

## Point Forecast Markov Switching Model for U.S. Dollar/ Euro Exchange Rate

(Ramalan Titik Menggunakan Model Peralihan Markov untuk Kadar Pertukaran

Wang Dolar US/Euro)

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### ABSTRACT

*This research proposes a point forecasting method into Markov switching autoregressive model. In case of two regimes, we proved the probability that  $h$  periods later process will be in regime 1 or 2 is given by steady-state probabilities. Then, using the value of  $h$ -step-ahead forecast data at time  $t$  in each regime and using steady-state probabilities, we present an  $h$ -step-ahead point forecast of data. An empirical application of this forecasting technique for U.S. Dollar/ Euro exchange rate showed that Markov switching autoregressive model achieved superior forecasts relative to the random walk with drift. The results of out-of-sample forecast indicate that the fluctuations of U.S. Dollar/ Euro exchange rate from May 2011 to May 2013 will be rising.*

*Keywords: Exchange rate; Markov switching; point forecast*

### ABSTRAK

*Kajian ini mencadangkan kaedah ramalan titik menggunakan model autoregresi peralihan Markov. Untuk situasi dua rejim, kebarangkalian sama ada ianya berada dalam rejim 1 atau 2 untuk proses  $h$  tempoh ke hadapan diberikan oleh kebarangkalian keadaan mantap. Seterusnya, dengan menggunakan keputusan yang diperolehi pada masa  $t$  untuk setiap rejim dan kebarangkalian keadaan mantap, kami mempersembahkan ramalan titik  $h$  langkah ke hadapan. Aplikasi empirikal kaedah ramalan ini dengan menggunakan kadar pertukaran wang US dolar/Euro menunjukkan bahawa model autoregresi peralihan Markov mampu memberikan ramalan yang lebih baik berbanding dengan model perjalanan rawak berserta hanyutan. Keputusan ramalan luar sampel menunjukkan bahawa kadar pertukaran asing US Dollar/Euro akan meningkat dari Mei 2011 hingga Mei 2013.*

*Kata kunci: Kadar pertukaran wang; ramalan titik; peralihan Markov*

### INTRODUCTION

Engle and Hamilton (1990) found that Markov switching model of exchange rate generates better forecasts than random walk. Yuan (2011) proposed an exchange rate forecasting model which combines the multi-state Markov-switching model with smoothing techniques. In this paper, we present a point forecasting method into Markov switching autoregressive model. Usually, two or three regimes were defined in this model. In case of two regimes, regime 1 describes the periods of downtrend of exchange rates and regime 2 denotes the periods of uptrend of exchange rates. In case of two regimes, we showed the probability that  $h$  periods later process will be in regime 1 or 2 is given by steady-state probabilities. Then, using the value of  $h$ -step-ahead forecast data at time  $t$  in each regime and using steady-state probabilities, we generate an  $h$ -step-ahead point forecast of data.

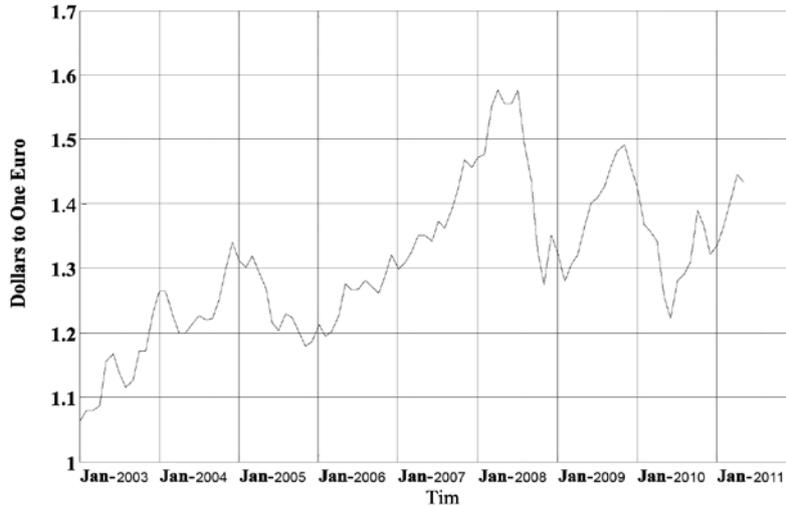
Markov Switching models by a change in their regimes themselves will up to date, when jumps arise in time series data. Therefore, these models will offer a better statistical fit to the data with jumps than the linear models.

Figure 1 shows series of the U.S. Dollars to One Euro. The fluctuations of U.S. Dollars to One Euro have jumps in their behavior. Therefore, Markov switching model can be a candidate for study of U.S. Dollar/ Euro exchange rate.

