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Investigation of Heat Mass Transfer for Combined Convective Slips Flow: A Lie Group Analysis

(Kajian Pemindahan Haba dan Jisim Bagi Aliran Berolak Gabungan Gelincir: Analisis Kumpulan Lie)

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ABSTRACT

The steady laminar combined convective flow with heat and mass transfer of a Newtonian viscous incompressible fluid over a permeable flat plate with linear hydrodynamic and thermal slips has been investigated numerically. The velocity of the external flow, the suction/injection velocity and the temperature of the plate surface are assumed to vary nonlinearly following the power law with the distance along the plate from the origin. Lie group analysis is used to develop the similarity transformations and the governing momentum, the energy conservation and the mass conservation equations are converted to a system of coupled nonlinear ordinary differential equations with the associated boundary conditions. The resulting equations are solved numerically using the Runge-Kutta-Fehlberg fourth-fifth order numerical method. The effects of hydrodynamic slip parameter (a), thermal slip parameter (b), suction/injection parameter (fw), power law parameter (m), buoyancy ratio parameter (N), Prandtl number (Pr) and Schmidt number (Sc) on the fluid flow, heat transfer and mass transfer characteristics are investigated and presented graphically. We have also shown the effects of the Reynolds number (Re) and the power law parameter (m) on the velocity slip and the thermal slip factors. Good agreement is found between the numerical results of the present paper and published results.

Keywords: Combined convective flow; heat and mass transfer; hydrodynamic and thermal slip; Lie group

ABSTRAK

Aliran berolak tergabung yang mantap dan berlamina dengan pemindahan haba dan jisim bagi suatu bendalir Newtonan likat tak mampat ke atas plat rata yang telap dengan gelinciran linear hidrodinamik dan haba dikaji secara berangka. Halaju aliran luar, halaju sedutan/semburan dan suhu permukaan plat diandaikan berubah terhadap jarak sepanjang plat dari asalan secara tak linear mengikut hukum kuasa. Analisis Kumpulan Lie digunakan untuk memperoleh penjelmaan keserupaan dan persamaan momentum, keabadian tenaga dan keabadian jisim ditukar kepada sistem persamaan pembezaan biasa tak linear dengan syarat sempadan yang sepadan. Persamaan yang terhasil diselesaikan menggunakan kaedah berangka Runge-Kutta-Fehlberg peringkat keempat-kelima. Kesan parameter gelincir halaju a, parameter gelincir haba b, parameter sedutan/semburan (fw), parameter hukum kuasa m, parameter keapungan N, nombor Prandtl (Pr) dan nombor Schmidt (Sc) terhadap aliran bendalir, pemindahan haba dan pemindahan jisim dikaji dan dibentangkan secara bergraf. Kami juga mempamerkan kesan nombor Reynolds Re dan parameter hukum kuasa (m) ke atas faktor gelincir halaju dan faktor gelincir haba. Keputusan berangka dalam makalah ini didapati menepati keputusan yang telah diperoleh dalam penerbitan sebelum ini.

Kata kunci: Gelincir hidrodinamik dan haba; kumpulan Lie; olakan campuran; pemindahan haba dan jisim gabungan

INTRODUCTION

Mixed or combined convection flow is formed when forced and free convection is of comparable magnitude. In nature and in many technological devices, situations arise where forced and free convection act in a simultaneous manner to establish the flow regime, temperature and concentration fields around a heated/cooled permeable/impermeable plate body or stretching sheet. Examples include flow in electronic equipment cooled by a fan and flows in the ocean and in the atmosphere (Ali et al. 2011; Moulic & Yao 2009; Sengupta et al. 2011). There are two types of mixed convection based on the direction of the forced flow, namely, aiding mixed and opposing mixed convection of the forced flow. They originated from buoyancy forces which are induced by density gradients, owing to imposed temperature or concentration differences in the presence of body force. It is found that body forces are usually gravitational force field, the centrifugal force field in rotating fluid machinery, the Coriolis force field in atmospheric and Oceanic rotational motion and in the electromagnetic force field (Incropera et al. 2007). It may change the flow velocity, the temperature and the concentration distributions and hence the wall shears stress, the rate of heat and the rate of mass transfer (Aydın &

Kaya 2007). The laminar boundary layer flow over a flat surface due to mixed convection has received considerable attention for both steady and unsteady situation in evaluating flow parameters for technical purposes and has practical importance in engineering devices. This includes atmospheric boundary layer flows, heat exchangers and electronic equipments. The study of similar solutions may be of use in applications or may give comparisons for approximate methods of calculating more complex non-similar cases (Subhashini et al. 2011).

Mixed convection boundary layer flow over a vertical plate has been studied by several investigators such as Lloyd et al. (1970), Wilks (1973), Yao (1987), Ramachandran et al. (1988), Moulic and Yao (1989). Moulic and Yao (2009) investigated mixed-convection boundary-layer flow over a heated semi-infinite vertical flat plate with a uniform surface heat flux placed in a uniform isothermal upward free stream by perturbation technique. Ishak et al. (2010) studied the steady magneto hydrodynamic mixed convection boundary layer flow of a viscous fluid which is electric conducting around the stagnation-point on a vertical permeable surface. Deswita et al. (2010) investigated the effects of suction/injection for a steady laminar mixed convection boundary layer flow over a permeable horizontal surface of a wedge in a viscous and incompressible fluid. Singh et al. (2011) discussed the effect of surface mass transfer on MHD mixed convection flow past a heated vertical flat permeable surface in the presence of thermophoresis, radiative heat flux and heat source/sink. The mixed convection flow with mass transfer over a stretching surface with suction/ injection was investigated by Jalil et al. (2010) using Lie group analysis. Recently, Subhashini et al. (2011) studied new similarity solution of steady mixed convection boundary layer flow over a permeable surface for convective boundary condition.

