

## Investigation of the Laminar Viscous Flow in a Semi-Porous Channel in the Presence of Uniform Magnetic Field using Optimal Homotopy Asymptotic Method

(Kajian tentang Aliran Likat Lamina dalam Terusan Semi-Berliang dengan Kehadiran Medan Magnet Seragam menggunakan Kaedah Homotopi Asimptot Optimum)

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### ABSTRACT

*In this paper, the problem of laminar viscous flow in a semi-porous channel in the presence of transverse magnetic field is studied. The Optimal Homotopy Asymptotic Method (OHAM) is employed to approximate the solution of the system of nonlinear differential equations governing the problem. The influence of the Hartmann number ( $Ha$ ) and the Reynolds number ( $Re$ ) on the flow was investigated. The results of the OHAM were compared with homotopy analysis method (HAM) and variation iteration method (VIM) results.*

*Keywords: Laminar viscous flow; magnetic field; optimal homotopy asymptotic method; porous channel*

### ABSTRAK

*Dalam makalah ini, masalah aliran likat lamina dalam terusan semi-berliang dengan kehadiran medan magnet dikaji. Kaedah homotopi asimptot optimum (KHAO) digunakan untuk mencari penyelesaian hampiran bagi sistem persamaan terbitan tak linear yang menakluk model itu. Kesan nombor Hartmann ( $Ha$ ) dan nombor Reynolds ( $Re$ ) terhadap aliran diselidiki. Hasil daripada KHAO dibandingkan dengan hasil daripada kaedah analisis homotopi (KAH) dan kaedah lelaran ubahan (KLU).*

*Kata kunci: Aliran likat lamina; kaedah homotopi asimptot optimum; medan magnet; terusan berliang*

### INTRODUCTION

The flow problem in porous tubes or channels has been under considerable attention in recent years because of its various applications in biomedical engineering, for example, in the dialysis of blood in artificial kidney (Wernert et al. 2005), in the flow of blood in the capillaries (Jafari et al. 2009), in the flow in blood oxygenators (Goerke et al. 2002), as well as in many other engineering areas such as the design of filters (Mneina & Martens 2009), in transpiration cooling boundary layer control (Andoh & Lips 2003) and gaseous diffusion (Runstedtler 2006). More recently, Chandran et al. (1996) analyzed the effects of a magnetic field on the thermodynamic flow past a continuously moving porous plate.

Since there are some limitations with the common perturbation method and also because of this fact that the basis of the common perturbation method is upon the existence of a small parameter, developing this method for different applications is very difficult. Therefore, different methods have recently introduced some ways to eliminate the small parameter, such as the Homotopy Perturbation Method (He 2005), Differential Transformation Method (Joneidi et al. 2009) and Homotopy Analysis Method (Ziabakhsh & Domairry 2009). Optimal Homotopy Asymptotic Method (OHAM) is another method which is a powerful one for solving nonlinear problems without

depending to the small parameter and is developed and examined by some authors (Marinca & Herişanu 2008).

In this study, OHAM is applied to find the approximate solutions of nonlinear differential equations governing non-Newtonian fluid flow in an axisymmetric channel with a porous wall for turbine cooling applications and have made a comparison with the Numerical solution results. The fourth order Runge-Kutta method has been used and considered as the numerical solution for validity of this method.

### PROBLEM STATEMENT AND MATHEMATICAL FORMULATION

Consider the laminar two-dimensional stationary flow of an electrically conducting incompressible viscous fluid in a semi-porous channel made by a long rectangular plate with length of  $L_x$  in uniform translation in  $x^*$  direction and an infinite porous plate.

The distance between the two plates is  $h$ . The physical fluid properties ( $\rho$ ,  $\mu$ ) are constant. We observed a normal velocity  $q$  on the porous wall. A uniform magnetic field  $B$  is assumed to be applied towards direction  $y^*$  (Figure 1) (Desseaux 1999).