We compare the in-sample forecasts between Markov switching autoregressive (MS-AR) and random walk with drift (RWd) processes. We find that MS-AR model achieves superior forecasts relative to the random walk with drift. Thereupon, we obtain the out-of-sample point forecasts for U.S. Dollar/ Euro exchange rate by Markov switching autoregressive model.

### DATA

In this study, we employed the U.S. Dollars to One Euro, which are collected monthly from January 2003 to April 2011. The data were obtained from the Board of Governors of the Federal Reserve System (<http://research.stlouisfed.org>). The variable under investigation is exchange rate returns in percentage:



RAJAH 1. The Exchange rate series of the U.S. Dollars to One Euro (Constructed by the authors using data obtained from Board of Governors of the Federal Reserve System, downloaded from <http://research.stlouisfed.org>.)

$$y_t = 100 \times [\ln(r_t) - \ln(r_{t-1})], \tag{1}$$

where  $r_t$  represent the monthly exchange rates.

THE MARKOV SWITCHING METHODOLOGY

The Markov switching model was introduced by Hamilton (1989). A Markov switching autoregressive model (MS-AR) of two states with an AR process of order  $p$  is written as:

$$y_t = \begin{cases} c_1 + \alpha_{11}\gamma_{t-1} + \dots + \alpha_{p1}\gamma_{t-p} + \varepsilon_t & S_t = 1 \\ c_2 + \alpha_{12}\gamma_{t-1} + \dots + \alpha_{p2}\gamma_{t-p} + \varepsilon_t & S_t = 2, \end{cases} \tag{2}$$

where regimes in model (2) are index by  $s_t$ . In this model, the parameters of the autoregressive part and intercept are depended on the regime at time  $t$ . The regimes are discrete unobservable variable. Regime 1 describes the periods of downtrend of exchange rates and regime 2 denotes the periods of uptrend of exchange rates. The transition between the regimes is governed by a first order Markov process as follows:

$$p_{ij} = \Pr(s_t = j | s_{t-1} = i) \quad \forall i, j = 1, 2, \sum_{i=1}^2 p_{ij} = 1.$$

It is normal to collect the transition probabilities in a matrix  $P$  known as the transition matrix:

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}.$$

Note that  $p_{11} + p_{12} = 1$  and  $p_{21} + p_{22} = 1$ .

We estimated the parameters of MS-AR model by MLE. The log likelihood function is given by:

$$\ln L = \sum_{t=1}^T \ln \left\{ \sum_{s_{t-1}=1}^2 f(y_t | s_t, \Psi_{t-1}) P[s_t | \Psi_{t-1}] \right\}. \tag{3}$$

Where  $\Psi_{t-1} = \{y_1, \dots, y_{t-1}\}$ . In Eq. (3),  $P[s_t | \Psi_{t-1}]$  are filtered probabilities. Using  $\gamma_t$  as observed at the end of the  $t$ -th iteration, we calculated filtered probabilities as:

$$P[s_t = j | \Psi_t] = \sum_{s_{t-1}=1}^2 P[s_t = j, s_{t-1} = i | \Psi_t] \quad ; \text{for } t = 1, \dots, T.$$

The next step, using all the information in the sample i.e.  $\Psi_T = \{y_1, \dots, y_T\}$ , we calculated smoothed probabilities:

$$P[s_t = j | \Psi_T] = \sum_{k=1}^2 P[s_t = j, s_{t+1} = k | \Psi_T] \\ = \sum_{k=1}^2 \frac{P[s_{t+1} = k | \Psi_T] P[s_t = j | \Psi_t] P[s_{t+1} = k | s_t = j]}{P[s_{t+1} = k | \Psi_t]} \quad \text{For } t = T-1, T-2, \dots, 1.$$

In addition,  $P[s_T | \Psi_T]$  at the last iteration of filter is calculated.

FURTHER DISCUSSION OF MARKOV SWITCHING MODELS

The Markov switching autoregressive models applied a great variety of specifications. These models can be applied where the autoregressive parameters, the mean or the intercepts, are regime-dependent (see Krolzing 1998 for further details). The Markov switching-mean according to the notation introduced by Krolzig (1998):

$$\gamma_t = \mu_{s_t} + \alpha_1 \gamma_{t-1} + \dots + \alpha_p \gamma_{t-p} + \varepsilon_t.$$

In this model, only the mean is depended on regime. Andel (1993) showed that Markov switching-mean and ARMA the processes have similar properties than a long memory

process. Kuswanto and Sibbertsen (2008) discussed and showed that model (4) is a candidate for a Markov switching process which is able to create a spurious long memory. Charfeddine and Guegan (2009) applied models that have changes in mean like the Markov switching model and the structural change model. They showed that when the data are weakly dependent with changes in mean, the hypothesis of long memory is accepted with a high power. Therefore, often Markov switching-mean model needs a survey for apparent of long memory.