It is known that for fluid flows in certain systems, for example, micro electro mechanical systems / nanoscale, the conventional no slip condition at the solid–fluid interface must be replaced by slip condition where the tangential components of the velocity at the surface equal to the velocity gradient normal to the surface (Aziz 2010; Hak 2002). It is also known that slip flow model describe the non-equilibrium region near the interface accurately. Slips may occur on a stationary and moving boundary when the fluid is in particulate form such as in the form of emulsions, suspensions, foams and polymer solutions (Rahman 2010). The effect of linear slip $|u| = l\left|\frac{\partial u}{\partial y}\right|$, where $l > 0$ is the slip length on boundary layers over different flow configurations for both Newtonian and non-Newtonian fluids have been studied widely in the literature on a stationary flat plate as well as moving plates and on a stretching surface by various authors. This includes Anderson (2002), Fang and Lee (2005), Martin and Boyd (2009, 2010), Mathews and Hill (2007), Wang (2009), Fang et al. (2009), Mahmoud (2010), Sahoo (2010). Abbas and Hayat (2009) have studied stagnation slip flow and heat transfer characteristics of a viscous fluid over a nonlinear stretching surface. Cao and Baker (2009) studied laminar mixed convection over an isothermal vertical plate with first-order momentum and thermal discontinuities at the wall. The hydrodynamic and thermal slip flow boundary layer over a flat plate with constant heat flux boundary condition has been investigated by Aziz (2010) who concluded that as the slip parameter increases, the slip velocity increases and the wall shear stress decreases.

Lie group analysis is a classical method to generate similarity solution. The numbers of the independent variables of the partial differential equations are reduced and the independent variables are converted into a single independent variable (called similarity variable). Also, the original initial and boundary conditions become boundary conditions in the new combined variable (White & Subramanian 2010). The main advantage of the Lie group method is that it can be successfully applied to non-linear differential equations. Lie group analysis has been used by various researchers to solve different convective flow phenomena over different geometries under various boundary conditions. Examples include Ibrahim et al. (2007), Pandey et al. (2009), Jalil et al. (2010) and Hamad et al. (2012).

We aim to develop similarity transformations of steady laminar incompressible Newtonian mixed convective flow with heat and mass transfer over a permeable plate taking into account variable slip boundary conditions using Lie group analysis. We further investigate the variation of the flow field, the temperature field and the concentration field within the boundary layers with the hydrodynamic slip parameter, thermal slip parameter, buoyancy ratio parameter, suction/injection parameter, the Prandtl number and the Schmidt number respectively. The similarity equations are solved using the Runge-Kutta-Fehlberg fourth-fifth order numerical method under Maple 13.

FORMULATION

We studied a steady two dimensional mixed convective flow of a viscous incompressible Newtonian fluid of constant density ρ (except in the body force term), temperature T_∞ and concentration C_∞ moving over a nano-porous plate with non-uniform velocity $\bar{u}_e(\bar{x})$ as shown in Figure 1. The field variables within boundary layers are the velocity components $\bar{u}$ along the plate ($\bar{x}$-axis) and $\bar{v}$ perpendicular to the plate ($\bar{y}$-axis), the temperature T and the concentration C. Assume that the viscous dissipation and Joule heating terms in the energy equation are negligible. The governing boundary layer equations in dimensional form are (Incropera et al. 2007)

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \bar{u}_e\frac{d\bar{u}_e}{d\bar{x}} + g\beta\left(T - T_\infty\right) + g\beta^*\left(C - C_\infty\right), \tag{2}$$

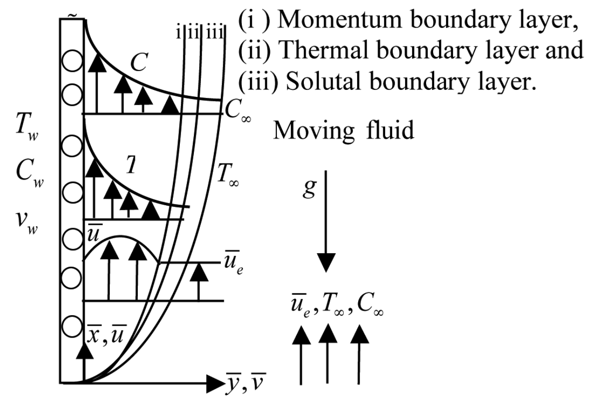


FIGURE 1. Physical configuration and coordinate system of the problem

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2}, \tag{3}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}} + \bar{v}\frac{\partial C}{\partial \bar{y}} = D\frac{\partial^2 C}{\partial \bar{y}^2}. \tag{4}$$

