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Temperature Dependency on Mixed Convection Boundary Layer Flow Over an Isoflux Horizontal Circular Cylinder

(Kebersandaran Suhu Terhadap Aliran Lapisan Sempadan bagi Olakan Campuran ke atas Silinder Bulat Mengufuk Isofluks)

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ABSTRACT

Steady laminar mixed convection boundary layer flow past a horizontal circular cylinder with constant wall heat flux, immersed in a viscous and incompressible fluid of temperature-dependent viscosity is considered in this study. The governing partial differential equations were transformed using non-similar transformation and then solved numerically by an implicit finite-difference scheme known as the Keller-box method. The effects of temperature-dependent viscosity parameter θ_r *on the flow and heat transfer characteristics were examined for various values of Prandtl number, Pr and the mixed convection parameter,* λ*. It was found that for both assisting and opposing flows, as* θ_r *increases, the local skin friction coefficient increases while the wall temperature decreases for air but for water, the local skin friction coefficient decreases then slightly increases while temperature decreases.*

Keywords: Boundary layer; horizontal circular cylinder; isoflux; mixed convection; temperature dependency

ABSTRAK

Aliran lapisan sempadan mantap berlamina bagi olakan campuran ke atas silinder bulat mengufuk dengan fluks haba malar, direndam dalam bendalir likat tak mampat dengan kelikatan bersandar-suhu dipertimbangkan. Persamaan menakluk dalam bentuk persamaan pembezaan separa dijelmakan menggunakan penjelmaan tak serupa dan kemudiannya diselesaikan menggunakan skim beza terhingga tersirat yang dikenali sebagai kaedah kotak Keller. Kesan parameter kelikatan bersandar-suhu θ_r *ke atas ciri-ciri aliran dan pemindahan haba dikaji bagi pelbagai nilai nombor Prandtl, Pr dan parameter olakan campuran,* λ*. Didapati untuk kedua-dua aliran membantu dan menentang, apabila* θ_r *meningkat, pekali geseran kulit meningkat sementara suhu dinding menurun bagi udara tetapi bagi air, pekali geseran kulit menurun kemudian meningkat sedikit sementara suhu dinding menurun.*

Kata kunci: Isofluks; kebersandaran suhu; lapisan sempadan; olakan campuran; silinder bulat mengufuk

INTRODUCTION

Mixed convection heat transfer exists when free convection are in the same order of magnitude as forced convection. In recent years, the study of mixed convective heat transfer has received much attention due to a large number of applications, which is frequently encountered in many industrial and technical processes including solar central receivers exposed to winds, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown, heat exchangers placed in a low-velocity environment (Kafoussias & Williams 1995). Several studies related to the analysis of mixed convection boundary layer flow over horizontal circular cylinder have been mentioned in detail in Ahmad et al. (2009). Ahmad et al. (2009) have studied the problem of mixed convection boundary layer flow past an isothermal horizontal circular cylinder with temperature-dependent viscosity. In that paper, the case of constant wall temperature was considered and it was reported that the flow and temperature characteristics are significantly affected by the temperature-dependent parameter. Studies related to temperature-dependent case have been considered by Ramani Mayappan and Zainal Arifin Ahmad (2009), Ahmad et al. (2010) and Karamdel et al. (2010). For brevity, we just mention a few and other representative studies in this area may be found in Ahmad et al. (2009).

Motivated by the work presented by Ahmad et al. (2009) and to get better understanding about the flow as well as temperature characteristics, we extended the problem by investigating the effect of temperature dependency on mixed convection boundary layer flow of viscous incompressible fluid over a horizontal circular cylinder for the case of constant wall heat flux. To the authors' present knowledge, this study has not been reported in literature and it is hoped that the results will serve as a complement to the previous studies as well as to provide useful information for real applications.