Ismail and Isa (2006) used structural change test to detect nonlinear feature in three ASEAN countries exchange rates. They find that the null hypothesis of linearity is rejected and there is evidence of structural breaks in the exchange rates series. Therefore, they apply regime-switching model in their study.

#### PARAMETER ESTIMATION

We followed Psaradakis and Spagnolo (2003) for selecting the number of regimes, who propose to use the value of the Akaike Information Criterion (AIC). Then, we compared the different types of Markov switching autoregressive models. Our comparison strategy follows Cologni and Manera (2009), who compared Markov

switching models using value of the log-likelihood function, values of means or intercepts estimated in any regime and estimated matrix of transition probabilities. Using these selection strategies, the best performance was obtained for model (2) with two regimes and one-lag autoregressive component. The details of the model fitted for MS-AR is presented in Table 1. All estimated coefficients were statistically significant at conventional significance levels. The transition probabilities suggest that regime 1 is highly persistent. When the process was in regime 1, there was a low probability that it switches to regime 2  $\{p(s_t = 2 | s_{t-1} = 1) = 0.06\}$ . The average duration of the two regimes were 17.81 and 7.50 months, respectively (Table 1).

Figure 3 shows time series of smoothed probabilities for fluctuations of the U.S. Dollar/ Euro exchange rate by MSAR model. This figure shows the probability of being in regime 1 or 2 at a specific time. In December 2008 and July 2010, the fluctuation for U.S. Dollar/ Euro exchange rate is ascendant (Figure 2), which causes the process in regime 2 with a high probability (Figure 3). In to be other years since, the fluctuations for exchange rate is low (Figure 2), therefore the process is in regime 1 with a high probability (Figure 3).

TABLE 1. Estimated of MS-AR model with details

		Coefficient	Stand. Error
Regime 1	$a_0$	0.5418	0.0303 (0.00)***
	$a_1$	0.2333	0.0225 (0.00)***
	$\sigma$	1.9312	
Regime 2	$a_0$	-0.4067	0.0307 (0.00)***
	$a_1$	0.2699	0.0385(0.00)***
	$\sigma$	3.3842	
	Regime 1	Regime 2	
Regime 1	0.94	0.06	
Regime 2	0.13	0.87	
The expected duration of regime 1		17.81	
The expected duration of regime 2		7.50	
Steady-state probability of regime 1 ( $\pi_1$ )		0.6842	
Steady-state probability of regime 2 ( $\pi_2$ )		0.3158	
Log. Likelihood		-224.3386	
AIC		468.6773	
BIC		494.5270	

P-values are reported in the parenthesis. \*\*\*, \*\*, \* denotes significance of the coefficient at the 0.1%, 1%, 5% level.

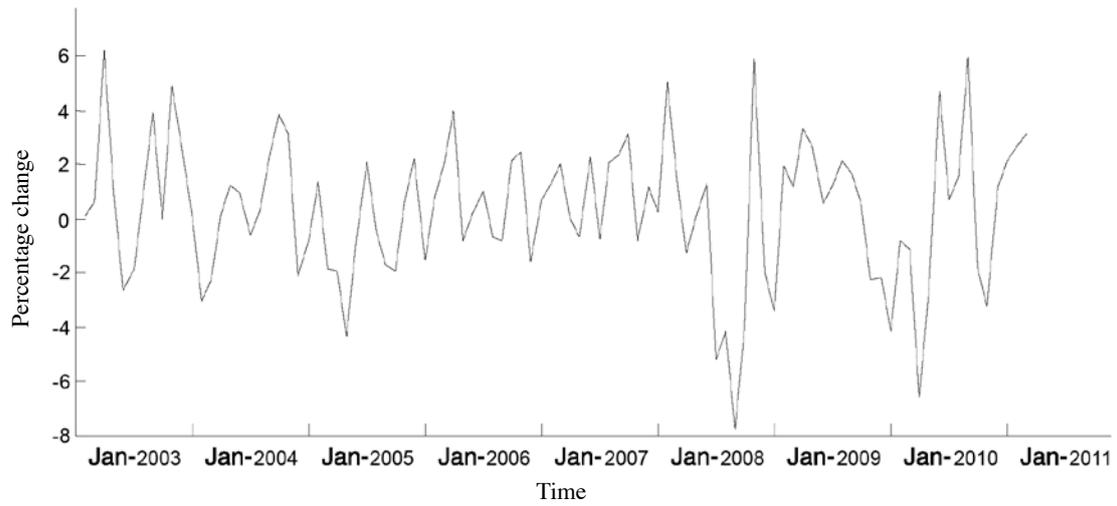


FIGURE 2. Percent changes series in the log of the U.S. Dollars to One Euro (Jan 2003-Apr 2011)