The boundary conditions are taken as (Mukhopadhyay & Anderson 2009)

$$\begin{aligned} &\bar{u} = N_1(\bar{x})\nu\frac{\partial \bar{u}}{\partial \bar{y}}, \bar{v} = -v_w(\bar{x}), T = T_\infty + b_0(\bar{x})(T_w - T_\infty) \\ &+D_1(\bar{x})\frac{\partial T}{\partial \bar{y}}, C = C_w \text{ at } \bar{y} = 0, \\ &\bar{u} \to \bar{u}_e(\bar{x}) = U\left(\frac{\bar{x}}{L}\right)^m, T \to T_\infty, C \to C_\infty \text{ as } \bar{y} \to \infty, \end{aligned} \tag{5}$$

where ν: the kinematic coefficient of viscosity, α: thermal diffusivity, D: concentration diffusivity N_1: the velocity slip factor having dimension (velocity)$^{-1}$, D_1: the thermal slip factor having dimension, v_w: transpiration velocity ($v_w < 0$ for injection, $v_w > 0$ for suction), κ: thermal conductivity, β: volumetric coefficient of thermal expansion, β^*:volumetric coefficient of mass expansion. The subscripts w, ∞ denote wall conditions and free stream conditions respectively.

NONDIMENSIONALIZATION

Introducing following dimensionless variables:

$$\begin{aligned} &x = \frac{\bar{x}}{L}, y = \frac{\bar{y}}{L}\sqrt{\text{Re}}, u = \frac{\bar{u}}{U}, v = \frac{\bar{v}}{U}\sqrt{\text{Re}}, \\ &\theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty}, u_e = \frac{\bar{u}_e(\bar{x})}{U}. \end{aligned} \tag{6}$$

Also introducing stream function ψ defined by:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}. \tag{7}$$

The momentum, the energy and the concentration equations reduce to,

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^3 \psi}{\partial y^3} - u_e\frac{du_e}{dx} - \theta - N\phi = 0, \tag{8}$$

$$\Pr\left(\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}\right) - \frac{\partial^2 \theta}{\partial y^2} = 0, \tag{9}$$

$$Sc\left(\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y}\right) - \frac{\partial^2 \phi}{\partial y^2} = 0. \tag{10}$$

The boundary conditions in equation (5) yield:

$$\begin{aligned} &\frac{\partial \psi}{\partial y} = a_0(x)\frac{\partial^2 \psi}{\partial y^2}, \frac{\partial \psi}{\partial x} = f_0(x), \\ &\theta = b_0(x) + \gamma_0(x)\frac{\partial \theta}{\partial y}, \phi = 1 \text{ at } y = 0, \\ &\frac{\partial \psi}{\partial y} \to x^m, \theta \to 0, \phi \to 0 \text{ as } y \to \infty. \end{aligned} \tag{11}$$

Here $a_0(x) = \frac{N_1(x)\nu\sqrt{\text{Re}}}{L}$, $\gamma_0 = \frac{D_1(x)\sqrt{\text{Re}}}{L}$, $f_0(x) = \frac{v_w(x)\sqrt{\text{Re}}}{U}$, $\Pr = \frac{\mu c_p}{\kappa}$, $Sc = \frac{\nu}{D}$, $N = \frac{\beta \Delta C}{\beta^* \Delta T}$ and $\lambda = \frac{g\beta\,\Delta T\, L}{U^2} = \frac{Gr}{\text{Re}^2} = 1$, L is the characteristic length, U is some reference velocity, is the Reynolds number, Pr stands for Prandtl number, Sc stands for Schmidt number, λ stands for mixed convection parameter and N stands for buoyancy ratio parameter. Note that $N = 0$ means no species diffusion, $N \to \infty$ means no thermal diffusion and $N > 0$ means that both thermal and solutal effects driven the flow.

SYMMETRIES ANALYSIS

Consider the following one parameter (ε) Lie group of infinitesimals (ξ´s, τ´s) transformations

$$\begin{aligned} x^* &= x + \varepsilon\xi_1(x^*, y^*, \psi^*, \theta^*, \phi^*) + O(\varepsilon^2), \\ y^* &= y + \varepsilon\xi_2(x^*, y^*, \psi^*, \theta^*, \phi^*) + O(\varepsilon^2), \\ \psi^* &= \psi + \varepsilon\tau_1(x^*, y^*, \psi^*, \theta^*, \phi^*) + O(\varepsilon^2), \\ \theta^* &= \theta + \varepsilon\tau_2(x^*, y^*, \psi^*, \theta^*, \phi^*) + O(\varepsilon^2), \\ \phi^* &= \phi + \varepsilon\tau_3(x^*, y^*, \psi^*, \theta^*, \phi^*) + O(\varepsilon^2). \end{aligned} \tag{12}$$

where ε is a real small parameter and the variables $(x, y, \psi, \theta, \phi)$ transformed into $(x^*, y^*, \psi^*, \theta^*, \phi^*)$, which keep the partial differential system (8)-(11) invariant. By carrying out long and rather tedious algebra, the form of the infinitesimals are found to be as follows:

$$\begin{aligned} &\xi_1 = c_1 x,\ \xi_2 = c_1\frac{1-m}{2}y + g(x),\ \tau_1 = c_1\frac{1+m}{2}\psi + c_2, \\ &\tau_2 = c_1(2m-1)\theta - c_3 N,\ \tau_3 = c_1(2m-1)\phi + c_3. \end{aligned} \tag{13}$$

