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A COMPARISON BETWEEN THE STANDARD DEVIATION OF THE RUN LENGTH (SDRL) PERFORMANCE OF OPTIMAL EWMA AND OPTIMAL CUSUM CHARTS

(Suatu Perbandingan Panjang Larian Sisihan Piawai (SDRL) antara Carta-carta EWMA Optimum dengan CUSUM Optimum)

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ABSTRACT

The Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) control charts are very effective in detecting small shifts in the process mean or variance. The average run length (ARL) has always been used as a sole measure of the performances of control charts. Generally, the performances of the EWMA and CUSUM charts are comparable and most comparative studies are based on the ARL. Therefore, this study is aimed at comparing the performances of the optimal EWMA and optimal CUSUM charts, based on their standard deviation of the run lengths (SDRLs). The Statistical Analysis System (SAS) software version 9.1.3 is used to conduct the simulation studies, for the optimal EWMA and optimal CUSUM charts. The SDRL results show that the optimal EWMA chart is slightly superior to the optimal CUSUM chart, when the process is out-of-control. However, when the process is in-control, the converse is true.

Keywords: EWMA chart; CUSUM chart; average run length (ARL); standard deviation of the run length (SDRL)

ABSTRAK

Carta kawalan purata bergerak berpemberat eksponen (EWMA) dan hasil tambah longgokan (CUSUM) amat berkesan untuk mengesan anjakan kecil dalam min atau varians proses. Panjang larian purata (ARL) selalu digunakan sebagai ukuran tunggal prestasi carta kawalan. Pada amnya, prestasi carta kawalan EWMA dan CUSUM adalah boleh banding dan kebanyakan kajian perbandingan adalah berdasarkan ARL. Justeru, kajian ini bertujuan untuk membandingkan prestasi carta-carta EWMA optimum dan CUSUM optimum berdasarkan panjang larian sisihan piawai (SDRL). Perisian Sistem Analisis Berstatistik (SAS) versi 9.1.3 digunakan untuk menjalankan kajian simulasi bagi carta EWMA optimum dan CUSUM optimum. Keputusan SDRL menunjukkan bahawa carta EWMA optimum adalah lebih baik sedikit daripada carta CUSUM optimum apabila proses berada di luar kawalan. Walau bagaimanapun, apabila proses berada dalam kawalan, hal yang sebaliknya adalah benar.

Kata kunci: carta EWMA; carta CUSUM; panjang larian purata (ARL); panjang larian sisihan piawai (SDRL)

1. Introduction

The Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) charts are memory charts that are used when the detection of small shifts in the process is of interest. To measure the performances of the EWMA and CUSUM charts, the ARL is usually used. However, some researchers suggested the median, standard deviation and other percentiles of the run length distribution as alternative measures of the charts' performances.

The EWMA chart was proposed by Roberts (1959). Since then, the performance of the EWMA chart has been extensively studied over the years. Crowder (1987) numerically evaluated the properties of the EWMA chart by formulating and solving a system of integral equations. The EWMA design procedures using solutions to these integral equations were given in Crowder (1989). These design procedures allow the practitioners to design charts with various in-control ARLs while providing the optimal choices of the smoothing constant needed to detect specified process changes. Lucas and Saccucci (1990) used a Markov chain approximation to study the run length distribution of the EWMA control chart. Gan (1993) proposed optimal design procedures based on the median run length (MRL). Chandrasekaran *et al.* (1995) used a Markov chain approximation to compute the ARL of the chart when the exact variance rather than the asymptotic variance, is used in computing the control limits. Montgomery *et al.* (1995) presented statistically constrained economic design procedures for the EWMA chart. Steiner (1999) studied the run length distribution of EWMA charts using the exact control limits, and reported the effect of the fast initial response (FIR) feature on the performance of the chart. Amin *et al.* (1999) developed an EWMA control scheme for monitoring the smallest and largest observations in a sample, known as the MaxMin EWMA control chart. Practically, parameters of control charts are usually unknown. Jones (2002) replaced these parameter estimates with design procedures for the EWMA control chart. Shu *et al.* (2007) proposed the one-sided EWMA chart for rapid detection of upward or downward changes in the process mean.