GOVERNING EQUATIONS

We consider a problem of mixed convection boundary layer flow past a horizontal circular cylinder of radius a, immersed in a viscous and incompressible fluid with temperature-dependent viscosity. It is assumed that the cylinder is placed in a constant free stream temperature T_∞ and is kept at a constant surface heat flux q_w where $q_w > 0$ for an assisting flow (heated cylinder) and $q_w < 0$ for opposing flow (cooled cylinder). As in Merkin (1977), the characteristic velocity is assumed to be equal to $(1/2)U_\infty$. It is also assumed that Boussinesq and boundary layer approximations are valid. Under these assumptions, the dimensional governing equations of the problem are (Merkin 1977):

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \rho \bar{u}_e(\bar{x})\frac{d\bar{u}_e(\bar{x})}{d\bar{x}} + \frac{\partial}{\partial \bar{y}}\left(\mu \frac{\partial \bar{u}}{\partial \bar{y}}\right) + \rho g \beta (T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right), \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha \frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

with boundary conditions:

$$\bar{u} = \bar{v} = 0, \quad \frac{\partial T}{\partial \bar{y}} = -\frac{q_w}{k} \quad \text{at } \bar{y} = 0, \qquad \bar{u} \to \bar{u}_e(x), \; T \to T_\infty \quad \text{as } \bar{y} \to \infty, \tag{4}$$

where $\bar{x}$ is the coordinate measured along the surface of the cylinder starting from the lower stagnation point ($\bar{x} \approx 0$) and $\bar{y}$ is the distance measured normal to it. ($\bar{u}$,$\bar{v}$) are the velocity components along the ($\bar{x}$,$\bar{y}$) axes, $\bar{u}_e$ (x) is dimensional velocity outside boundary layer, T is the local fluid temperature, g is the acceleration due to gravity, α is thermal diffusivity, ρ is fluid density, β is thermal expansion coefficient and k is thermal conductivity and ($\bar{x}/a$) is the angle of the cylinder. The physical model and coordinate system of the problem is shown in Figure 1.

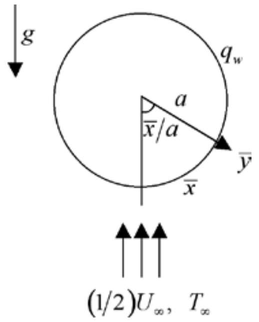

FIGURE 1. Physical model and coordinate system

The dynamic viscosity, μ is assumed to be an inverse linear function of temperature, T as given in Lings and Dybbs (1987), viz:

$$\mu = \frac{\mu_\infty}{1 + \gamma(T - T_\infty)}. \tag{5}$$

Here, u_∞ and γ is a constant dynamic viscosity in the fluid free stream and a constant thermal property of the fluid, respectively.

Now, we introduce non-dimensional variables:

$$x = \bar{x}/a, y = \mathrm{Re}^{1/2}(\bar{y}/a), u = \bar{u}/U_\infty, \quad v = \mathrm{Re}^{1/2}(\bar{v}/U_\infty), \tag{6}$$

where $\mathrm{Re}(=U_\infty a/v)$ is the Reynolds number and the non-dimensional temperature

$$\theta = \mathrm{Re}^{1/2}\frac{(T - T_r)}{(aq_w/k)} + \theta_r. \tag{7}$$

where:

$$\theta_r = \mathrm{Re}^{1/2}\frac{(T_r - T_\infty)}{(aq_w/k)} = -\frac{\mathrm{Re}^{1/2}}{\gamma(aq_w/k)} = \text{constant}, \tag{8}$$