where c_i (i = 1,2,3) are constants and $g(x)$ is arbitrary function. It is noticed from (13) that the parameter c_2 corresponds to the translation in the variable ψ, while the parameters c_3 corresponds to the translation in the variables θ and ϕ, respectively. It is further noticed that the parameter c_1 corresponds to the scaling in the variables x, y, ψ, θ and ϕ, respectively. The generators corresponding to the infinitesimal (13) are as follows:

$$X_1 = x\frac{\partial}{\partial x} + \left(\frac{1-m}{2}y + g(x)\right)\frac{\partial}{\partial y} + \frac{1+m}{2}\psi\frac{\partial}{\partial \psi} + (2m-1)\theta\frac{\partial}{\partial \theta} + (2m-1)\phi\frac{\partial}{\partial \phi},$$
$$X_2 = g(x)\frac{\partial}{\partial y} + \frac{\partial}{\partial \psi}, X_3 = g(x)\frac{\partial}{\partial y} + \frac{\partial}{\partial \psi} - N\frac{\partial}{\partial \theta} + \frac{\partial}{\partial \phi}. \tag{14}$$

The infinitesimals in Equation (13) give the following transformations:

$$\eta = \frac{y}{x^{\frac{1-m}{2}}} - \int \frac{g(x)}{2c_1 x^{\frac{3-m}{2}}}dx, \psi = x^{\frac{1+m}{2}}F(\eta) - \frac{c_2}{c_1(1+m)},$$
$$\theta = x^{2m-1}G(\eta) - \frac{c_3 N}{c_1(1-2m)}, \phi = x^{2m-1}H(\eta) + \frac{c_3}{c_1(1-2m)}. \tag{15}$$

As the stream function is determined up to an arbitrary additive constant, we may set $c_2 = 0$. Here *F, G,* and *H* stand for the dimensionless velocity, the temperature and the concentration functions, respectively.

BOUNDARY CONDITIONS

Applying the transformations in (15) to the boundary conditions (11), we have found that to transform $y = 0$ to $\eta = 0$, we have to set $g(x) = 0$. To ensure that $G \to 0$, $H \to 0$ as $\eta \to \infty$, we set $c_3 = 0$.

SIMILARITY EQUATIONS

Substituting (15) into (8) - (10) leads to the following nonlinear system of similarity equations:

$$F''' + \frac{m+1}{2}F F'' + m\left(1 - F'^2\right) + G + N H = 0, \tag{16}$$

$$G'' + \frac{m+1}{2}F G' + (1-2m)F'G = 0, \tag{17}$$

$$H'' + \frac{m+1}{2}F H' + (1-2m)F'H = 0. \tag{18}$$

The boundary conditions become:

$$F(0) = \frac{2}{m+1}fw, F'(0) = aF''(0),$$
$$G(0) = 1 + bG'(0), H(0) = 1,$$
$$F'(\infty) - 1 = G(\infty) = H(\infty) = 0. \tag{19}$$

For similarity solutions to exist we get:

$$f_0(x) = fw\, x^{\frac{1-m}{2}}, a_0(x) = a\, x^{\frac{1-m}{2}},$$
$$b_0(x) = x^{2m-1}, \gamma_0(x) = b\, x^{\frac{1-m}{2}}. \tag{20}$$

where the constant a stands for hydrodynamic slip parameter, constant b stands for thermal slip parameter and the constant fw stands for suction parameter. The form of the velocity slip factor $N_1(\bar{x})$ and the thermal slip factor $D_1(\bar{x})$ are:

$$N_1(\bar{x}) = \frac{aL}{v\sqrt{\mathrm{Re}}}\left(\frac{\bar{x}}{L}\right)^{(1-m)/2}, \; D_1(\bar{x}) = \frac{bL}{\sqrt{\mathrm{Re}}}\left(\frac{\bar{x}}{L}\right)^{(1-m)/2}. \tag{21}$$

From (21) we can discuss the variation of the slip factors versus the axial distance $\bar{x}$ for different values of the physical parameters, namely, Reynolds number, kinematic viscosity. In our study we show the effect of Reynolds number Re and the index parameter *m*. The quantities of physical interest are the local skin friction factor, local Nusselt number and the local Sherwood number which are proportional to the numerical values of $F''(0)$, $-G'(0)$ and $-H'(0)$.