The Cumulative Sum control chart was initially proposed by Page (1954) and has been widely used to monitor the quality of products from manufacturing processes for detecting small process shifts. Gan (1991) presented plots of chart parameters which enable the chart parameters of an optimal CUSUM chart to be determined easily. Gan (1994) interpreted the optimal CUSUM chart, where the run length distribution can vary from a highly skewed distribution to an almost symmetric distribution with respect to the shift, based on the median run length (MRL). Sparks (2000) suggested an adaptive CUSUM chart to detect a broader range of mean shifts. Arnold and Reynolds (2001) developed CUSUM chart statistics with the variable sample size (VSS) feature and with both variable sampling interval and variable sample size (VSSI) features. Jones *et al.* (2004) discussed the run length distribution of the CUSUM chart with estimated parameters and provided a method for approximating this distribution and moments. Luceno and Puig-Pey (2006) provided an algorithm to compute the in-control and out-of-control average run lengths and run-length probability distributions for one-sided CUSUM charts initialised using random intrinsic fast initial response (RIFIR) starting policy. The same RIFIR starting policy procedures for the two-sided CUSUM chart was applied by Luceno and Cofino (2006). Wu and Wang (2007) proposed a single CUSUM chart which uses a single observation in each sampling to detect mean and variance shifts.

Several studies have been made on a comparison between the EWMA and CUSUM charts. Hunter (1986) commented that the differences between the Shewhart, CUSUM and EWMA charts have to do with the way each charting technique uses the data generated by the production process. Lucas and Saccuci (1990) compared the ARLs of the EWMA and CUSUM control schemes over a wide range of parameter values and found that the ARL of the former is slightly smaller than that of the latter. However, Woodall and Maragah (1990) noticed that the EWMA chart can be slower to react than the CUSUM chart, for some changes in the process. Gan (1991) compared the ARLs of the optimal EWMA and optimal CUSUM charts with and without headstarts. When compared with the optimal EWMA, the optimal CUSUM chart without headstart is less effective in detecting small shifts in the mean but more effective in detecting large shifts in the mean. The CUSUM with headstarts is best for detecting a shift that is larger than one sigma. Srivastava and Wu (1993) studied the properties of the EWMA procedure under the continuous time model and compared it with the CUSUM and

Shiryayev-Roberts procedure. The results show that the EWMA procedure is less efficient than the other two procedures when the $ARL_0 \to \infty$. An interesting result, however, is that the EWMA procedure is less sensitive to the reference value when the shift amount is unknown. Reynolds and Stoumbos (2006) presented a comparative study on the performances of the CUSUM, as well as the EWMA charts.

From the many comparative studies between the performances of the EWMA and CUSUM charts, the performances of both the charts are generally comparable. Since most of these studies are based on the ARL and MRL, the aim of this study is to determine which chart performs better, in terms of the SDRL. The organisation of this paper is as follows: Section 2 and 3 explain the design of the optimal EWMA and optimal CUSUM charts, respectively. Section 4 studies and compares the performances of the optimal EWMA and optimal CUSUM charts. Conclusions are drawn in Section 5.

2. Design of an Optimal EWMA Control Chart

An optimal EWMA chart is defined as the chart with a fixed in-control ARL (ARL_0) and having the smallest ARL for a specified shift in the mean. The EWMA chart's statistics is as follows (Crowder 1989):

$$Z_t = (1-\lambda)Z_{t-1} + \lambda \bar{X}_t, 0 < \lambda < 1,\ t = 1,2,\ldots \tag{1}$$

Here, λ ($0 < \lambda < 1$) is a smoothing constant and $\bar{X}_t$ is the sample average observed at time t, assumed to be normally distributed. Note that for $\lambda = 1$, the value of Z_t depends only on the most recent observation, just as in the case of the $\bar{X}$ chart.
The control limits of the EWMA chart are

$$\mu_0 \pm K\sigma_Z, \tag{2}$$

where $\sigma_Z^2 = \left[\frac{\lambda}{2-\lambda}\right]\frac{\sigma^2}{n}$, K is the control limits constant chosen by the user and s is the standard deviation of an observation from the process.

The steps to design an optimal EWMA chart to detect process shifts are as follows (Crowder 1989):

Step 1: Choose a nominal ARL_0 when the process shift is zero.

Step 2: Choose the magnitude of a shift in the process mean, where a quick detection is needed and determine the optimal λ corresponding to the shift.

Step 3: From the optimal λ value determined, find the control limits constant K which corresponds to the ARL_0 value fixed in Step 1.

