

## Uncertainty Factors of a Finite Element Model using the Fuzzy Analysis Method

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### ABSTRACT

*The recent advancement in manufacturing technology in the automotive and aerospace sectors has led to the invention of advanced structured material, which is lightweight and a complex geometry model that can be manufactured. As it is related to human safety and hazards, the need for uncertainty analysis in a structure before and after a manufacturing process is a primary concern. Thus, this paper analyzes the uncertainty parameters of a meshed finite element model in the geometry, boundary condition, load, and material properties. An uncertainty analysis numerical tool, the fuzzy analysis method, is applied in Excel-VBA as the simulation platform. Each uncertainty parameter is in a range of numbers, with a maximum and minimum value as the limit. The  $\alpha$ -cuts determine the fuzzy analysis output on the membership function. The deterministic value of the variable is implemented for comparison purposes. The simulation result for the von-Mises stress analysis has significantly impacted the uncertainty analysis as its curve has surpassed the yield strength limit of the material. The simulation output for the displacement has a more considerable uncertainty dispersion when compared to the other results. This study helps to find a better security margin of a structure for its sustainability in the future.*

*Keywords:*  $\alpha$ -cut; finite element method; fuzzy analysis; structural integrity; uncertainty

### INTRODUCTION

The latest technological advancement in manufacturing methods has been implemented, especially in the automotive and aeronautic sectors. These new approaches, for example, the rapid prototyping method, are gaining popularity among large manufacturing companies as it helps to produce lightweight end products and can manufacture complex geometry components (Seharing et al. 2020). Giant aeronautic producers such as Boeing, Airbus, and many other automotive companies are establishing this method to produce their high-end products (Jin et al. 2022) as it helps to increase production rate and decrease manufacturing time and cost (Vasilescu et al. 2020).

Despite these technological advancements, structural longevity and sustainability play a vital part, especially

when it involves the safety of the users. This is crucial for large structures such as buildings and bridges as they are surrounded by uncertain environments and conditions. These uncertainties are due to the scarcity of information during the manufacturing and modeling phase (Stritih et al. 2019), the law of nature, and human heuristics (Li et al. 2018). Thus, errors are generated when dealing with material attributes, boundaries, and initial states in experimental and engineering scenarios (Hariri-Ardebili et Sudret, 2021).

To overcome these uncertainties, researchers have come out with numerical and mathematical methods such as Neuro-Fuzzy, Artificial Neural Network (ANN), and Fuzzy analysis methods. Each method applies a range of random numbers as the uncertainty parameters. These random numbers of minimum and maximum values are easily obtained compared to specific deterministic values

(Qiu and Ju 2022; Sudin et al. 2020). According to Dahri et al. (2022), ANN can produce output with incomplete knowledge and store information on the network. Neuro-fuzzy method has a greater choice of membership function and a better convergence time (Pezeshki et Mazinani 2022; Alas 2022). However, these methods are hardware dependent, as a complex neural network requires a vast amount of computational effort and simulation time (Ghenai et al. 2022). Interval-based fuzzy analysis has the smallest computational cost and acceptable output accuracy for a problem with a low level of uncertainty (Pham et al. 2020).

Moreover, Patle et al. (2018) have indicated a good correlation between the fuzzy analysis approach in the finite element method, which includes uncertainty properties in the model. Furthermore, the fuzzy analysis approach has significantly improved the accuracy of the finite element method output (Li et al. 2022). Uncertainties in geometric parameters such as Young's Modulus and Poisson's ratio are easily quantified and propagated in stochastic non-probabilistic approach, for example, the fuzzy sets (Xu et al. 2019). Thus, this paper aims to analyze and determine the uncertainty factors that exist in the boundary condition and a load of a finite element model by applying the fuzzy analysis. The model's material attributes and applied force are the fuzzy parameters. The analysis output helps to determine the safety margin and failure of a finite element structure. The influence of fuzzy parameters on the resulting output is discussed in this paper.