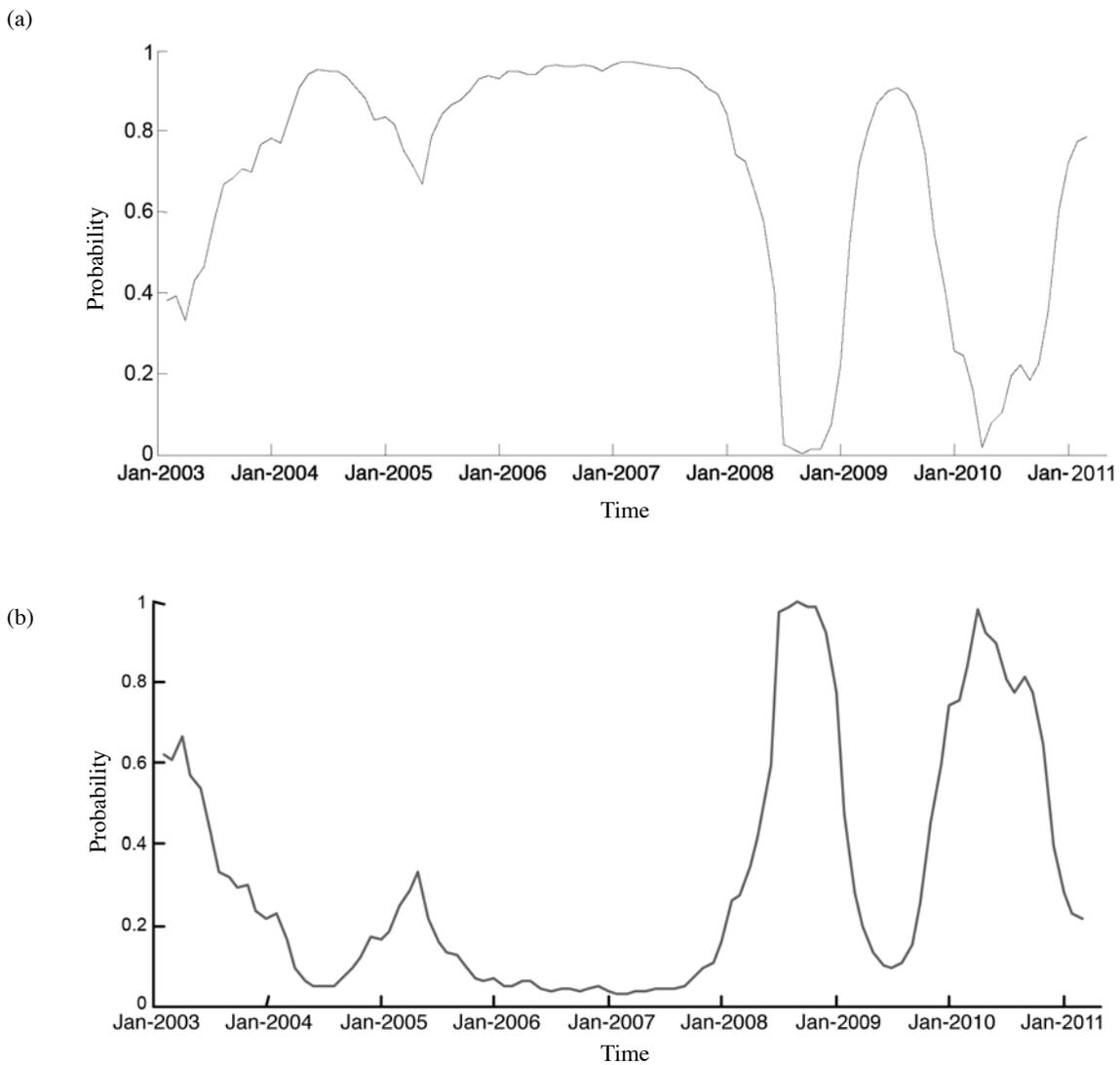


FIGURE 3. Smoothed probabilities of (a) Regime 1 and (b) Regime 2

FORECAST

Yuan (2011) predicted fluctuations of the dollar by Markov switching model of K regimes, which implied according to the following formula:

$$\hat{y}_{t+h} = E(y_{t+h} | \Psi_t) = \hat{\pi}_{t|t} \cdot P^h \cdot \hat{\mu},$$

where

$$y_t = \mu_{s_t} + \sigma_{s_t} \varepsilon_t \text{ and } \hat{\pi}_{t|t} = [\text{pr}(s_t = 1 | \Psi_t), \dots, \text{pr}(s_t = k | \Psi_t)].$$