3. Design of an Optimal CUSUM Control Chart

The optimal CUSUM chart is defined as the chart with a fixed ARL_0 which has the smallest ARL for a specified shift in the mean. The CUSUM chart's statistics is as follows (Gan 1991):

$$T_t = \min\{0, T_{t-1} + (Y_t + k)\} \tag{3a}$$

and

$$S_t = \max\ \{0, S_{t-1} + (Y_t + k)\}, \tag{3b}$$

respectively, for $t = 1, 2, \ldots$, where the chart's parameter $k \geq 0$, $Y_t = \sqrt{n}\left(\bar{X}_t - \mu_0\right)/\sigma$, and $\bar{X}_t$ is the sample mean observed at sample t. A lower-sided CUSUM chart intended for detecting downward shifts in the mean issues an out-of-control when $T_t < -h$. Similarly, an upper-sided CUSUM chart intended for detecting upward shifts in the mean issues an out-of-control when $S_t > h$. Here, h is the limit of the CUSUM chart, determined based on a desired ARL_0 value.

The steps to design an optimal CUSUM chart to detect process shifts are as follows (Gan 1991):

Step 1: Choose an acceptable ARL_0 when the process is in-control.

Step 2: Choose the optimal shift ($\boldsymbol{\delta}_{opt}$), where a quick detection is needed and determine the value of k as $k = \delta_{opt}/2$.

Step 3: Based on the k value obtained, determine h such that the CUSUM produces the ARL_0 fixed in Step 1.

4. A Comparison between the EWMA and CUSUM Charts based on SDRL

A comparison between the SDRL performances of the optimal EWMA and optimal CUSUM charts is discussed in this section. The Statistical Analysis System (SAS) program is used to calculate the SDRLs via the simulation method. The optimal parameters of the EWMA and CUSUM charts are chosen to obtain a desired ARL_0 and at the same time minimizing the out-of-control ARL (ARL_1), for several specified shifts in the process mean, where quick detections are needed. The $\text{ARL}_0 \in \{50, 100, 250, 500, 1000, 1500\}$ and $\boldsymbol{\delta}_{opt} \in \{0.5, 1.0, 1.5\}$ are considered. Here, $\boldsymbol{\delta}_{opt}$ denotes the optimal shift in multiples of standard deviation.

The optimal smoothing constant l for the EWMA chart is determined based on the $\boldsymbol{\delta}_{opt}$ value. Then from this optimal l value, the control limit constant K, corresponding to the desired ARL_0 value is calculated using the method described in Crowder (1989). For the optimal CUSUM chart, the $\boldsymbol{\delta}_{opt}$ value is used to compute k. From the k value computed, the limit h such that the CUSUM chart produces the desired ARL_0 is computed following the procedures in Gan (1991). The optimal values of l and K, used in computing the SDRLs for the optimal EWMA chart, are shown in Table 1. The optimal values of k and h for the CUSUM chart, used to compute the SDRLs are shown in Table 2.

Table 1: Optimal (λ, *K*) values of the EWMA chart for $ARL_0 \in \{50, 100, 250, 500, 1000, 1500\}$

ARL_0	50		100		250		500		1000		1500	
δ_{opt}	λ	*K*	λ	*K*	λ	*K*	λ	*K*	λ	*K*	λ	*K*
0.5	0.08	1.72	0.07	2.02	0.05	2.32	0.05	2.63	0.04	2.83	0.04	2.98
1.0	0.21	2.07	0.18	2.34	0.15	2.66	0.13	2.88	0.12	3.10	0.11	3.22
1.5	0.38	2.22	0.32	2.47	0.27	2.78	0.24	2.98	0.22	3.20	0.20	3.31

Table 2: Optimal (*k, h*) values of the CUSUM chart for $ARL_0 \in \{50, 100, 250, 500, 1000, 1500\}$

ARL_0	50		100		250		500		1000		1500	
δ_{opt}	*k*	*h*	*k*	*h*	*k*	*h*	*k*	*h*	*k*	*h*	*k*	*h*
0.5	0.25	4.394	0.25	5.650	0.25	7.244	0.25	8.594	0.25	9.944	0.25	10.581
1.0	0.50	2.830	0.50	3.490	0.50	4.390	0.50	5.074	0.50	5.740	0.50	6.148
1.5	0.75	2.038	0.75	2.500	0.75	3.070	0.75	3.526	0.75	4.000	0.75	4.246

The SDRLs for the EWMA and CUSUM charts are computed, for shifts from the in-control mean $\mu = \mu_0$ to the out-of-control mean $\mu = \mu_0 + \delta\sigma$. For simplicity, μ_0 = 0 and σ = 1 are considered but it is found that the results remain the same for any value of μ_0 and s. Note that $\delta \in \{0, 0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0\}$ were employed. The SDRL performances of the optimal EWMA and optimal CUSUM charts for the various mean shifts d are studied and compared. The SDRLs for the EWMA chart are shown in Tables 3 and 4 while that for the CUSUM chart are given in Tables 5 and 6.