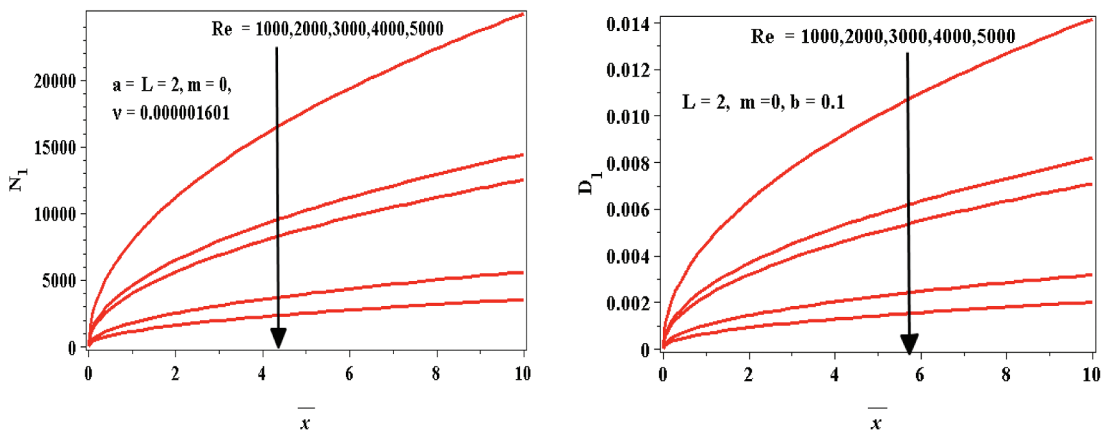
COMPARISONS

If the exponent $m = 1$, velocity slip parameter $a = 0$ (no slip) and the thermal slip parameter $b = 0$ (isothermal plate), the equations (16)-(18) and the boundary conditions (19) reduce to that of Ishak et al. (2010), if we ignore the magnetic field effect in that paper and to that of Lok et al. (2005), if we ignore the microrotation equation and $K = 0$ in that paper.

RESULTS AND DISCUSSION

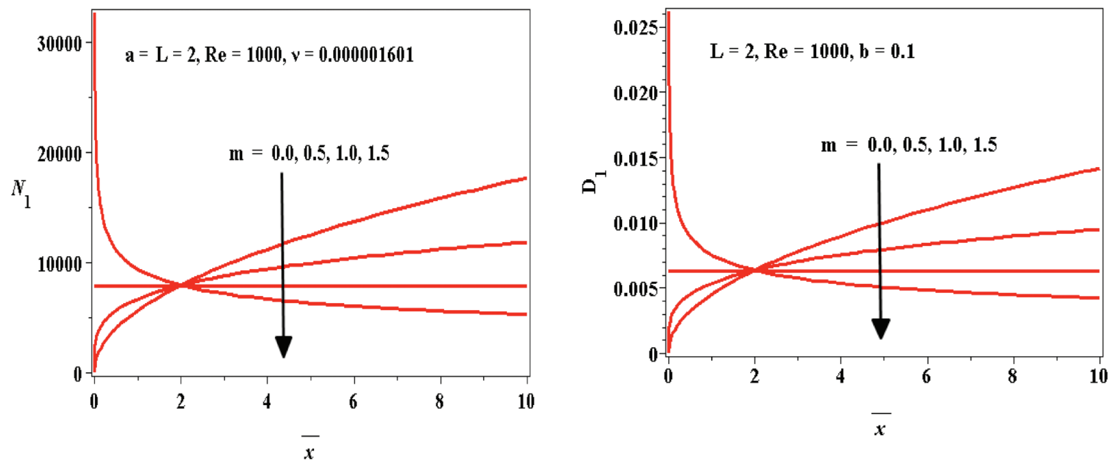
The numerical solution of (16)-(18) subject to boundary conditions in (19) is obtained for various values of the controlling parameters Pr, *Sc, b, N, m, fw* and *a* by fourth-fifth order Runge-Kutta-Fehlberg numerical method under Maple 13 proposed by Aziz (2009). Results are plotted in Figures 2 to 9 to show the influence of flow parameters on the flow, heat and mass transfer characteristics. To justify the accuracy of our results a comparison of the skin friction factor and the rate of heat transfer coefficient for $fw = 0$, $N = 0$, $b = 0$ and $m = 1$ at different values of Prandtl number Pr are made with that of Hassanien and Gorla (1990), Devi et al. (1991), Ramachandran et al. (1998) and Lok et al. (2005). From Tables 1 and 2 we notice that there is a close agreement with these approaches and thus verifies the accuracy of our numerical. Further, the values of $F'(0)$, $F''(0)$, $G(0)$, $G'(0)$ and $-H'(0)$ for different nonzero values of the governing parameters are summarised in Tables 3 and 4 by taking $\eta_{max} = 10$.

Figures 2 and 3 illustrate the Reynolds number and the index parameter *m* effect on the hydrodynamic slip and the



(a) (b)

FIGURE 2. Effects of Reynolds number on (a) the velocity slip factor and (b) on the thermal slip factor for $v = 16.01 \times 10^{-6}$ m^2/s for air at 30°C



(a) (b)

FIGURE 3. Effects of index m on (a) the velocity slip factor and (b) on the thermal slip factor

TABLE 1. Comparison of the values of $F'(0)$ for various values of Pr when $fw = 0$, $m = 1$, $a = 0$, $b = 0$ and $N = 0$ with previous published work

Pr	Ramachandran et al. (1988)	Devi et al. (1991)	Lok et al. (2005)	Hassanien and Gorla (1990)	Present results
0.7	1.7063	1.7064	1.706376	1.70632	1.706324
1.0	-	-	-	-	1.675437
7.0	1.5179	1.5180	1.517952	-	1.517913
10	-	-	-	1.49284	1.492839
20	1.4485	1.4485	1.448520	-	1.448483
40	1.4101	-	1.410094	-	1.410058
50	-	-	-	1.40686	1.398930
60	1.3903	1.3903	1.390311	-	1.390274
80	1.3774	-	1.377429	-	1.377392
100	1.3680	1.3680	1.368070	1.38471	1.368033

