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SMOOTHING ROPE SKIPPING DATA USING GAUSSIAN SCALE-SPACE METHOD

(Melicinkan Data Lompat Tali Menggunakan Kaedah Ruang-Skala Gaussan)

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ABSTRACT

The scale-space method has been widely used in handling image data at multiple scales. Application of Gaussian filtering in different field includes human vision problem, medical data, financial data and electroencephalogram (EEG) signal. The main purpose of this paper is to apply the Gaussian scale-space method by determining a suitable σ value in order to smooth rope skipping data. Smoothing technique using a Gaussian kernel with a selection of bandwidth (σ) and time (x) is applied. It is found that the tolerance value of σ can be used to smooth not only one set of data, but also other biomechanical data of different anatomical body landmarks.

Keywords: Gaussian scale-space method; rope skipping; smoothing

ABSTRAK

Kaedah ruang-skala telah digunakan dengan meluas dalam pengendalian data imej pada skala yang berganda. Aplikasi penapisan Gaussan telah digunakan dalam pelbagai bidang termasuk masalah penglihatan manusia, data perubatan, data kewangan dan isyarat elektroensefalogram (EEG). Tujuan utama kajian ini adalah untuk menggunakan kaedah ruang-skala Gaussan dengan memilih satu nilai σ yang bersesuaian untuk melicinkan data aktiviti lompat tali. Teknik pelicinan menggunakan inti Gaussan digunakan dengan memilih jalur lebar (σ) dan masa (x). Hasil kajian mendapati nilai σ yang dipilih boleh digunakan bukan hanya pada satu set data sahaja, tetapi juga boleh digunakan untuk data biomekanik yang lain pada kedudukan anatomi badan yang berbeza.

Kata kunci: kaedah ruang-skala Gaussan; lompat tali; pelicinan

1. Introduction

The Gaussian scale-space method has been used for analysing various kinds of data such as human and computer vision problems (Lindeberg 1994a; Romeny 2003; Young 1987), financial Kuala Lumpur Composite Index (KLCI) and electroencephalogram (EEG) data (Karim & Kong 2011), volumetric tumor characterisation (Okada *et al*. 2004), blood glucose concentrations (Skrøvseth & Godtliebsen 2011),vessels structures (Yi & Hayward 2002) and many more. None has been done on biomechanical data.

Therefore, the main purpose of this paper is to apply the Gaussian scale-space method in analysing rope skipping data. The work intends to determine a suitable σ value in order to smooth rope skipping data. So far, most of biomechanical data use Butterworth filtering method (Roithner *et al*. 2000; van den Bogert & de Koning 1996; Challis 1999), splines (Woltring 1986) and other techniques (Wood 1982).

2. Method

2.1. *Linear scale-space concept*

If we have *N*-dimensional continuous signal f_C, which is denoted as

$$f_C(x_1, x_2, ..., x_N, t) \tag{1}$$

with *x* is time and *t* is scale, and *N*-dimensional Gaussian kernel, G_N,

$$G_N(x_1, x_2, ..., x_N, t), \tag{2}$$

then the linear scale space *L*, is obtained through a convolution of the two signals f_C and G_N which can be written as,

$$L(x_1, x_2, ..., x_N, t) = \int_{u_1=-\infty}^{\infty} \int_{u_2=-\infty}^{\infty} \cdots \int_{u_N=-\infty}^{\infty} f_C(x_1 - u_1, x_2 - u_2, ..., x_N - u_N, t) \bullet G_N(u_1, u_2, ..., u_N, t)\, du_1\, du_2 \ldots du_N. \tag{3}$$

However, to apply this to real discrete data such as biomechanical data, this definition is impractical. When applying the scale space concept to a discrete signal, f_D, different approaches can be taken. The following summarises the Gaussian scale-space method of discrete data (Lindeberg 1994b).