## RESEARCH DEVELOPMENT

### UNCERTAINTY MODEL APPROACH

Uncertainty solution technics are distinguished into two types; the probabilistic and non-probabilistic methods (Karsch et al. 2019). The fuzzy analysis method is a non-probabilistic uncertainty approach that involves fuzzy numbers mapping via  $\alpha$ -cut based computational process and has fuzzy sets as the output (Baykasoğlu et Gölçük 2021). Based on the mapping procedures, the output is defined as lower and upper bound values representing the fuzzy sets. These fuzzy sets can interpret numerical parameters into membership functions representing a precise solution of a complex system. Figure 1 shows  $f(P)$  as the fuzzy member function and the lower and upper bounds of the fuzzy output of element P.

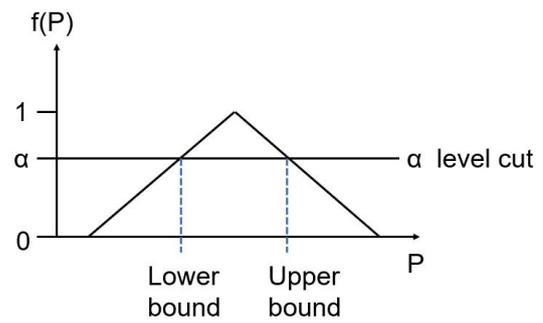


FIGURE 1. Fuzzy Output

A steel plate with a hole in the middle part is applied for Finite Element Method (FEM) purposes. This method analyses the finite element model of the plate to determine its stress and displacement. Excel-VBA is the platform for this FEM simulation as it can run uncertainty analysis simulations by including fundamental loops, logical statements, input and output, user functions, and subroutines (Kalwar et al. 2022; Abdullah, 2021). Due to the symmetrical effect, the simulation work only requires a quarter of the plate. Figure 2 represents the plate with its boundary state, dimension, and a distributed applied force on top of it. The model is a two-dimensional (2D) model; the quarter plate model is meshed and has 22 nodes and 26 elements. Figure 3 illustrates the FEM model of the plate. The number of mesh and nodes is the optimum number for the computational effort in this work.

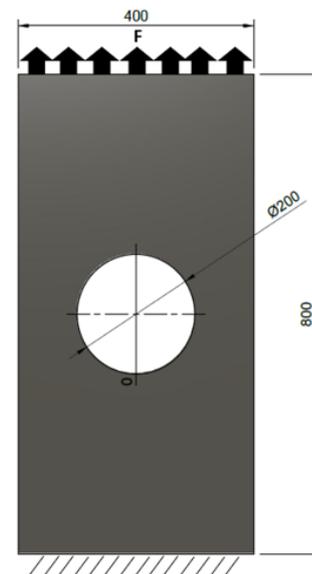


FIGURE 2. Plate Model

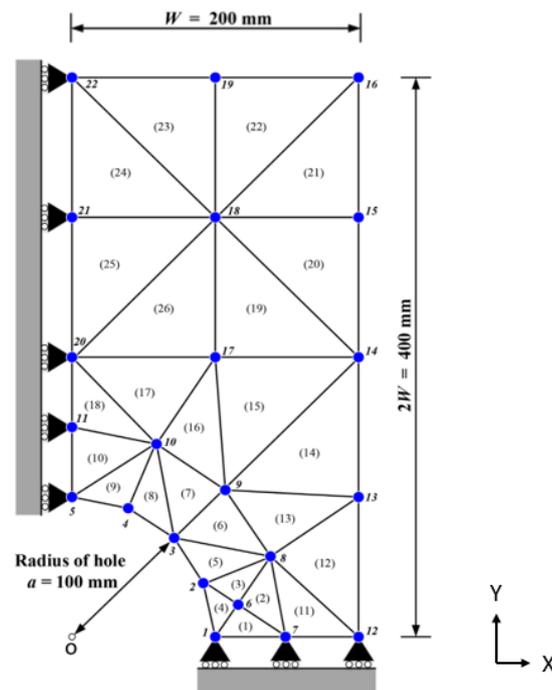


FIGURE 3. FEM Model

### FUZZY PARAMETERS SETTING

The plate's material attributes and load are the varied parameters in this work. The uncertainty values are fixed in a range of maximum and minimum numbers. Table 1

represents the specification of the steel plate; Young's Modulus,  $E$ ; Poisson's Ratio,  $\nu$ ; length,  $L$ ; width,  $H$ ; hole radius,  $r$ ; thickness,  $t$ ; and distributed load,  $F$ . The deterministic numbers and the uncertainty range of the parameters are shown in Table 1.