In the following lemma, we proof  $\hat{\pi}_{t|t} \cdot P^h$  is equal to  $\hat{\pi}_{t|t}$  when Markov chain has two regimes.

Lemma: For a 2-regime Markov chain, we have:

$$\hat{\pi}_{t|t} \cdot P^h = \hat{\pi}_{t|t}.$$

Proof: For a two-state Markov chain, the transition matrix is.

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}.$$

Then, the matrix of m-period-ahead transition probabilities for an ergodic two-state Markov chain is given by:

$$P^h = \begin{bmatrix} \frac{(1-p_{22}) + \lambda_2^h (1-p_{11})}{2-p_{11}-p_{22}} & \frac{(1-p_{11}) + \lambda_2^h (1-p_{11})}{2-p_{11}-p_{22}} \\ \frac{(1-p_{22}) - \lambda_2^h (1-p_{22})}{2-p_{11}-p_{22}} & \frac{(1-p_{11}) + \lambda_2^h (1-p_{22})}{2-p_{11}-p_{22}} \end{bmatrix} = \begin{bmatrix} p_{11}^h & p_{12}^h \\ p_{21}^h & p_{22}^h \end{bmatrix} \tag{5}$$

where  $\lambda_2 = -1 + p_{11} + p_{22}$  (see Hamilton 1994 for more details, note that Hamilton defines matrix P from the matrix with each column sum equal to 1. However, in this paper and many other literatures, matrix P is defined from the matrix with each row sum equal to 1).

The steady-state probabilities is given by

$$\pi = \begin{bmatrix} \frac{1-p_{22}}{2-p_{11}-p_{22}} \\ \frac{1-p_{11}}{2-p_{11}-p_{22}} \end{bmatrix} = \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix}.$$

Thus with this details, we have

$$\pi' P^h = [\pi_1 \ \pi_2] \cdot \begin{bmatrix} p_{11}^h & p_{12}^h \\ p_{21}^h & p_{22}^h \end{bmatrix} = [\pi_1 p_{11}^h + \pi_2 p_{21}^h \quad \pi_1 p_{12}^h + \pi_2 p_{22}^h]. \tag{6}$$

The first element of (6) becomes

$$\begin{aligned} \pi_1 p_{11}^h + \pi_2 p_{21}^h &= \frac{1-p_{22}}{2-p_{11}-p_{22}} \times \frac{(1-p_{22}) + \lambda_2^h (1-p_{11})}{2-p_{11}-p_{22}} \\ &+ \frac{1-p_{11}}{2-p_{11}-p_{22}} \times \frac{(1-p_{22}) - \lambda_2^h (1-p_{22})}{2-p_{11}-p_{22}} \\ &= \frac{(1-p_{22})\{(1-p_{22}) + \lambda_2^h (1-p_{11})\} + (1-p_{11})\{(1-p_{22}) - \lambda_2^h (1-p_{22})\}}{(2-p_{11}-p_{22})^2} \\ &= \frac{(1-p_{22})^2 + (1-p_{11})(1-p_{22})}{(2-p_{11}-p_{22})} \\ &= \frac{(1-p_{22})\{(1-p_{22}) + (1-p_{11})\}}{(2-p_{11}-p_{22})^2} \\ &= \frac{1-p_{22}}{2-p_{11}-p_{22}} \\ &= \pi_1. \end{aligned}$$