Here, T_r is a constant depending on the reference state and the constant γ. Meanwhile, the value of θ_r is determined by the viscosity/temperature characteristics of the fluid and the wall heat flux q_w. Using (5) to (7), the basic equations (1) to (3) become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e(x)\frac{du_e(x)}{dx} + \frac{\theta_r}{(\theta - \theta_r)^2}\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} + \frac{\theta_r}{(\theta_r - \theta)}\frac{\partial^2 u}{\partial y^2} + \lambda\theta \sin x, \tag{10}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2}, \tag{11}$$

with boundary conditions:

$$u = v = 0, \quad \frac{\partial \theta}{\partial y} = -1 \quad \text{at } y = 0, \qquad u \to u_e(x), \; \theta \to 0 \quad \text{as } y \to \infty, \tag{12}$$

where Pr is the Prandtl number and λ is the mixed convection parameter defined as:

$$\lambda = \frac{Gr}{\mathrm{Re}^{5/2}}. \tag{13}$$

Here $\lambda > 0$, $\lambda < 0$ and $\lambda = 0$ represent the assisting flow, the opposing flows and the forced convection flow, respectively, with the Grashof number, $Gr = g\beta(aqw/k)a^3/v^2$.

Following Merkin (1977), we take $u_e(x) = \sin x$ where $u_e(x)$ is velocity outside boundary layer in non-dimensional form. To solve equations (9) - (11) along with the boundary condition (12), we assume the following variables:

$$\psi = xf(x, y), \theta = \theta(x, y), \tag{14}$$

where the stream function, ψ is defined as:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \tag{15}$$

and therefore, equation (9) is satisfied automatically. Substituting (14) and (15) into (10) and (11), we obtain:

$$\frac{\partial^3 f}{\partial y^3} = \frac{\theta - \theta_r}{\theta_r} f \frac{\partial^2 f}{\partial y^2} + \frac{\theta - \theta_r}{\theta_r}\left(\frac{\partial f}{\partial y}\right)^2 - \frac{1}{(\theta - \theta_r)}\frac{\partial \theta}{\partial y}\frac{\partial^2 f}{\partial y^2} - \frac{\sin x \cos x}{x}\frac{(\theta - \theta_r)}{\theta_r} - \lambda \frac{\sin x}{x}(\theta - \theta_r)\frac{\theta}{\theta_r} = x\left(\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2} - \frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y}\right)\frac{\theta - \theta_r}{\theta_r}, \tag{16}$$

$$\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + f\frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y}\right), \tag{17}$$

subject to the boundary conditions (12) which become:

$$f = \frac{\partial f}{\partial y} = 0, \ \frac{\partial \theta}{\partial y} = -1 \quad \text{at} \quad y = 0$$
$$\frac{\partial f}{\partial y} \to \frac{\sin x}{x}, \theta \to 0 \quad \text{as} \quad y \to \infty. \tag{18}$$

The physical quantities of practical interest in this study are the skin friction coefficient C_f and the wall temperature θ_w which are given by:

$$C_f = \frac{x\theta_r}{\theta_r - 1}\frac{\partial^2 f}{\partial y^2}(x,0), \theta(x,0) = \theta_w(x). \tag{19}$$

It can be seen at the lower stagnation point of the cylinder, i.e. ($x \approx 0$), (16) and (17) reduce to the ordinary differential equations as follows:

$$f''' - \frac{\theta - \theta_r}{\theta_r} ff'' + \frac{\theta - \theta_r}{\theta_r} f'^2 - \frac{1}{(\theta - \theta_r)}\theta' f'' - \frac{\theta - \theta_r}{\theta_r} - \lambda(\theta - \theta_r)\frac{\theta}{\theta_r} = 0, \tag{20}$$

$$\frac{1}{\Pr}\theta'' + f\theta' = 0, \tag{21}$$

and the boundary conditions (18) become:

$$f(0) = f'(0) = 0. \ \theta'(0) = -1, f'(\infty) = 1, \theta(\infty) = 0, \tag{22}$$

where primes denote the differentiation with respect to y.

Referring to Kafoussias and Williams (1995), we notice that θ_r cannot take values between 0 and 1 for physically realizable situation. In this region, no solution satisfying the governing equations and the boundary conditions can be obtained. It is also important to highlight that the viscosity variation in the boundary layer is negligible when $|\theta_r|$ is large ($|\theta_r| \to \infty$) and (16) - (17) and (20) - (21) reduce to those found by Nazar et al. (2004). However, the viscosity variation becomes increasingly significant as $\theta_r > 1$ is for gases or $\theta_r < 0$ is for liquids (Ahmad et al. 2009).