Table 3: SDRLs for the Optimal EWMA chart when $ARL_0 \in \{50, 100, 250\}$

δ	$ARL_0 = 50$			$ARL_0 = 100$			$ARL_0 = 250$		
	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$
0.00	46.9497	48.2923	49.7552	98.3979	100.9678	99.2420	244.9985	251.0671	249.4488
0.25	21.3820	25.9889	31.4464	32.3144	43.8230	54.2804	47.1019	76.5158	103.4015
0.50	8.6627	10.9281	14.5171	11.0931	14.8604	20.3410	13.5535	20.9873	30.8882
0.75	4.4705	5.5559	7.3245	5.4081	7.1575	9.4047	6.3190	8.7475	12.3031
1.00	2.7943	3.2938	4.2960	3.2334	3.8743	5.0080	3.7806	4.6675	6.2042
1.50	1.4550	1.5607	1.8378	1.6421	1.7502	2.0647	1.8669	2.0341	2.3867
2.00	0.9250	0.9573	1.0584	1.0384	1.0498	1.1445	1.1684	1.1795	1.2832
2.50	0.6505	0.6840	0.7409	0.7643	0.7204	0.7731	0.8375	0.8114	0.8206
3.00	0.5074	0.5816	0.5706	0.5821	0.5547	0.6234	0.6533	0.5964	0.6132
4.00	0.4889	0.4469	0.3263	0.3548	0.5091	0.4478	0.5058	0.4134	0.5173

Table 4: SDRLs for the Optimal EWMA chart when $ARL_0 \in \{500, 1000, 1500\}$

δ	$ARL_0 = 500$			$ARL_0 = 1000$			$ARL_0 = 1500$		
	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$
0.00	513.9446	502.3902	470.4961	1023.2400	1019.5400	973.0752	1582.8300	1571.7900	1534.9100
0.25	67.4725	114.9171	159.8205	83.6420	175.0589	256.4970	101.6902	214.4133	328.2965
0.50	16.6157	26.1384	42.3200	18.3814	33.9384	59.4140	20.7303	38.4708	67.5379
0.75	7.3073	10.1585	15.0974	7.9943	11.7698	18.7197	8.4783	12.6412	19.9069
1.00	4.1930	5.1783	7.3440	4.5542	5.7641	8.5466	4.7670	6.0063	8.8054
1.50	2.0405	2.1813	2.5719	2.2149	2.3584	2.8281	2.2985	2.4338	2.9304
2.00	1.2780	1.2729	1.3650	1.3811	1.3543	1.4913	1.4126	1.3916	1.5240
2.50	0.8969	0.8636	0.8799	0.9659	0.9146	0.9380	1.0013	0.9486	0.9715
3.00	0.6771	0.6551	0.6225	0.7544	0.7000	0.6663	0.7670	0.7172	0.7032
4.00	0.5242	0.3730	0.4896	0.4922	0.4390	0.4328	0.5167	0.4985	0.4087

Table 5: SDRLs for the Optimal CUSUM chart when $ARL_0 \in \{50, 100, 250\}$

δ	$ARL_0 = 50$			$ARL_0 = 100$			$ARL_0 = 250$		
	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$
0.00	43.8299	46.1420	48.5798	96.9595	94.7654	101.4951	244.8228	251.0935	246.2925
0.25	23.3493	28.4252	33.4785	37.0716	48.5262	61.3460	57.6040	90.7270	121.3931
0.50	9.7930	12.7948	16.4961	12.4195	17.6855	24.4364	15.6239	25.0392	38.8283
0.75	4.9812	6.4278	8.4775	5.8111	7.8447	10.9965	6.8397	9.8973	14.6052
1.00	3.0245	3.6443	4.7838	3.4938	4.2246	5.6958	3.9924	4.9769	6.9886
1.50	1.5362	1.6995	2.0030	1.7475	1.8635	2.2208	1.9485	2.0843	2.5192
2.00	0.9865	1.0014	1.1124	1.1074	1.0834	1.2005	1.2184	1.2126	1.3247
2.50	0.7012	0.7156	0.7631	0.8013	0.7380	0.8061	0.8729	0.8312	0.8491
3.00	0.5200	0.5893	0.5724	0.6257	0.5814	0.6326	0.6648	0.6011	0.6445
4.00	0.4582	0.4383	0.3169	0.3507	0.5095	0.4248	0.5165	0.4438	0.5088