TABLE 1. Plate model attributes

Model specification	Value	Uncertainty Range
Young's modulus, $E$	225 GPa	$\pm 35$ GPa
Distributed applied force, $F$	3 kN	$\pm 2$ kN
Poisson's ratio, $\nu$	0.3	-
Length, $L$	200 mm	-
Width, $H$	200 mm	-
Hole radius, $r$	100 mm	-
Thickness, $t$	8 mm	-

Source: Yanase, 2017

### FUZZY ANALYSIS AND SIMULATION APPROACH

Fuzzy numbers are fuzzy arithmetic operations. Seresht and Fayek (2019) mentioned that there are two mathematical approaches for implementing this method; the  $\alpha$ -cut approach and the principal extension approach. In this work, the  $\alpha$ -cut method was chosen due to its coherence (Singh et al. 2022). This fuzzy approach requires a range

of uncertainty parameter numbers, the maximum and minimum numbers. The fuzzy parameters applied are Young's Modulus,  $E$ , and Applied force,  $F$ . Each parameter simulation run was performed separately, while the other parameters were constant.

In a fuzzy analysis, a membership function is known as a curve that maps the input data points into the degree of membership between 0 and 1 interval. Figure 4 represents several types of membership functions;

trapezoid, singleton, sigmoid, triangular, and Gaussian. Akramin et al. (2020) and Khairuddin et al. (2021) suggested that the triangular, trapezoid, and Gaussian

membership functions perform better due to their simplicity in computational application. Therefore, the triangular membership function is chosen in this paper.

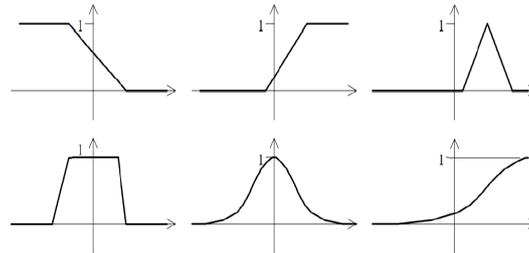


FIGURE 4. Different membership functions  
 Source: Mahajan et Gupta (2021)

According to Pourabdollah et al. (2020), the output of the fuzzy analysis is significantly affected by the number of  $\alpha$ -cuts in a fuzzy simulation. Thus, a total of 3 to 10

$\alpha$ -cuts were performed in this work due to computational cost and time. Figure 5 shows the flow for each task in this work.

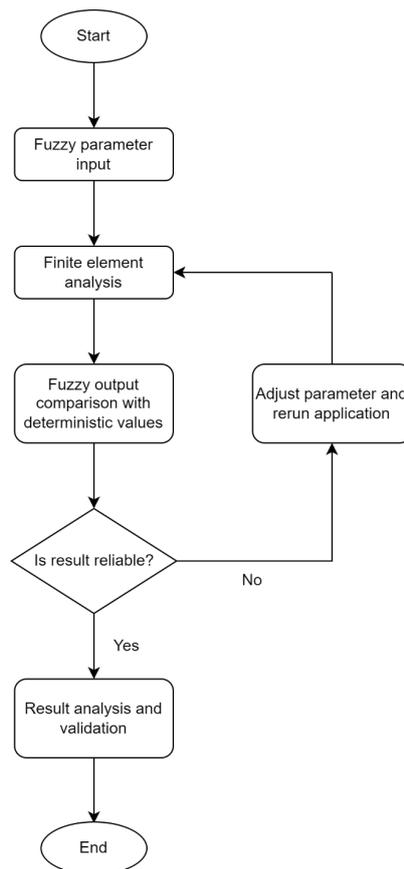


FIGURE 5. Fuzzy Analysis Flowchart

## RESULTS AND DISCUSSION

### SIMULATION SETTING

The von-Mises stress,  $\sigma$ , and displacement,  $d_y$ , are the output of the simulation work. The effect of fuzzy input on the output is analyzed. Figure 6 shows the angle direction on the hollow part of the quarter plate for simulation results. The angle is variable and will be evaluated in the range of 0 to 90 degrees.