Results and Discussion

The system of {(16),(17)} and {(20),(21)} with the boundary conditions (18) and (22), respectively, were solved via an implicit finite difference scheme known as the Keller box method as described in detail by Cebeci and Bradshaw (1988). Both cases of $\lambda > 0$ (assisting flow) and $\lambda < 0$ (opposing flows) are considered. The edge of boundary layer thickness, y_∞ has been adjusted for different range of parameters with the step size of $\Delta y = 0.02$ and $\Delta x = 0.02$ has been used for the computation. The iterations were continued until an accuracy of 10^{-7} was achieved.

The numerical solutions starts at the lower stagnation point of the cylinder, $x \approx 0$, with the initial profiles as given by equations (20) and (21) with boundary conditions (22) and proceed around the cylinder up to the boundary layer separation point x_s. Numerical results for the skin friction coefficient C_f and the non dimensional temperature distribution, $\theta_w(x)$ have been obtained for Prandtl number, Pr = 1, 0.7 (air) and 7 (water) at different positions of x around the cylinder with various values of mixed convection parameter λ and the viscosity/temperature parameter θ_r. In order to ascertain the accuracy of the present results, in Figure 2 and 3, the values of C_f and $\theta_w(x)$ for the case $\theta_r \to \infty$ (temperature independent/constant viscosity) with Pr = 1 are compared with those reported by Nazar et al. (2004). It is observed that the agreement with those found by Nazar et al. (2004) is good. Therefore, we confident that the present results are accurate and this is an encouragement to further study this problem.

Hence, numerical results for the skin friction coefficient and wall temperature for Pr = 0.7 at different positions x with various values of θ_r for both assisting ($\lambda = 0.3$) and opposing ($\lambda = -0.5$) flows cases are presented in Figure 4 and 5, respectively. It is seen from Figure 4 that at each point of x, the skin friction coefficient increases as θ_r increases for both, the opposing and the assisting flows. Meanwhile, Figure 5 shows that each point of x, wall temperature decreases as θ_r increases.

Figures 6 and 7, respectively, show the numerical results for skin friction coefficient and wall temperature for Pr = 7 with various values of θ_r at different positions x for both assisting ($\lambda = 0.3$) and opposing ($\lambda = -0.5$) flows cases. It is noticed from Figure 6 that for the assisting

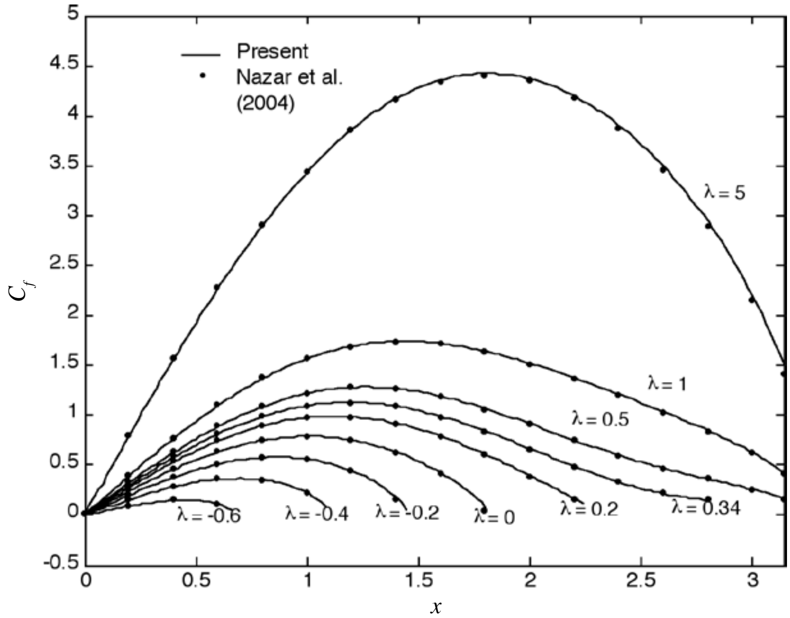

FIGURE 2. Variation of the local skin friction coefficient C_f for various values of λ when Pr = 1 (case of constant viscosity)

