solid line represents the fitting from Equation (13) with the error bars shown. The red dashed line represents the fitting from the NFW model

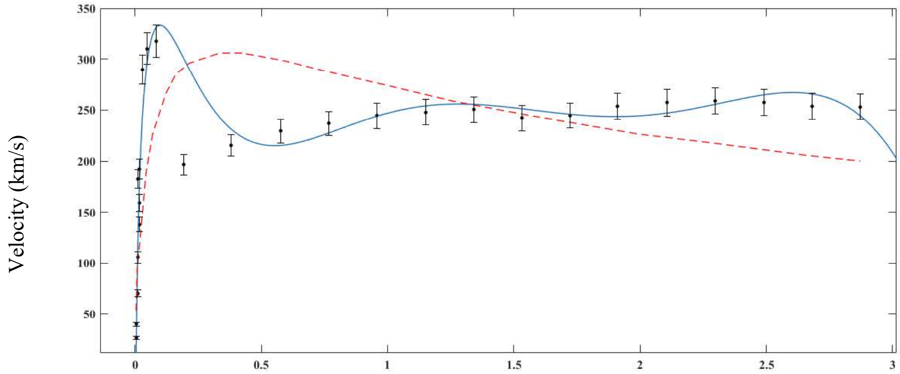

FIGURE 5. The rotation curve for the galaxy NGC 1097. The experimental data are represented in dots. The solid line represents the fitting from Equation (13) with the error bars shown. The red dashed line represents the fitting from the NFW model

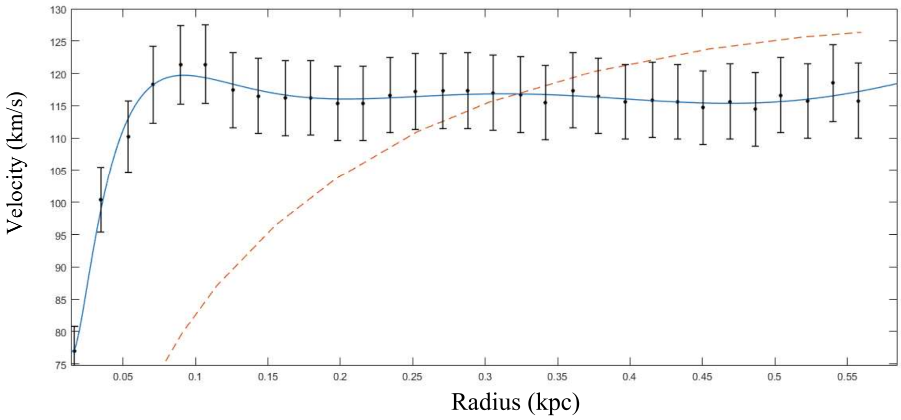

FIGURE 6. The rotation curve for the galaxy NGC 6503. The experimental data are represented in dots. The solid line represents the fitting from Equation (13) with the error bars shown. The red dashed line represents the fitting from the NFW model

Regarding the galaxy NGC 1097, many works indicated a strong interaction between the lanes and the nuclear region, where the gas sometimes moves from both lanes to the nuclear region and part of it continues moving to the opposite lane (Lin et al. 2013; Sormani et al. 2023). This agrees with the results of the present model as noticed with high values for the terms α_{77} and α_{55}. The galaxy NGC 6503 in spite of its simple rotation mechanism, the studies indicated that there is a bar for the Type II surface brightness profile interpretation (Kuzio de Naray et al. 2012). The last three galaxies have a common factor having a bar which leads to an increase in the interaction between the bulge and its surrounding regions.

In all Figures from (2) to (6) we have used the NFW model equation (Navarro, Frenk & White 1997) of the rotation curve for comparison with the present model. The NFW model applies well only for the IC 2574 galaxy due to its simple profile that can be described through the cuspy halo approximation. For the rest of the five galaxies, the NFW model predictions were far from the experimental data. This is due to the complicated profiles of these galaxies. The adjusted R-square from Table (1) shows good applicability of the predicted fitting curve of the present derived model with the experimental results for all galaxies. For the IC 2574, the adjusted R-square value of the present model is better than the NFW model. It is clear from Figures (2) to (6) that the NFW model becomes farther from the experimental results in the central and halo regions, while the present model predictions are close to observed results through all regions within the galaxy.

CONCLUSIONS

In the present work, a new model for rotation curves was derived and used to fit the rotation curve for five galaxies. The model is mainly depending on three parameters, where each one is related to a certain factor or region in the galaxy. The $r^{(5/2)}$ is related to the transition region between the bulge and halo region, the second term the r is mainly related to the central region, and the third term $r^{(-1/2)}$ is related to the halo region where dark matter is the main influencer. All three terms interact with each other; therefore, another four interaction terms are assumed. One of the advantages of the present work is its ability to relate each term to its main region of action. At the bulge region, the major effective terms are the first and the second (due to the foundation of high mass regions) and the fourth term from the interaction of the first and the second. At the disk region, and because it represents a transition region between the bulge and the halo, the effective terms are the first and one that is yielded from the interaction between the first and the third term. The halo is mostly affected by the third and the sixth terms. Applying the derived model to the five galaxies brings interesting results. In the galaxies that have extended arms IC 2574 and NGC 3198, bulges have minor effects on the galaxy dynamics excluding the central region. While with the bared galaxies NGC 1068, NGC 1097, and NGC 6503, the transition region between bulge and halo plays a crucial role in the galaxy dynamics and its evolution. The present model showed an excellent description of the experimental data for all galaxies, while the NFW model applied well only for the IC 2574 galaxy.

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