By similar reasoning, the second element of (6) becomes

$$\pi_1 p_{12}^h + \pi_2 p_{22}^h = \pi_2.$$

Next step, we show the true of above lemma by using our empirical finding. Using details of Table 1, the matrix of steady-state probabilities is estimated as

$$\hat{\pi} = \begin{bmatrix} 0.6842 \\ 0.3158 \end{bmatrix}.$$

Using (5), the matrix of 2-period-ahead transition probabilities is estimated as

$$\begin{aligned} \hat{P}^2 &= \begin{bmatrix} \frac{(1-0.87) + 0.81^2 (1-0.94)}{2-0.94-0.87} & \frac{(1-0.94) - 0.81^2 (1-0.94)}{2-0.94-0.87} \\ \frac{(1-0.87) - 0.81^2 (1-0.87)}{2-0.94-0.87} & \frac{(1-0.94) + 0.81^2 (1-0.87)}{2-0.94-0.87} \end{bmatrix} \\ &= \begin{bmatrix} 0.8914 & 0.1086 \\ 0.2353 & 0.7647 \end{bmatrix}. \end{aligned}$$

Hence, for two periods ahead

$$\begin{aligned} \hat{\pi}' \hat{P}^2 &= [0.6842 \quad 0.3158] \begin{bmatrix} 0.8914 & 0.1086 \\ 0.2353 & 0.7647 \end{bmatrix} \\ &= [0.68420362 \quad 0.31579638] \end{aligned}$$

A similar result holds for three periods ahead:

$$\begin{aligned} \hat{\pi}' \hat{P}^3 &= [0.6842 \quad 0.3158] \begin{bmatrix} 0.8520 & 0.1480 \\ 0.3206 & 0.6794 \end{bmatrix} \\ &= [0.684218388 \quad 0.31581612]. \end{aligned}$$

Finally, for  $n$  periods ahead

$$\hat{\pi}' \hat{P}^n = [0.6842 \quad 0.3158] \begin{bmatrix} 0.6842 & 0.3158 \\ 0.6842 & 0.3158 \end{bmatrix} \\ = [0.6842 \quad 0.3158].$$

when  $n \geq 42$ . Consequently, our empirical finding confirms the true of above lemma.

Therefore, in case of two regimes; process  $h$  periods later will be in regime 1 with probability  $\pi_1 = pr(s_t=1|\Psi_t)$  and in regime 2 with probability and  $\pi_2 = pr(s_t=2|\Psi_t)$   $\pi_1$  and  $\pi_2$  are steady-state probabilities i.e. with changes in the time, they are steady.

Now, we rewrite the model (2) as

$$y_t = \begin{cases} y_{t,1} \\ y_{t,2} \end{cases}.$$

where

$$y_{t,1} = c_1 + \alpha_{11}y_{t-1} + \dots + \alpha_{p1}y_{t-p} + \varepsilon_t,$$

and

$$y_{t,2} = c_2 + \alpha_{12}y_{t-1} + \dots + \alpha_{p2}y_{t-p} + \varepsilon_t,$$

Let  $y_{t,1}$  and  $y_{t,2}$  denote the value of process at time  $t$  in the regime 1 and regime 2, respectively. The 1-step-ahead forecast at time  $t$  of  $y_{t,1}$  and  $y_{t,2}$  are

$$\hat{y}_{t+1,1|t} = E[c_1 + \alpha_{11}y_t + \dots + \alpha_{p1}y_{t+1-p} + \varepsilon_{t+1} | \Psi_t] \\ = c_1 + \alpha_{11}y_t + \dots + \alpha_{p1}y_{t+1-p},$$

and

$$\hat{y}_{t+1,2|t} = E[c_2 + \alpha_{12}y_t + \dots + \alpha_{p2}y_{t+1-p} + \varepsilon_{t+1} | \Psi_t].$$

The process is in regime 1 with probability  $\pi_1$  and in regime 2 with probability  $\pi_2$ . Therefore, the point forecast of  $y_{t+1}$  given  $\Psi_t$  is

$$\hat{y}_{t+1,1|t} = \hat{\pi}_1 \hat{y}_{t+1,1|t} + \hat{\pi}_2 \hat{y}_{t+1,2|t}.$$

Of course, one can use  $\hat{y}_{t+1,1|t}$  to calculate a 2-step-ahead forecast of  $y_{t,1}$  and  $y_{t,2}$ . Then use  $\pi_1$  and  $\pi_2$  to calculate the point forecast of  $y_{t+2|t}$ . The above procedure can be iterated to obtain the point forecast of the future value of the time series, i.e.  $y_{t+h}$ .

#### FORECAST PERFORMANCE

The standard for measuring forecastability in context of exchange rates is whether the proposed model can be well in forecasting relative to a random walk (Yuan 2011). Usually, comparison between forecasting models is based on mean squared errors (MSE) as

$$MSE = \frac{1}{h} \sum_{t=1}^h \hat{\varepsilon}_t^2,$$

where  $\hat{\varepsilon}_t = y_{t+i} - \hat{y}_{t+i|t}$ . We compared the in-sample MSE of the forecasts from February 2010 to April 2011 between Markov switching autoregressive and random walk with drift (RWd) processes. The results showed that MSE are 18.59 and 10.73 for RWd and MS-AR, respectively. Therefore, MS-AR achieves superior forecasts relative to the random walk with drift.