TABLE 2. Comparison of the values of $-G'(0)$ for various values of Pr when $fw = 0$, $m = 1$, $a = 0$, $b = 0$, $N = 0$ with previous published work

Pr	Ramach-andran et al. (1988)	Devi et al. (1991)	Lok et al. (2005)	Hassanien and Gorla (1990)	Present results
0.7	0.7641	0.7641	0.764087	0.76406	0.7640633803
1	-	-	-	-	0.8707785644
7	1.7224	1.7223	1.722775	-	1.7223816041
10	-	-	-	1.94461	1.9446172814
20	2.4576	2.4574	2.458836	-	2.4575897227
40	3.1011	-	3.103703	-	3.1010930548
50	-	-	-	3.34882	3.3414576259
60	3.5514	3.5517	3.555404	-	3.5514059167
80	3.9095	-	3.914882	-	3.9094810380
100	4.2116	4.2113	4.218462	4.23372	4.2116454962

thermal slip factors. It is apparent that as Reynolds number Re and index parameter m rise, both the velocity and the thermal slip factors fall while as the axial distance $\bar{x}$ rises for fixed m both the velocity and the thermal slip factor rise, as expected. The reason for this type of behaviour is that both slip factors is inversely proportional to the Reynolds number and axial distance for $m > 1$.

Figure 4 depicts the hydrodynamic slip effects on the dimensionless velocity, the temperature and the concentration profiles for nonzero values of the remaining flow controlling parameters. It is apparent that the dimensionless velocity profiles $F'(\eta)$ rises monotonically with the rising of the hydrodynamic slip parameter (a). The fluid dimensionless axial velocity at the plate surface is 0 when hydrodynamic slip is zero (conventional no-slip case). The slip velocity at the wall increases with the increase in linear momentum slip parameter (a).The velocity of fluid at the vicinity of the plate has some positive value because of the hydrodynamic slip boundary condition at the plate and accordingly the thickness of the hydrodynamic boundary layer decreases. Physically, as the momentum slip parameter enhances, the penetration of the stagnant surface through the fluid domain decreases leading to a reduction in the hydrodynamics boundary layer. Increasing slip factor may be looked at as a miscommunication between the stationary plate and the moving fluid. Dimensionless temperature $H(\eta)$ is found to be decreased with the increasing of the momentum slip parameter a. The physics behind it, is, as the momentum slip parameter enhances, more flow will penetrate through the boundary layer due to slipping effect. As a result hot plate heats more amounts of fluid and this leads to decrease in the temperature profiles. Further, dimensionless concentration is decreased with the increasing value of the momentum slip parameter. Physically, the fluid next to the wall surface of the plate is heated up by the hot fluid on the wall surface of the plate, making it become lighter and flow faster.

Suction fw effects on the dimensionless velocity, the temperature and the concentration distributions are exhibited in the graphs of Figure 5. As the suction fw increases the velocity as well as the thickness of the

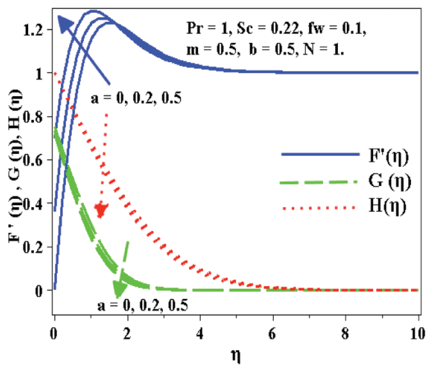


FIGURE 4. Effects of hydrodynamic slip a on the dimensionless velocity $F'(\eta)$, the temperature $G(\eta)$ and the concentration $H(\eta)$ profiles

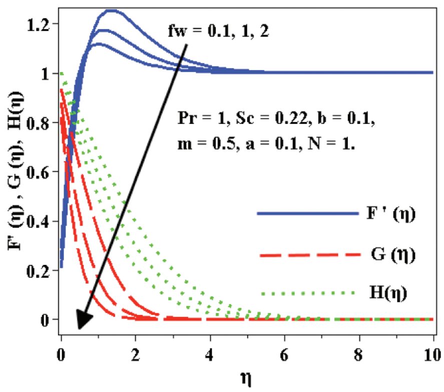


FIGURE 5. Effects of suction on the dimensionless velocity $F'(\eta)$, the temperature $G(\eta)$ and the concentration $H(\eta)$ profiles

momentum boundary layer decrease. The physics behind this is, in case of suction, the heated fluid is pushed towards the wall where the buoyancy forces can act to retard the fluid due to high influence of the viscosity. We further observe that the thermal and the concentration boundary layer thicknesses decrease with the increasing of suction parameter which causes an increase in the rate of the heat transfer and the rate of the mass transfer; this is what we can see in Table 4. The physics behind this type of behaviour is that the fluid is brought closer to the surface and reduces the thermal boundary layer thickness. As such the presence of suction decreases the velocity, the thermal and the concentration boundary layer thicknesses.