2.2. *Separability Property*

Using the separability property of the Gaussian kernel,

$$G_N(u_1, u_2, ..., u_N, t) = G(u_1, t) G(u_2, t) ... G(u_N, t) \tag{4}$$

the *N*-dimensional convolution operation can be decomposed into a set of separable smoothing steps with a one-dimensional Gaussian kernel *G* along each dimension

$$L(x_1, x_2, ..., x_N, t) = \int_{u_1=-\infty}^{\infty} \int_{u_2=-\infty}^{\infty} \cdots \int_{u_N=-\infty}^{\infty} f_C(x_1 - u_1, x_2 - u_2, ..., x_N - u_N, t) \bullet G(u_1, t)\, du_1 G(u_2, t)\, du_2 ... G(u_N, t)\, du_N \tag{5}$$

where

$$G(x, t) = \frac{1}{\sqrt{2\pi t}} e^{-x^2/2t}. \tag{6}$$

2.3. *The Sampled Gaussian Kernel*

In this paper, the Gaussian scale space algorithm focuses on the one-dimensional case. When implementing the one-dimensional smoothing step in practice, the presumably simplest approach is to convolve the discrete signal f_D with a sampled Gaussian kernel,

$$L(x,t) = \sum_{n=-\infty}^{\infty} f(x-n)G(n,t) \tag{7}$$

where

$$G(n,t) = \frac{1}{\sqrt{2\pi t}} e^{-n^2/2t} \tag{8}$$

(with $t = \sigma^2$) which in turn is truncated at the ends to give a filter with finite impulse response

$$L(x,t) = \sum_{n=-M}^{M} f(x-n)G(n,t) \tag{9}$$

for M chosen sufficiently large such that

$$2\int_{u=M}^{\infty} G(u,t)\,du = 2\int_{v=M/\sqrt{t}}^{\infty} G(v,1)\,dv < \varepsilon. \tag{10}$$

A common choice is to set M to a constant C times the standard deviation of the Gaussian kernel

$$M = C_\sigma + 1 = C\sqrt{t} + 1 \tag{11}$$

where C is often chosen somewhere between 3 and 6.

Using the sampled Gaussian kernel can lead to implementation problems, in particular when computing higher-order derivatives at finer scales by applying sampled derivatives of Gaussian kernels. When accuracy and robustness are primary design criteria, alternative implementation approaches should therefore be considered.

2.4. *Gaussian Scale-Space*

Smoothing technique using a Gaussian kernel extracts important features of the original discrete data (Wand & Jones 1995). However, this approach is heavily dependent on the choice of bandwidth or scale and space thus is difficult to know which features are significant or not. However, it is true that significant structure may emerge at a variety of scales and that the significant features may disappear again at different scales. We treat a scale space in time, where each location is denoted by a time x and a scale or bandwidth, σ.

Considering values recorded at times $x_i, i = 1, 2, ..., N$, we have a smoothed density estimator (Wand & Jones 1995),

$$\hat{f}_{\sigma}(x) = \frac{1}{N}\sum_{i=1}^{N} G_{\sigma}(x - x_i), \tag{12}$$

where $G_{\sigma}(x)$ is the Gaussian kernel,

$$G_{1D}(x,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}. \tag{13}$$

The Gaussian kernel is used as a standard for density smoothing due to the fact that it is unique in that it has a monotone decrease of zero crossings of the derivative smooth with increasing bandwidth (Silverman 1981; Babaud *et al.* 1986). This means that features are monotone in scale space. It should be noted that the value of σ must be greater than 0 ($\sigma > 0$).

2.5. *Data Aquisition*

In this study, we conducted an experiment of rope skipping activity in order to obtain the data. Subjects were required to skip until they complete five cycle of rope skipping for each trial and repeated the trial for six times. One cycle of rope skipping starts from the moment a subject is ready to skip until both of the legs touch the ground again for landing.

After obtaining the data, we apply Gaussian scale-space as a method of filtering to the biomechanic data. The data of interest are data at the ankle, knee and hip joints. The data consisted of 216 frames (x) and contaminated with noise. The source of error or noise may be resulted from misalignment of the cameras, different types of cameras used, lenses, calibration objects, skin movement, incorrect digitisation or other factors (Wood 1982; Winter 2005; Allard *et al*. 1995; Robertson *et al*. 2004).

3. Results

In this section, we will discuss the application of Gaussian scale-space for data smoothing. Taking all trials into account, we analyse repeated skipping cycles throughout the skipping activity. Figure 1 illustrates one complete cycle of the rope skipping activity. Raw data of ankle angles for five cycles during skipping performance is shown in Figure 2. This data was smoothed by a Gaussian kernel with a data-driven bandwidth computed using the Gaussian scale-space algorithm (Karim & Kong 2011).