In the case of a short circuit to neglect the electrical field and perturbations to the basic normal field and without

any gravity forces, according to Ziabakhsh and Domairry (2009), the governing equations are:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0, \tag{1}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial P^*}{\partial x^*} + \nu \left( \frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right) - u^* \frac{\sigma \beta^2}{\rho}, \tag{2}$$

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial P^*}{\partial y^*} + \nu \left( \frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right). \tag{3}$$

The appropriate boundary conditions for the velocity are:

$$y^* = 0 : u^* = u_0^*, v^* = 0. \tag{4}$$

$$y^* = h : u^* = 0, v^* = -q. \tag{5}$$

Calculating a mean velocity  $U$  by the relation:

$$y^* = 0 : u^* = u_0^*, v^* = 0. \tag{6}$$

We consider the following transformations:

$$x = \frac{x^*}{L_x}; y = \frac{y^*}{h}, \tag{7}$$

$$u = \frac{u^*}{U}; v = \frac{v^*}{q}, P_y = \frac{P^*}{\rho q^2}. \tag{8}$$

Then, we can consider two dimensionless numbers: the Hartman number  $Ha$  for the description of magnetic forces (Desseaux 1999) and the Reynolds number  $Re$  for dynamic forces:

$$Ha = Bh \sqrt{\frac{\sigma}{\rho \nu}}. \tag{9}$$

$$Re = \frac{hq}{\nu}. \tag{10}$$

Introducing equations (6) and (10) into equations (1) and (3) leads to the dimensionless equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{11}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\varepsilon^2 \frac{\partial P_y}{\partial x} + \frac{\nu}{hq} \left( \varepsilon^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - u \frac{Ha^2}{Re}, \tag{12}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial P_y}{\partial x} + \frac{\nu}{hq} \left( \varepsilon^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right). \tag{13}$$

Quantity of  $\varepsilon$  is defined as the aspect ratio between distance  $h$  and a characteristic length  $L_x$  of the slider. This ratio is normally small. Berman's similarity transformation is used to be free from the aspect ratio of  $\varepsilon$ :

$$v = -V(y); u = \frac{u^*}{U} = u_0 U(y) + x \frac{dV}{dy}. \tag{14}$$

Introducing equation (14) in the second momentum equation (13) shows that quantity  $\partial P_y / \partial y$  does not depend on the longitudinal variable  $x$ . With the first momentum equation, we also observe that  $\partial^2 P_y / \partial x^2$  is independent of  $x$ . We omit asterisks for simplicity. Then a separation of variables leads to (Desseaux 1999):

$$V'^2 - VV'' - \frac{1}{Re} V''' + \frac{Ha^2}{Re} V' = \varepsilon^2 \frac{\partial^2 P_y}{\partial x^2} = \varepsilon^2 \frac{1}{x} \frac{\partial P_y}{\partial x}. \tag{15}$$

$$UV' - VU' = \frac{1}{Re} [U'' - Ha^2 U]. \tag{16}$$

The right-hand side of equation (15) is constant. So, we derived this equation with respect to  $x$ . This gives:

$$V^{IV} = Ha^2 V'' + Re[V'V'' - VV''']. \tag{17}$$

where primes denote differentiation with respect to  $y$  and asterisks have been omitted for simplicity. The dynamic boundary conditions are:

$$y = 0: U = 1; V = 0; V' = 0. \tag{18}$$

$$y = 1: U = 0; V = 1; V' = 0. \tag{19}$$

SOLUTION WITH HOMOTOPY ASYMPTOTIC METHOD

In this section, OHAM is applied to nonlinear ordinary differential equations (16) and (17). According to the OHAM, equations (16) and (17) lead to:

$$(1-p)[V^{IV}] - H_1(p)[V^{IV} - Ha^2 V'' - Re[V'V'' - V''']], \tag{20}$$

$$(1-p)[U''] - H_1(p)[UV' - VU' = \frac{1}{Re} [U'' - Ha^2 U]].$$

where primes denote differentiation with respect to  $y$ . We consider  $V, U, H_1(p)$  and  $H_2(p)$  as following:

$$V = V_0 + pV_1 + p^2V_2, U = U_0 + pU_1 + p^2U_2. \tag{21}$$

Substituting  $V, U, H_1(p)$  and  $H_2(p)$  from equations (21) into (20) and some simplification and rearranging based on powers of  $p$ -terms, we have:

$$p^0: V^{IV} = 0, V_0(0) = 0, V_0'(0) = 0, V_0(1) = 1, V_0'(1) = 0$$

$$U' = 0, U_0(0) = 1, U_0(1) = 0. \tag{22}$$

$$p^1: V_1^{IV} + C_{11}V_0^V - C_{11}ReV_0''V_0' + C_{11}ReV_0''V_0 - C_{11}Ha^2V_0'' - V_0^{IV} = 0,$$

$$V_1(0) = 0, V_1'(0) = 0, V_1(1) = 0, V_1'(1) = 0$$

$$U_1'' - C_{21}ReV_0'U_0 + C_{22}U_0'' - U_0'' + C_{21}Ha^2U_0 = 0,$$

$$U_1(0) = 0, U_1(1) = 0. \tag{23}$$

Solving equations (22) and (23) with boundary conditions:

$$f_0(y) = -2y^3 + 3y^2, \quad q_{n_0}(y) = -y+1, \quad (24)$$

$$\begin{aligned} f_1(y) = & C_{11}(0.05714285714 \operatorname{Re} y^7 - 0.2y^6 \\ & - 0.1 Ha^2 y^5 + 0.25 Ha^2 y^4 \\ & - 0.3857142857 \operatorname{Re} y^3 \\ & - 0.2 Ha^2 y^3 + (0.22855714286 \operatorname{Re} + 0.05) y^2) \end{aligned}$$

$$\begin{aligned} g(y) = & C_{21}(0.2 \operatorname{Re} y^5 - 0.75 \operatorname{Re} y^4 + \operatorname{Re} y^3 \\ & - 0.1667 Ha^2 y^3 + 0.5 Ha^2 y^2 \\ & - 0.45 \operatorname{Re} y - 0.3333 Ha^2 y) \end{aligned} \quad (25)$$

The terms of  $V_2(y)$  and  $U_2(y)$  are too large that mentioned graphically. Therefore the final expression for  $V(y)$  and  $U(y)$  is:

$$\begin{aligned} V(y) &= V_0(y) + V_1(y) + V_2(y) \\ U(y) &= U_0(y) + U_1(y) + U_2(y). \end{aligned} \quad (26)$$

Substituting  $V(y)$  and  $U(y)$  into equation (20),  $R_1(\eta)$ ,  $C_{11}$ ,  $C_{12}$  and  $R_2(\eta)$ ,  $C_{21}$ ,  $C_{22}$  are obtained then  $J_1$  and  $J_2$  are obtained in the flowing manner:

$$\begin{aligned} J_1(C_{11}, C_{12}) &= \int_0^\infty R_1^2(\eta, C_{11}, C_{12}) d\eta, \\ J_2(C_{21}, C_{22}) &= \int_0^\infty R_2^2(\eta, C_{21}, C_{22}) d\eta. \end{aligned} \quad (27)$$

The constants  $C_{11}$ ,  $C_{12}$ ,  $C_{21}$  and  $C_{22}$  obtain from equation (26). In the particular cases:

$\operatorname{Pr} = 1$ ,  $K_1 = 0.01$ ,  $n = 0$  and  $\operatorname{Re} = 0.5$ :

$$\begin{aligned} C_{11} &= 1.099358714, \quad C_{12} = -0.01672969157, \\ C_{21} &= 1.0801896690, \quad C_{22} = 4.263548365. \end{aligned} \quad (28)$$

By substituting equation (28) into (26), an expression for  $V(y)$  and  $U(y)$  is obtained.