Table 2 presents the out-of-sample of the forecasts from May 2011 to October 2011 by MS-AR model. Use  $\hat{\pi}_1$ ,  $\hat{\pi}_2$  (table 1),  $\hat{y}_{t+h,1|t}$  and  $\hat{y}_{t+h,2|t}$  (column 2 and 3 of table 2) to calculate the point forecast of  $y_{t+h|t}$  (column 4 of Table 2). Then using (1) to forecast the exchange rate series in each regimes, i.e.  $\hat{r}_{t+h,2|t}$  and  $\hat{r}_{t+h,1|t}$ , and also the point forecast of  $r_{t+h|t}$  (see column 7 of Table 2).

Figure 4 shows actual data spans from January 2003 to April 2011 and out-of-sample point forecasts spans from May 2011 to May 2013 for fluctuations of U.S. Dollar/ Euro exchange rate by the MSAR model (also, panel (a) of this figure shows forecasts from May 2011 to May 2013 in each regime). The results indicated that the fluctuations of U.S. Dollar/ Euro exchange rate from May 2011 to May 2013 will be rising.

TABLE 2. The out-of-sample forecast from May 2011 to October 2011

$\hat{r}_{t+h,1 t}$	$\hat{r}_{t+h,2 t}$	$\hat{r}_{t+h,1 t}$	$y_{t+h t}$	$\hat{y}_{t+h,2 t}$	$\hat{y}_{t+h,1 t}$	date
1.4605	1.4522	1.4644	0.9989	0.4273	1.2627	May 2011
1.4676	1.4585	1.4719	0.4869	-0.1371	0.7748	Jun 2011
1.4730	1.4636	1.4773	0.3615	-0.2753	0.6554	Jul 2011
1.4778	1.4684	1.4822	0.3308	-0.3091	0.6261	Aug 2011
1.4826	1.4731	1.4870	0.3233	-0.3174	0.6190	Sep 2011
1.4874	1.4779	1.4918	0.3214	-0.3194	0.6172	Oct 2011