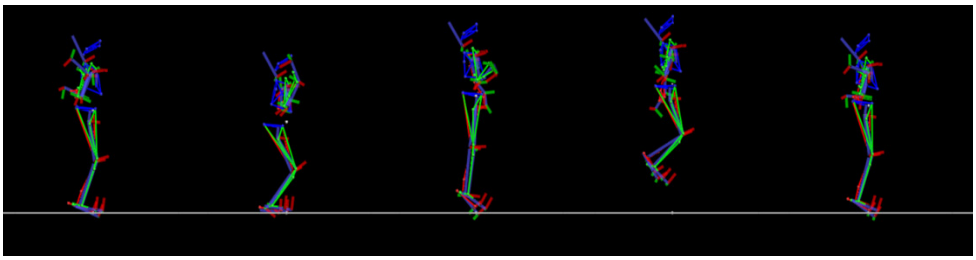


Figure 1: Rope skipping activity in one complete cycle

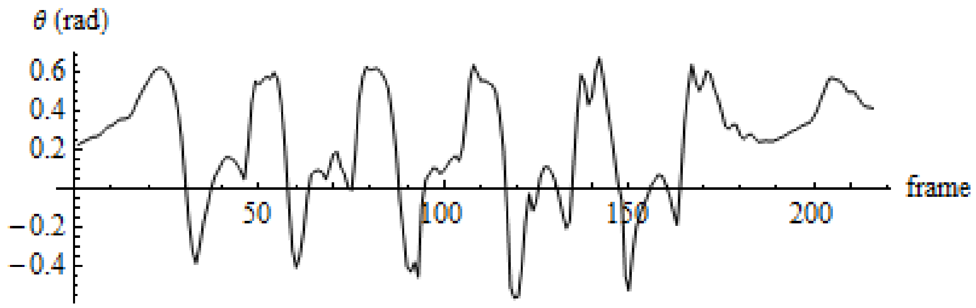
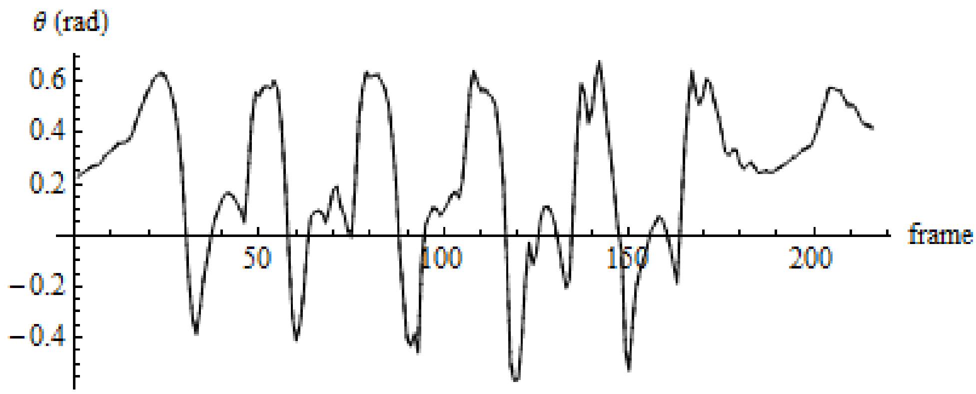
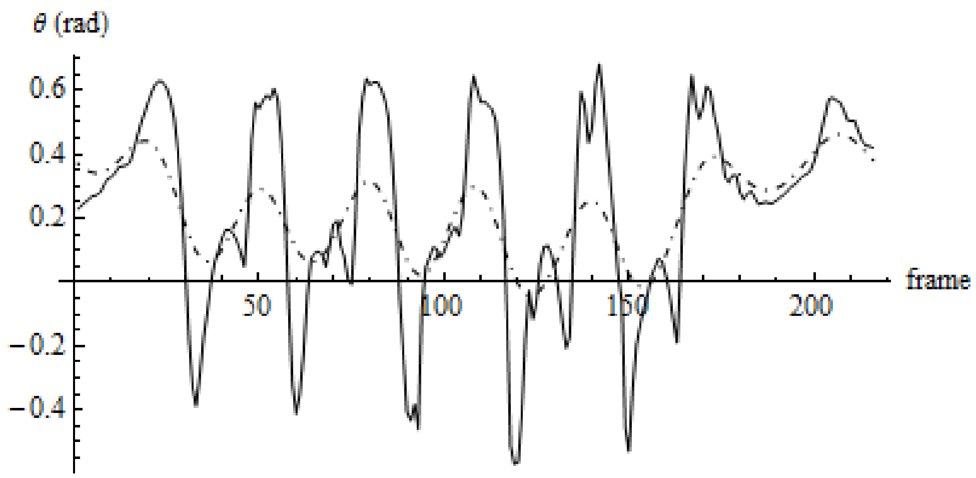