## RESULTS AND DISCUSSION

The objective of the present study was to apply Optimal Homotopy Asymptotic Method to obtain an explicit analytic solution of the laminar viscous flow in a semi-porous channel in the presence of uniform magnetic field (Figure 1). The results obtained by Optimal Homotopy Asymptotic Method were well matched with the results carried out by the numerical solution obtained by fourth-order Runge-Kutta method as shown in Table 1. In this table, the error percentage is introduced as followed:

$$\% \text{ Error} = \frac{|f(\eta)_{NM} - f(\eta)_{analytical}|}{f(\eta)_{NM}} \times 100. \quad (39)$$

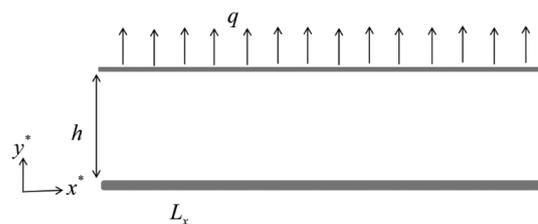


FIGURE 1. Schematic diagram of the system

Optimal Homotopy Asymptotic Method (OHAM) provides a highly accurate numerical solution in comparison with other methods such as homotopy analysis method (HAM) and variation iteration method (VIM), as shown in Table 1.

This accuracy in results gives us high confidence about the validity of this method and reveals an excellent accuracy. This investigation is completed by depicting the effects of some important parameters to evaluate how these parameters effect this fluid behavior. The velocities  $V(y)$ ,  $V'(y)$  and  $U(\eta)$  profiles for various parameters Reynolds number and Hartmann number are illustrated in Figures 2 and 3. Figure 2 shows the effect of Reynolds number on profiles at constant Hartmann number. With increasing Reynolds number,  $V(y)$  increases. At low Reynolds numbers the  $V'(y)$  exhibit center line symmetry indicating a poisuille flow. At higher Reynolds numbers the maximum  $V'(y)$  point shifted to the solid wall where shear stress becomes larger as the Reynolds number grows. Also it shows that increasing Reynolds number leads to increasing the curve of temperature profile and decreasing of  $U(y)$  values. These effects become less at higher Hartmann numbers. Figure 3 shows the effect of Hartmann number on profiles at constant Reynolds number. Increasing Hartmann number leads to increasing of  $V(y)$  when  $y < y_m$  and decreasing of  $V(y)$  when  $y > y_m$ ,  $y_m$  is a meeting point that all curves joint together at this point. Meeting point shifts to the solid wall as the Hartmann number grows. Values of  $V'(y)$  and  $U(y)$  decrease as a result of Hartmann number increment, especially at low Reynolds numbers.

## CONCLUSION

In this paper laminar viscous flow in a semi-porous channel in the presence of a uniform magnetic field (Figure 1) has been solved via a sort of analytical method, Optimal Homotopy Asymptotic Method, Also this problem has been solved by a numerical method (the Runge-Kutta method of order 4) and the results has been compared with VIM and HAM solution results. The observed good agreement between the present method and numerical method results shows that Optimal Homotopy Asymptotic Method is a powerful approach for solving nonlinear differential equations such as this problem. Some facts were observed through the results. All those effects can be also observed through figures and diagrams.

TABLE 1. Comparison between error percentages of OHAM, VIM, and HAM results for  $V(y)$  and  $U(y)$  when  $Re = 1$ ,  $Ha = 1$

$y$	$V$			$U$		
	OHAM	VIM	HAM	OHAM	VIM	HAM
0	0	0	0	0	0	0
0.1	0.000758	0.010614	0.002586	0.000274	0.014006	0.135962
0.2	0.000722	0.025813	0.005332	0.000662	0.01284	0.128277
0.3	0.000679	0.048113	0.007683	0.001228	0.01146	0.112836
0.4	0.000628	0.081155	0.009288	0.002076	0.009813	0.092527
0.5	0.000566	0.128826	0.010071	0.003397	0.007862	0.070073
0.6	0.000491	0.191855	0.010196	0.005568	0.005636	0.047948
0.7	0.000397	0.260207	0.009965	0.00945	0.003314	0.028314
0.8	0.000276	0.299971	0.009668	0.017534	0.001297	0.012963
0.9	0.000122	0.234666	0.009447	0.039281	0.000141	0.003261
1	0	0	0	0	0	0