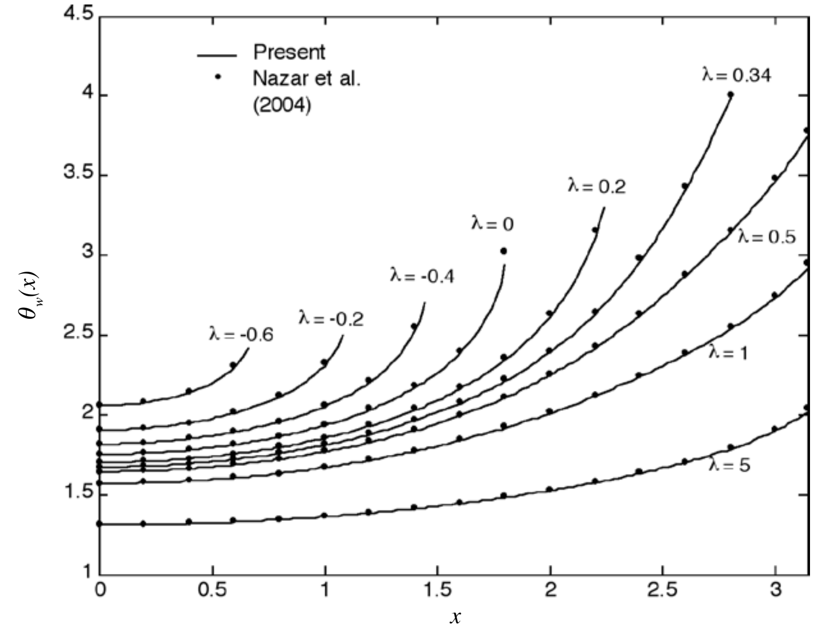

FIGURE 3. Variation of the wall temperature distribution $\theta_w(x)$ for various values of λ when Pr = 1 (case of constant viscosity)

flow as well as opposing flows cases, the skin friction coefficient decreases as θ_r increases at each point of x and this behaviour occurs to a certain value of x. Further, Figure 7 shows that for both assisting and opposing flows cases, the wall temperature decreasing with increasing θ_r at each point of x.

It is observed in Figures 4 and 6 that for each θ_r, the point of x where the skin friction is maximum for the assisting flow is greater than the opposing flows, for Pr = 0.7 and otherwise for Pr = 7. It also can be seen from Figures 4 and 6 that the skin friction coefficient when Pr = 0.7 (air) is greater than when Pr = 7 (water) for assisting flow. On the other hand, the skin friction when Pr = 0.7 is smaller than when Pr = 7 for opposing flows. Figures 5 and 7 show that for both assisting and opposing flows cases, the wall temperature increases as x increases up to the separation point x_s for all values of θ_r. It is also can be observed from these figures that for both assisting and opposing flows, the wall temperature for air happens to be greater than water. This is physically due to that air with lower Pr number than water, has higher thermal diffusivity compare to the water. This leads to more energy transfer ability and in consequence, air is easier to get hot than the water. Further, Figures 5 and 7 show that the wall temperature for the assisting flow are greater than the opposing flow for Pr = 7 and the wall temperature for the assisting flow are smaller than the opposing flows for Pr = 0.7 near the wall. However, the opposite happens after a certain point of x where the wall temperature continues to increase for assisting flow but in opposing flows boundary layer separation occurs. This is as a result of the fluid motion is being accelerated in assisting flow but in opposing flows, the fluid motion is being retarded.