The influence of the Prandlt number Pr on the dimensionless velocity, the temperature and the concentration profiles is shown in Figure 6. In the presence of the hydrodynamic slip, this figure shows that as Prandtl number Pr increases, the velocity and the temperature reduces at each location in the boundary layers whilst the concentration profiles increases, as in the classical case with conventional no slip boundary condition. Physically, lower Pr means fluids having higher thermal diffusivity α and lower dynamic viscosity μ. The heating effect of the hot wall is able to penetrate deeper and deeper through the fluid as α increases and this yields higher fluid temperatures. Also, more fluid penetrates through the boundary layer as μ decreases and the hot plate is supposed to heat less amount of fluid. This in turns yields higher temperatures for the plate at lower values of Pr. Consequently, the heat transfer rate increases with the rising of Pr; this is what we noticed from Table 3.

Figure 7 presents the dimensionless velocity, the temperature and the concentration profiles for Schmidt number $Sc = 0.22, 0, 78, 1$ with nonzero values of the other controlling parameters. It is found that as Schmidt number Sc increases, the velocity and the concentration decreases whilst the temperature increases as expected.

Physically, higher Schmidt number implies either lower solutal diffusivity for a uniform fluid dynamic viscosity or

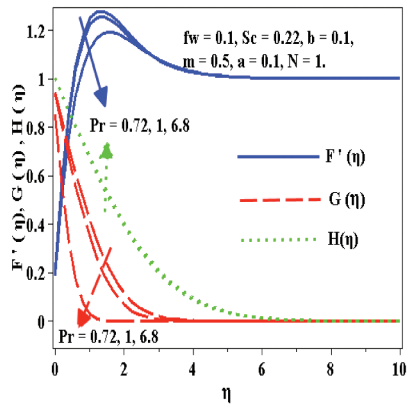


FIGURE 6. Effect of Prandtl number Prbon the dimensionless velocity $F'(\eta)$, the temperature $G(\eta)$ and the concentration $H(\eta)$ profiles

TABLE 3. Values of $F'(0)$, $F''(0)$, $G(0)$,$-G'(0)$ and $-H'(0)$ for $Sc = 0.22$, $fw = 0.1$, $m = 0.5$ and $N = 1$

Pr	a	b	$F'(0)$,	$F''(0)$	$G(0)$	$-G'(0)$	$-H'(0)$
0.7	0.5	0.2	0.729654	1.459308	0.883363	0.583184	0.36282
7			0.645853	1.291708	0.698225	1.5088758	0.349840
10			0.6355694	1.2711388	0.6532273	1.7338634	0.3487865
100			0.5967331	1.1934662	0.2755018	3.6224909	0.3465569
0.7	1	0.2	1.037889	1.037889	0.877764	0.611180	0.376132
	2		1.299216	0.649608	0.873263	0.633687	0.386959
	0.5	0.6	0.705177	1.410355	0.718407	0.469321	0.35982
		1.4	0.675885	1.351770	0.525531	0.33891	0.356188

TABLE 4. Values of $F'(0)$, $F''(0)$, $G(0)$, $-G'(0)$ and $-H'(0)$ for $Pr = 1$, $Sc = 0.22$, $b = 0.1$, $a = 0.1$

N	m	fw	$F'(0)$	$F''(0)$	$G(0)$	$-G'(0)$	$-H(0)$
0.8	0.4	0.3	0.207515	2.075153	0.933948	0.660523	0.293822
1.0			0.2211188	2.2111883	0.9329879	0.6701205	0.2983436
1.5			0.2537247	2.5372474	0.9308254	0.6917460	0.308594
0.8	0.6		0.199260	1.992602	0.9160289	0.8397107	0.4122794
	1		0.1981996	1.9819960	0.8878591	1.121408	0.4048476
	0.4	1	0.235380	2.3538003	0.8878591	1.121408	0.4048476
		3	0.314121	3.141218	0.7625443	2.37455	0.7732095

higher dynamic viscosity for uniform solutal diffusivity. Solutal diffusivity depends on the concentration and hence increasing Schmidt number reduces the concentration and velocity, as expected.

Figure 8 shows the influence of buoyancy ratio parameter *N* on the dimensionless velocity, the temperature and the concentration distributions within the boundary layers. It is found that as *N* increases, the velocity increases due to favourable slip velocity near the plate surface whilst the temperature and the concentration contribution in immigration of fluid particles from the vertical surfaces in other words the temperature and the concentration decreases. The physical reason is that an increase in *N* implies an addition of buoyancy-induced flow onto the external flow, as a results the velocity is increased. We observe that the temperature and the concentration is maximum at the plate surface and reduces exponentially to zero value far away from the plate, which fulfils the asymptotic boundary condition.

Figure 9 illustrates the variation of the thermal slip *b* on the dimensionless velocity and the temperature profiles. We notice that *b* leads to a reduction in both the velocity and the temperature profiles. Physically, as the thermal slip parameter *b* increases, the fluid in the boundary layer will not sense the heating effects of the plate and less amount of heat will be transferred from the hot plate to the fluid and hence the velocity, wall shear stress, the temperature and the rate of heat transfer decrease (Table 3).

The effects of the flow controlling parameters Pr, *a* and *b* on the wall velocity, the skin friction factor or wall shear stress, the wall heat transfer, the rate of heat transfer and the rate of mass transfer are shown in Table 3 whilst the effects of controlling parameters *N, m* and *fw* on the wall velocity, the skin friction factor or wall shear stress, the wall heat transfer, the rate of heat and the rate of mass transfers are shown in Table 4.