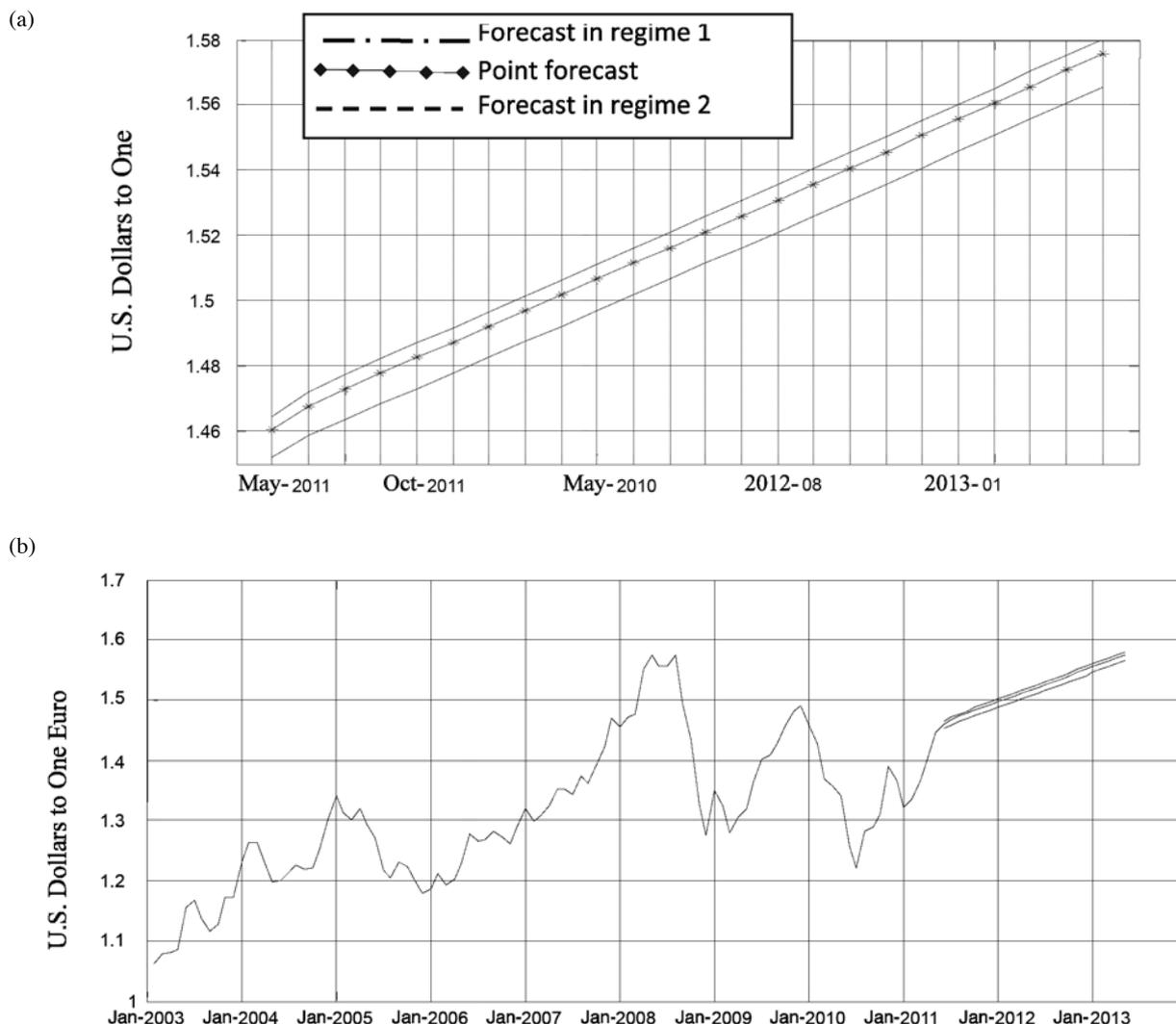


FIGURE 4. Forecast for U.S. Dollars to One Euro Panel (a) appears to a blow-up of the forecast regime (May 2011-May 2013). Panel (b) reports actual data from January 2003 to April 2011 and out-of-sample point forecasts from May 2011 to May 2013 for fluctuations of U.S. Dollar/ Euro exchange rate

## CONCLUSION

This paper outlines techniques for point forecasting into Markov switching autoregressive model. In case of two regimes, using the value of  $h$ -step-ahead forecast data at time  $t$  in each regime and using steady-state probabilities, we present an  $h$ -step-ahead point forecast of data.

Our applications focused on fluctuations of U.S. Dollar/ Euro exchange rate. The fluctuations of U.S. Dollar/ Euro exchange rate have jumps in their behavior. Markov Switching models by a change in their regimes themselves will up to date, when jumps arise in time series data. Hence, this model can be useful for modeling and forecasting this data, which is also confirmed by this study. Our finding demonstrated that MS-AR achieved superior forecasts relative to the random walk with drift. The results of out-of-sample forecast indicated that the fluctuations of U.S. Dollar/ Euro exchange rate from May 2011 to May 2013 will be rising.

## REFERENCE

- Andel, J. 1993. A time series model with suddenly changing parameters. *Journal of Time Series Analysis* 14: 111-123.
- Charfeddine, L. & Guegan, D. 2009. Breaks or long memory behaviour: An empirical investigation. *Documents de travail du Centre d'Economie de la Sorbonne*.
- Cognigni, A. & Manera, M. 2009. The asymmetric effects of oil shocks on output growth: A Markov-Switching analysis for the G-7 countries. *Economic Modelling* 26: 1-29.
- Engel, C., & Hamilton, J.D. 1990. Long switching in the dollar: Are they the data and do Markets know it? *American Economic Review* 80: 689-713.
- Hamilton, J.D. 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica* 57: 357-384.
- Hamilton, J.D. 1994. *Time Series Analysis*. Princeton: Princeton University Press.
- Ismail, M. T. & Z. Isa. 2006. Modelling exchange rates using regime switching models. *Sains Malaysiana* 35(2): 55 - 62.

- Krolzig, H.M. 1998. *Econometrics modelling of Markov switching vector autoregressions using MSVAR for Ox*. Institute of Economics and Statistics and Nuffield College, Oxford University.
- Kuswanto, H. & Sibbertsen, P. 2008. A study on spurious long memory in nonlinear time series models. *Applied Mathematical Science* 2(55): 2713-2734.
- Psaradakis, Z. & Spagnolo, N. 2003. On the determination of the number of regimes in Markov-Switching autoregressive models. *Journal of Time Series Analysis* 24: 237-252.
- Yuan, C. 2011. Forecasting exchange rates: The multi-state Markov-switching model with smoothing. *International Review of Economics and Finance* 20: 342-362.
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