Finally, it is worth mentioning that in the case $|\theta_r| \to \infty$ (constant viscosity), the present study shows that $C_f \to 0$ and $\theta \to \theta_s (\neq 0)$ as $x \to x_s$ when the boundary layer separates, as was found by Nazar et al. (2004).

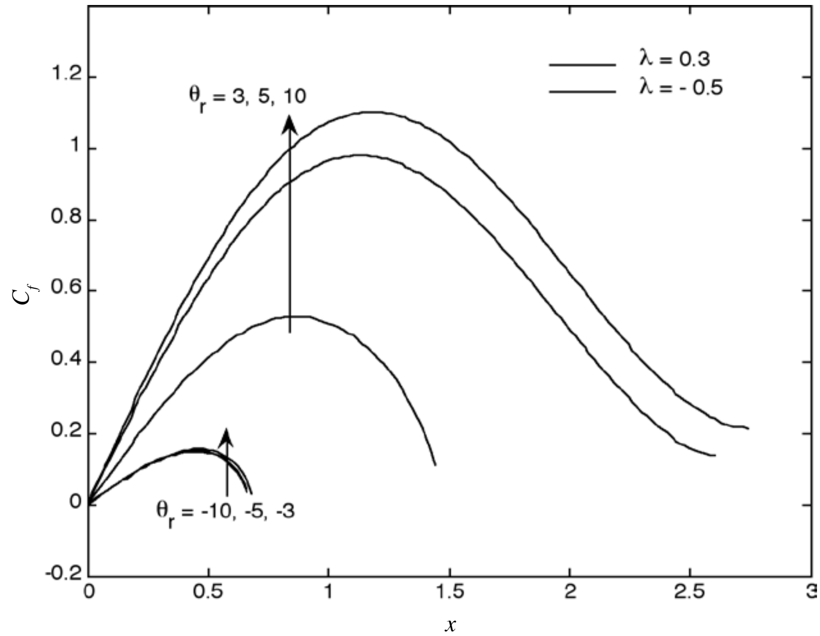

FIGURE 4. Variation of the local skin friction coefficient C_f for various values of θ_r when Pr = 0.7 and λ = 0.5 (assisting flow), λ = -0.5 (opposing flow)

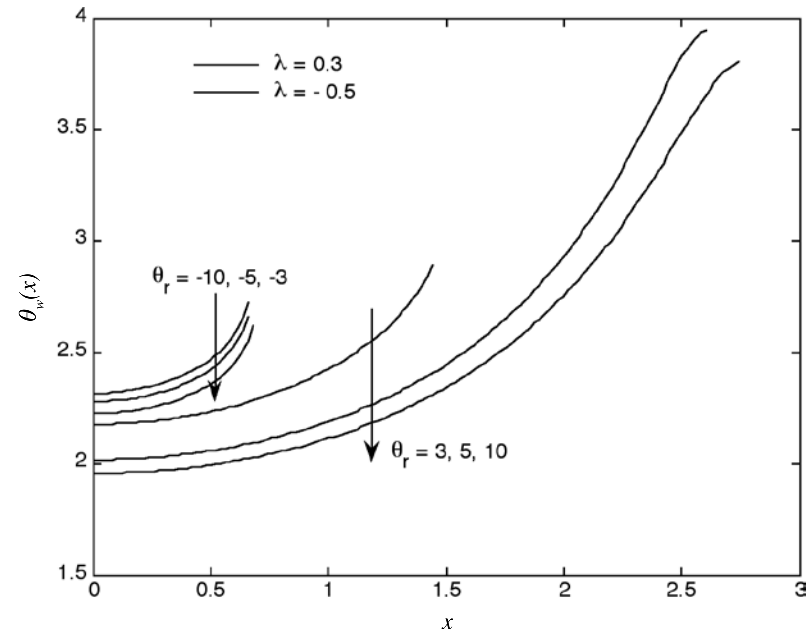

FIGURE 5. Variation of the wall temperature distribution $\theta_w(x)$ for various values of θ_r when Pr = 0.7 and λ = 0.3 (assisting flow), λ = -0.5 (opposing flow)