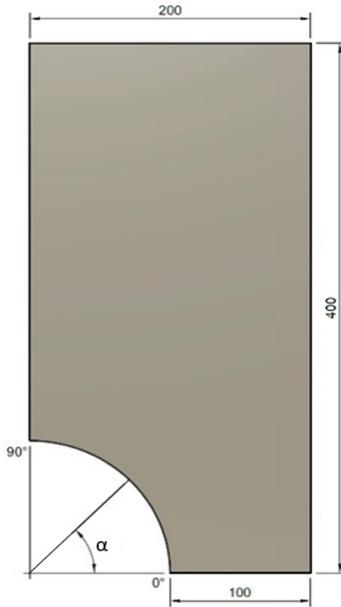


FIGURE 6. Angle direction of the hollow part

The fuzzy simulation has generated vast information and data from each  $\alpha$ -cuts. Wu et al. (2020) highlighted that the  $\alpha$ -cuts help to determine accurate result outputs to perform structural failure analysis. Therefore, each fuzzy output is defuzzified to obtain a real number output. These real number outputs are then analyzed.

### FUZZY SIMULATION OUTPUT

Figure 7 shows the stress distribution when a load is applied on top of the plate. Based on the figure, the stress intensity is maximum around the hollow section in the middle of the plate, where the deformation takes place as the force is applied perpendicularly to the deformation, and the stress distribution is intense around the hole. The red-colored mark in the figure shows the plate's highest stress and deformation area. Based on this finding, the output of the fuzzy simulation focuses on the nodes and elements around the hollow space of the plate.

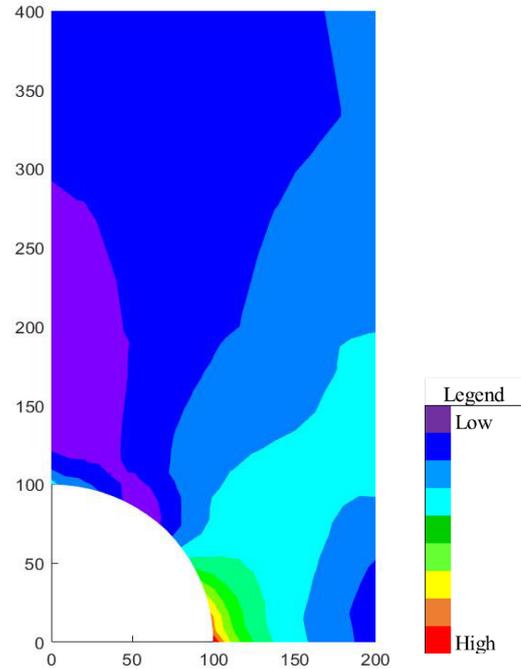


FIGURE 7. Stress distribution of the plate

The von-Mises stress is a criterion state that determines if a material will fracture. If the material's von-Mises stress is equal or larger than its yield limit under tension, it will yield. By obtaining stress X,  $\sigma_x$ , stress Y,  $\sigma_y$ , and the shear stress XY,  $\tau_{xy}$ , the calculation of von-Mises stress,  $\sigma_{vm}$  is as follows;

$$I_1 = \sigma_x + \sigma_y \quad (1)$$

$$I_2 = \sigma_x * \sigma_y - \tau_{xy}^2 \quad (2)$$

$$\tau_{xy} = \frac{F}{A} \quad (3)$$

where  $F$  is the force applied and  $A$  is the cross-section, that is parallel to the force,  $F$ .