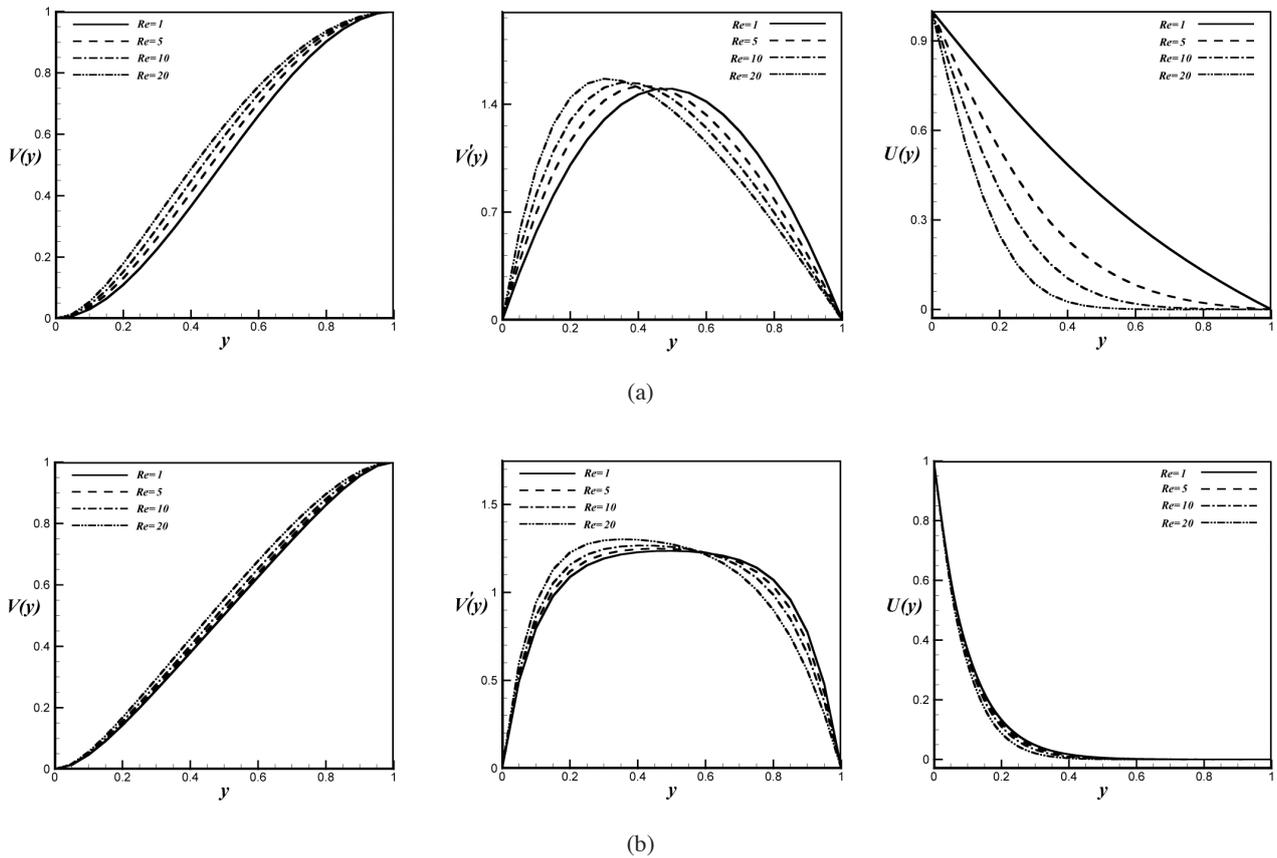


FIGURE 2. Effects of various values of Reynolds numbers ( $Re$ ) on  $V(y)$ ,  $V'(y)$  and  $U(y)$  when (a)  $Ha = 0$  and (b)  $Ha = 10$