Table 3 has been constructed to exhibit the effect of the index parameter hydrodynamic slip *a*, thermal slip *b* and Prandtl number Pr, on fluid flow, heat and mass transfer characteristics of the flow. The values of wall velocity $F'(0)$, the skin friction factor $F''(0)$, the wall temperature $G(0)$, the temperature gradient at the plate $-G'(0)$, the concentration gradient at the plate $-H'(0)$, are shown in Tables 3 for different values of Prandtl number, hydrodynamic and thermal slip parameter. We noticed that the shear stress $F''(0)$ decreases with increasing of the momentum slip parameter *a*. As the slip parameter increases in magnitude, permitting more fluid to slip past the plate, the flow accelerates adjacent to the plate and consequently, the shear stress $F''(0)$ decreases. The rate of heat and mass transfer is rising with the rising of the velocity slip parameter. The enhanced velocity due to linear hydrodynamic slip adjacent to the plate is the reason of rising heat transfer $-G'(0)$ and mass transfer $-H'(0)$. The wall velocity, the wall shear stress, the wall heat transfer, the rate of mass transfer decrease with the increasing Pr.

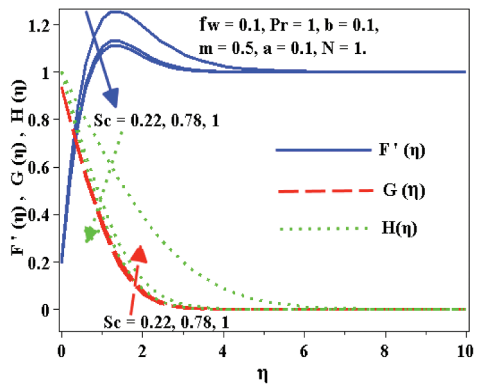


FIGURE 7. Effect of Schmidt number *Sc* on the dimensionless velocity $F'(0)$, the temperature $G(\eta)$ and the concentration $H(0)$ profiles

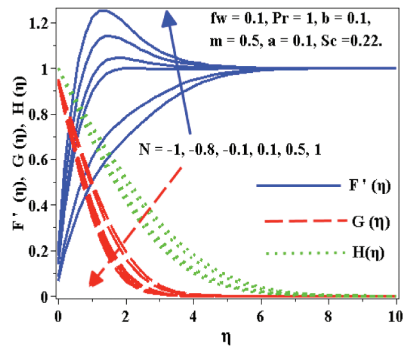


FIGURE 8. Effects of buoyancy ratio *N* on the dimensionless velocity $F'(\eta)$, the temperature $G(\eta)$ and the concentration $H(\eta)$ profiles

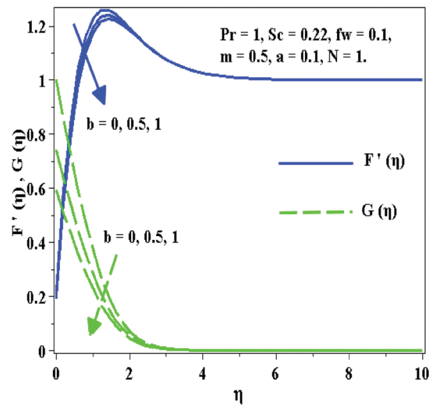


FIGURE 9. Effects of thermal slip *b* on the dimensionless velocity $F'(\eta)$ and the temperature $G(\eta)$ profiles

Also the rate of heat transfer increases with the increase in Pr, as discussed before.

Table 4 has been made to highlight the effect of the index parameter *m*, suction parameter *fw* and buoyancy ratio parameter *N* on fluid flow, heat and mass transfer characteristics of the flow. Note that we have tabulated the values for positive *N* (aiding flow). It is found form

Figure 3(a) that velocity slip factor N_1 is decreasing with the increasing of power law parameter m, leads to decrease in the momentum slip and hence decrease in the slip velocity at the wall, which in turn increase the wall shear stress. This is what we can see in Table 4. A parallel explanation is true for heat transfer rate. With the increase in the buoyancy ratio parameter, the slip velocity, the wall shear stress and the rate of heat transfer increase whilst the wall heat transfer and the rate of mass transfer decrease, as expected (see Lai 1991). We further noticed that suction increases slip velocity, the wall shear stress, the rate of heat transfer, the rate of mass transfer but it decreases the wall heat transfer, the reason behind it is already discussed before.

Conclusion

Heat and mass transfer due to steady mixed convection of a viscous incompressible fluid flow near a vertical permeable flat plate subject to variable hydrodynamic and thermal slip boundary conditions has been examined by Lie group analysis. The numerical solutions have been reported for various governing parameters. It is found that the velocity, the velocity slip factor as well as the temperature and thermal slip factors reduce as the index parameter, the Reynolds number and the thermal slip parameter increases. The velocity profiles increases while the temperature and the concentration profiles decreases as the thermal slip factor and the buoyancy parameter increase. The wall velocity, the wall shear stress and the rate of heat transfer increase whilst the wall heat transfer, the rate of mass transfer decrease with increasing buoyancy parameter. The wall velocity, the wall shear stress, the wall heat transfer, the rate of mass transfer decrease as Prandtl and the thermal slip increases. The slip velocity, the rate of heat transfer and the rate of mass transfer increase whilst the shear stress and the wall heat transfer decrease as the velocity slip parameter increases.

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