Thus, to obtain the von-Mises stress,  $\sigma_{vm}$ ,

$$\sigma_{VM} = \sqrt{I_1^2 + 3I_2} \quad (4)$$

STRESS SIMULATION OUTPUT

Figure 8 shows the von-Mises stress result of the plate. The yield strength of the material of the plate is indicated in the figure, which is 420 MPa. All curves have the same shape trend, and the peak point is at 38° approximately for each curve. The points of the upper bound curve between 30° to 50° have surpassed the yield strength value, which is already in an unsafe structure region. The determinant

curve is approximately at the mean position between the other two curves, the upper and lower bound curves. Fuzzy values between the determinant and lower bound values are considered safe to implement in a structure as they remain below the yield strength boundary. The upper bound fuzzy values have surpassed the yield strength limit up to 500 MPa between 30° to 50°, which left a difference of 80 MPa with the mentioned limit.

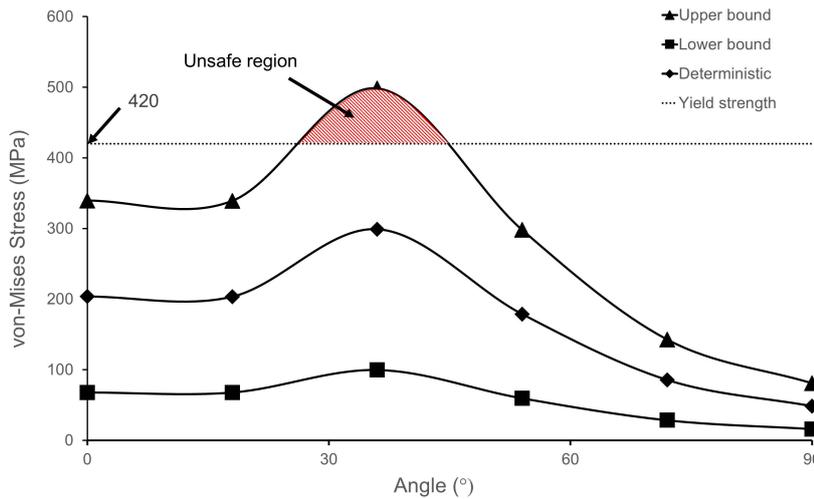


FIGURE 8. von-Mises stress output

Tables 2, 3, and 4 show the uncertainty range dispersion percentage between the deterministic values, the upper and lower bound values with the yield strength limit for the von-Mises stress output. The uncertainty range dispersion is the percentage of the difference between the deterministic, upper, and lower bound values with the yield strength limit of the material.

TABLE 2. Uncertainty range dispersion for upper bound values (von-Mises)

Value (MPa)		Uncertainty dispersion (%)
Upper bound	Yield strength	
339.57	420.00	19.2
339.14	420.00	19.3
498.34	420.00	-18.7
298.08	420.00	29.0
142.92	420.00	66.0
80.94	420.00	80.7

TABLE 3. Uncertainty range dispersion for lower bound values (von-Mises)

Value (MPa)		Uncertainty dispersion (%)
Lower bound	Yield strength	
67.91	420.00	83.8
67.83	420.00	83.9
99.67	420.00	76.3
59.62	420.00	85.8
28.58	420.00	93.2
16.19	420.00	96.1

TABLE 4. Uncertainty range dispersion for deterministic values (von-Mises)

Value (MPa)		Uncertainty dispersion (%)
Deterministic	Yield strength	
203.74	420.00	51.5
203.49	420.00	51.6
299.01	420.00	28.8
178.85	420.00	57.4
85.75	420.00	79.6
48.57	420.00	88.4

Table 2 represents the uncertainty dispersion of the upper bound values with the yield strength limit. Most of the points are below 30% of dispersion as the upper bound curve is the closest to the yield limit. Moreover, one point (498.34 MPa) has surpassed the limit (-18.7%) and enters the unsafe region of the material. This curve has a high risk of being considered a security margin.

Table 3 and 4 show the uncertainty dispersion of the lower bound and deterministic values with the yield strength limit. Most of the points in both tables indicate the same average uncertainty dispersion value, which is > 50%. This percentage shows a considerable gap between

the deterministic and lower bound values with the yield limit. An uncertainty dispersion of > 50% is considered a good security margin criterion as it helps to avoid the danger zone (rupture) when stress is applied to the structure (Wang et al. 2018).