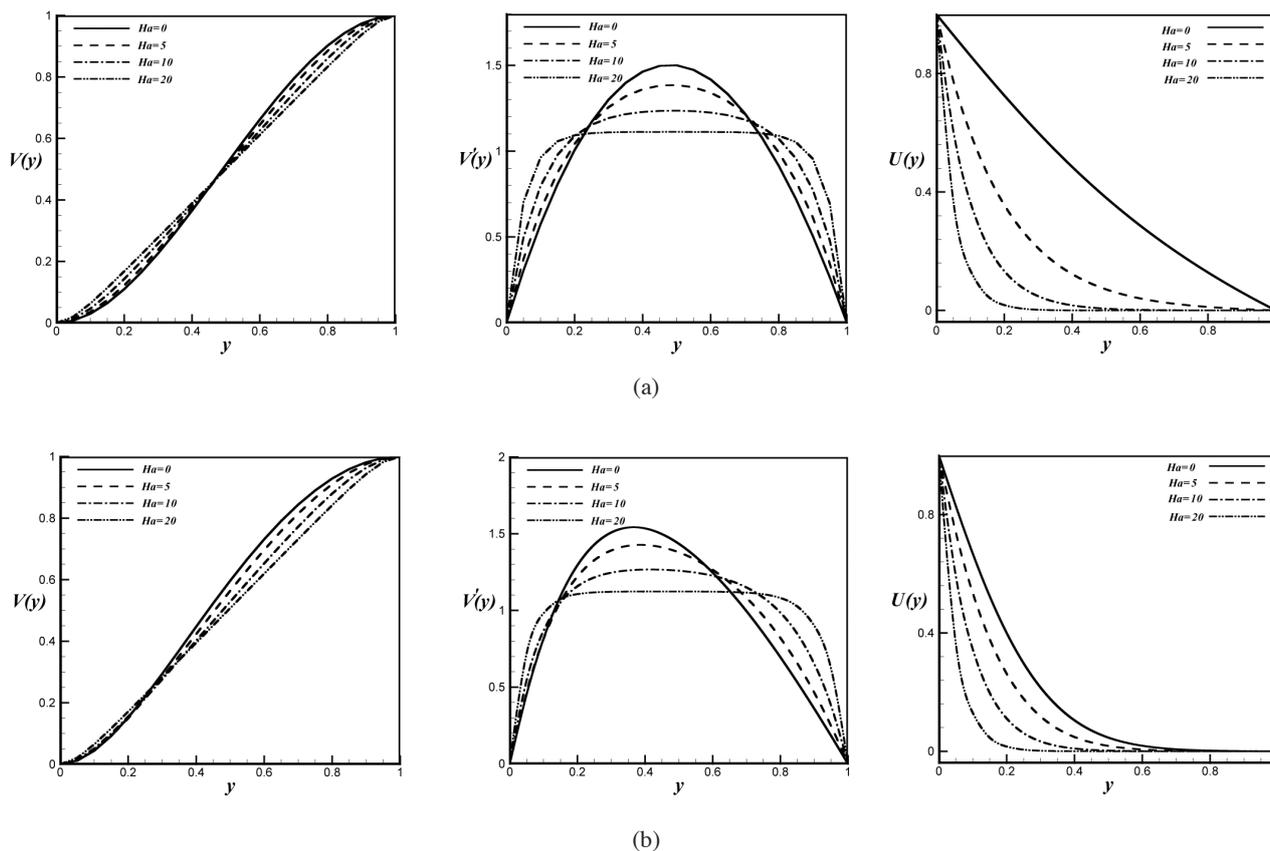


FIGURE 3. Effects of various values of Hartmann numbers ( $Ha$ ) on  $V(y)$ ,  $V'(y)$  and  $U(y)$  when (a)  $Re = 0$  and (b)  $Re = 0$

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