Table 6: SDRLs for the Optimal CUSUM chart when $ARL_0 \in \{500, 1000, 1500\}$

δ	$ARL_0 = 500$			$ARL_0 = 1000$			$ARL_0 = 1500$		
	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$	$\delta_{opt} = 0.5$	$\delta_{opt} = 1$	$\delta_{opt} = 1.5$
0.00	497.2714	495.8519	465.5332	980.9235	960.3861	980.9272	1387.2700	1444.7500	1444.6200
0.25	76.4440	141.4961	196.1555	99.7914	209.8340	329.9956	113.6390	262.9398	423.4151
0.50	17.5755	32.0338	52.4901	19.8496	39.4021	72.1718	20.9211	44.2022	85.1813
0.75	7.5337	11.3277	17.8522	8.2508	12.6411	21.9155	8.5417	13.3107	23.9202
1.00	4.3356	5.6264	7.9675	4.6709	5.8588	8.9817	4.8781	6.1086	9.6056
1.50	2.1069	2.2638	2.6866	2.2552	2.3800	2.8721	2.3569	2.4592	2.9799
2.00	1.3084	1.2921	1.4078	1.4197	1.3683	1.5062	1.4433	1.3890	1.5194
2.50	0.9430	0.8853	0.8940	0.9866	0.9212	0.9368	1.0102	0.9591	0.9633
3.00	0.7150	0.6618	0.6523	0.7591	0.7015	0.6631	0.7845	0.7227	0.6874
4.00	0.5082	0.3899	0.5201	0.4973	0.4228	0.4754	0.5345	0.4680	0.4559

4.1. *A comparison between the optimal EWMA and optimal CUSUM charts when δ > 0*

The results in Tables 3 - 6 show that the out-of-control SDRLs (SDRL1s) of the optimal CUSUM chart are larger than that of the optimal EWMA chart for $\delta_{opt} \in \{0.5, 1.0, 1.5\}$ when the mean shift is not greater than three standard deviations ($\delta \leq 3$). Therefore, the optimal EWMA chart performs better than the optimal CUSUM chart when $\delta \leq 3$, in terms of SDRL1. There exist only small differences between the SDRL1s of the optimal EWMA and optimal CUSUM charts when $0.75 \leq \delta \leq 3$. This suggests that the optimal EWMA chart performs only slightly better than the optimal CUSUM chart, in terms of the SDRL1, for mean shifts in this interval.

4.2. *A comparison between the optimal EWMA and optimal CUSUM charts when δ = 0*

From Tables 3 - 6, we observe that as ARL_0 increases, the in-control SDRL ($SDRL_0$) increases also, for both the optimal EWMA and optimal CUSUM charts. In general, the differences in the $SDRL_0$s, for different δ_{opt} values increase as ARL_0 increases, for the optimal EWMA and optimal CUSUM charts. The control chart with a smaller SDRL performs better. Thus, the optimal CUSUM chart performs better than the optimal EWMA chart as the $SDRL_0$s of the former are generally lower than that of the latter.

5. Conclusions

In this study, the SDRLs of the optimal EWMA and optimal CUSUM charts are compared. The SAS version 9.1.3 software is used to compute the SDRLs of the optimal EWMA and optimal CUSUM charts, for different magnitudes of mean shifts.

We conclude that the optimal EWMA and optimal CUSUM charts give different SDRL performances under different situations as discussed. The optimal EWMA chart surpasses the optimal CUSUM chart when $\delta \leq 3$, in terms of $SDRL_1$. However, when the process is in-control, the optimal CUSUM chart outperforms the optimal EWMA chart, as the $SDRL_0$s of the former are found to be lower than that of the latter.

In conclusion, the results indicate that the optimal EWMA chart is slightly superior to the optimal CUSUM chart, in terms of the SDRL, when the process is out-of-control. However, when the process is in-control, the converse is true.

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