#### DISPLACEMENT SIMULATION OUTPUT

Figure 9 represents the displacement simulation result of the plate. All curves show a significant increasing trend over the angle, although the displacement values for all curves are very small. The deterministic output is situated between the upper and lower bound and in the same trend. The lower bound curve increases slowly over the angle compared to the other curves. This creates a large gap dispersion, especially with the upper bound curve. The upper bound curve shows a significant and fast increase rate over angle compared to the other two curves. This creates a huge gap with the deterministic value curve. The elasticity of the material's behavior is shown by the increasing trend of the curves. The upper bound curve has the highest yield strength value as its increasing rate is higher than the other two curves (Simonovski et al. 2017).

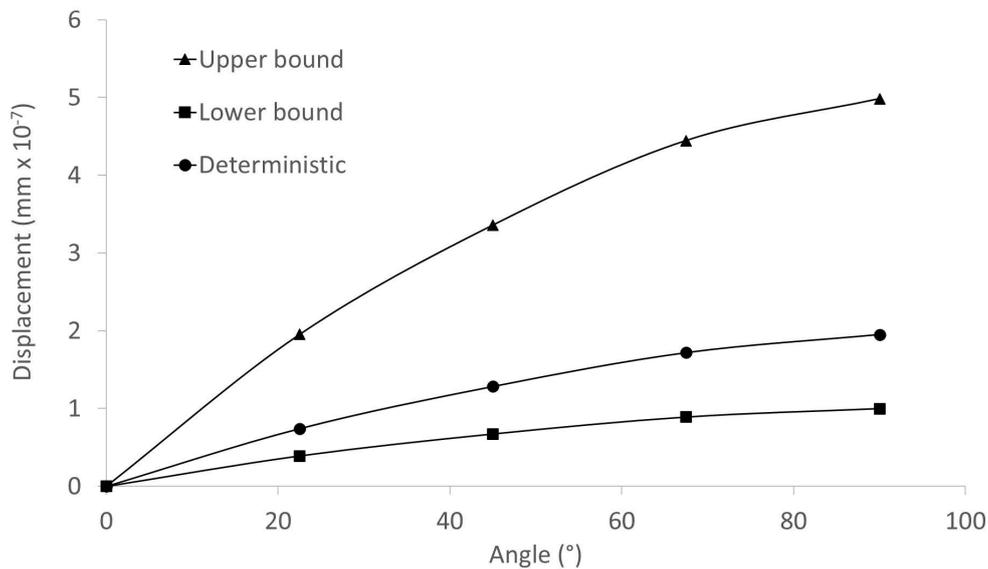


FIGURE 9. Displacement curves

Table 5 and 6 show the uncertainty range dispersion percentage between the deterministic values and the upper and lower bound values for the displacement output. The uncertainty dispersion is the percentage

of difference between the upper and lower bound values with the deterministic values.

TABLE 5. Uncertainty range dispersion for lower bound values (displacement)

Value (mm)		Uncertainty dispersion (%)
Deterministic	Lower bound	
1.39x10-12	7.37x10-13	46.89
7.42x10-08	3.90x10-08	47.36
1.28x10-07	6.72x10-08	47.68
1.72x10-07	8.89x10-08	48.28
1.95x10-07	9.96x10-08	48.95

TABLE 6. Uncertainty range dispersion for upper bound values (displacement)

Value (mm)		Uncertainty dispersion (%)
Deterministic	Upper bound	
1.39x10-12	3.69x10-12	165.56
7.42x10-08	1.95x10-07	163.21
1.28x10-07	3.36x10-07	161.59
1.72x10-07	4.45x10-07	158.60
1.95x10-07	4.98x10-07	155.26

The mean value for the uncertainty dispersion of the lower bound is around 47%, which is a slightly big difference gap with the deterministic value. It represents a good security margin with the deterministic value as the curve has the lowest increase rate and reaches the fracture point at a slower pace. However, the uncertainty dispersion gap for the upper bound with the deterministic values has exceeded the 100% boundary. There is a very large gap with the deterministic values, and its curve increase trend is very fast. Thus, there is a high possibility that the upper bound values reach the unsafe region and fracture point faster. This shows that the upper bound curve has higher

ductility properties. The deterministic and lower bound curves have slower ductility rates and are suitable to be selected for security margin purposes (Antwi et al. 2018).