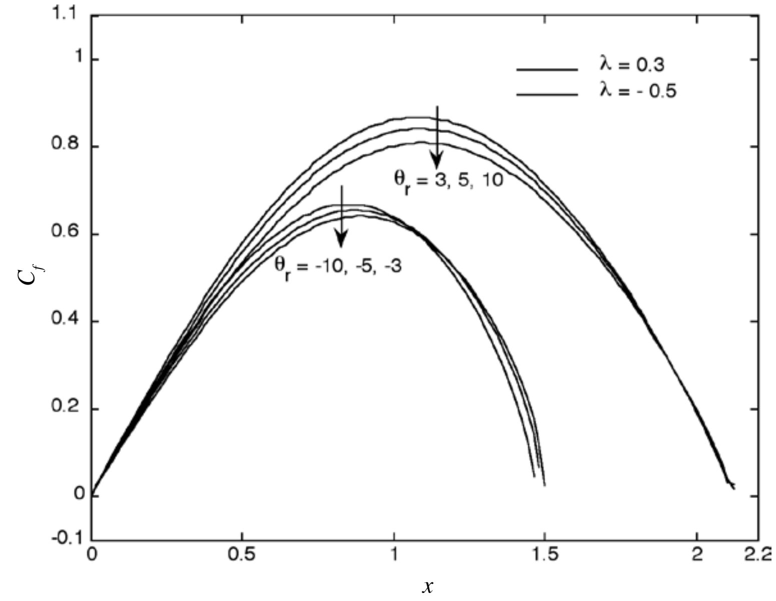

FIGURE 6. Variation of the local skin friction coefficient C_f for various values of θ_r when Pr = 7 and λ = 0.3 (assisting flow), λ = -0.5 (opposing flow)

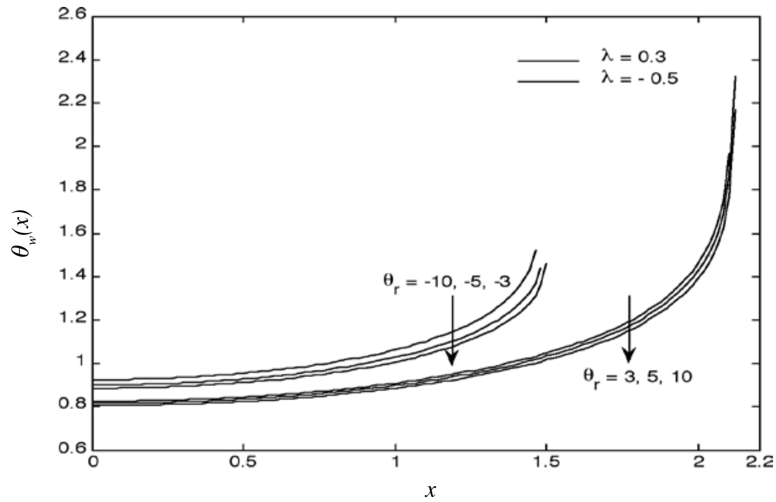

FIGURE 7. Variation of the wall temperature distribution $\theta_w(x)$ for various values of θ_r when Pr = 7 and λ = 0.3 (assisting flow), λ = -0.5 (opposing flow)

CONCLUSION

In the present paper, the problem of steady laminar mixed convection boundary layer flow past a horizontal circular cylinder with constant heat flux placed in a viscous and incompressible fluid of temperature-dependent viscosity have been studied theoretically. Numerical computations are carried out to examine the effects of temperature-dependent viscosity on the flow and heat transfer for different values of Prandtl number, Pr and the mixed convection parameter, λ. The obtained results show that the changes of viscosity with temperature may yield relevant effects on the flow and thermal characteristics. Therefore, consideration on the influence of temperature-dependent viscosity is essential especially when the viscosity of a fluid is sensitive to temperature variations.

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