### SHEAR STRESS SIMULATION OUTPUT

Figure 10 illustrates the shear stress simulation output of the plate. The graph shows that all three curves almost superposed each other and showed the same trend. This indicates that the uncertainty dispersion range for all angles is small compared with the deterministic values ( $\approx 20\%$  of

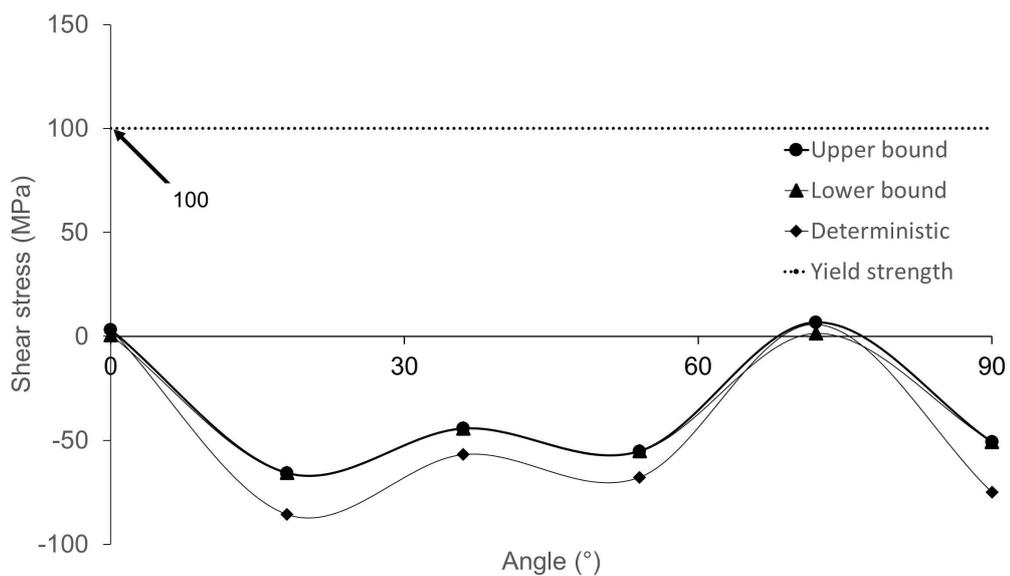


FIGURE 10. Shear stress curves

dispersion for both bounds). Moreover, most of the points are negative values as the plate is under compression along the X-axis when the plate is being pulled upwards along the Y-axis, as indicated in Figure 2. The maximum point observed is at 6.77 MPa at 72° for the upper bound, while the minimum point observed is at -85.52 MPa at 18° for the lower bound. All curves are far below the material's shear stress limit at 100 MPa, which is in a safe zone. This proves that the failure of the plate will not occur because of the shear stress, as the curves are within the elastic range. However, the shear stress effect can be increased if the hollow part of the plate is enlarged to a bigger size (Khan et al. 2018). This modification may produce a different fuzzy output values dispersion compared to the deterministic values.

### CONCLUSION

The fuzzy finite element method is simulated numerically, and the upper and lower bound values of the input data are analyzed and compared with the deterministic values. This method helps to widen the perspective to determine the security margin of the fuzzy uncertainty output, which is reliable for the longevity and sustainability of a structure. The  $\alpha$ -cuts from this analysis enable the possibility of finding uncertainty output values (upper and lower bounds) from a range of random data. In this work, the yield strength limit threshold is the main reference point for the safety factor of the FEM structure, and the upper bound von-Mises stress values have surpassed the limit. This shows that the fuzzy uncertainty output can interfere with a structure's safety limit, especially when it involves an industrial application, for example, in building construction. For future research, a mesh convergence simulation could be implemented on the finite element model in this research. This is to observe the influence of different mesh types on the convergence of the fuzzy simulation output result.

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### DECLARATION OF COMPETING INTEREST

None

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