Figure 2: Raw ankle angle data for 5-cycle of skipping performance



(a)



(b)

Figure 3: The filtered (dot-dashed line) and unfiltered (line) data when (a) $\sigma = 0.5$ and (b) $\sigma = 6.5$

Based on trial and error basis, we selected 20 different values of σ ranged from 0.5 to 6.5 in order to obtain a suitable σ value for the rope skipping data. By increasing the value of σ, the smoothed graph will diverge from the original data. Hence, the suitable σ might lie between these ranges of values. Figure 3 shows the differences between filtered (dot-dashed line) and unfiltered (line) data when $\sigma = 0.5$ and $\sigma = 6.5$.

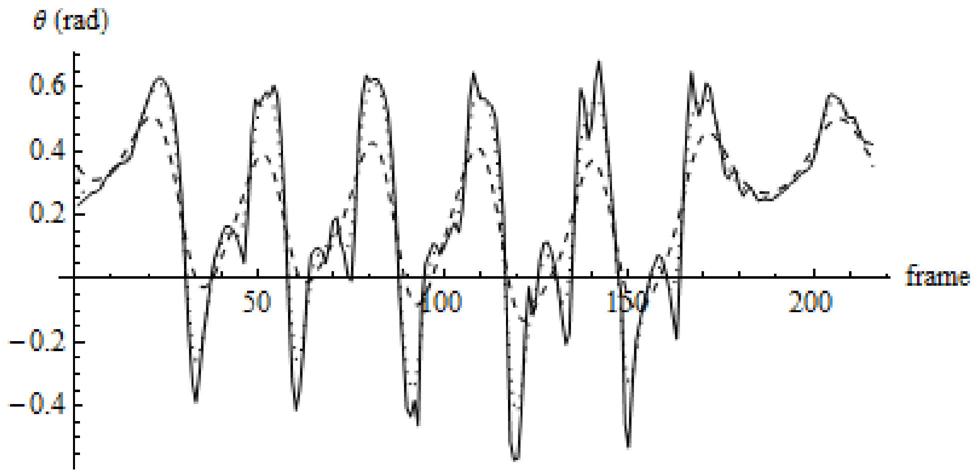


Figure 4: Combinations of unfiltered data (line), and filtered data when σ = 1.65 (dotted line) and σ = 4.5 (dashed line)

Figure 4 shows a family plot of selected readings through the entire trial period using a kernel density estimator with Gaussian kernel smooth and different bandwidth (σ). It shows a combination of unfiltered data (line), and filtered data with σ = 1.65 (dotted line) and with σ = 4.5 (dashed line).

We choose a range of bandwidth such that we capture all the relevant scales, that is, from the smallest scale at which a significant feature was preserved to a larger scale which caused a divergent from the original data.

In order to obtain the preferred value of σ, we perform a residual analysis of the difference between filtered and unfiltered signals. The term residuals are referred to what is left over when the filtered data is subtracted from the raw data. When we filtered only noise, some of the residual values should be greater than zero and some are less than zero. The sum of all residuals should equal to zero or at least close to zero (Robertson *et al.* 2004; Christodoulakis *et al.* 2010).

The residuals are calculated as follows for a signal of N sample points in time,

$$R = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(X_i - \hat{X}_i)^2} \tag{13}$$

where X_i = raw data for the ith sample, and $\hat{X}_i$ = filtered data for the ith sample.

For the rope skipping data, we selected a suitable σ based on the residual analysis which gives the sum of all residuals that is closer to zero. From the result obtained, the residuals appear randomly scattered around zero which indicates that the chosen parameter describes the data well. Since all data are slightly equals to zero, we take the smallest value which is closer to zero.

Table 1 shows 20 different values of σ and the sum of residuals for three sets of rope skipping data (ankle, knee and hip joints).

Table 1: Twenty different value of σ ranged from 0.5 to 6.5

Values of σ	Sum of residuals (ankle)	Sum of residuals (knee)	Sum of residuals (hip)
0.50	4.11752E-09	-2.13838	-1.07016
1.00	-2.62753E-07	-7.95449E-07	-3.98085E-07
1.10	-4.16188E-09	-1.25993E-08	-6.30546E-09
1.20	-4.43618E-11	-1.34096E-10	-6.72066E-11
1.30	-2.64178E-13	-6.10956E-13	-3.81695E-13
1.40	6.3366E-14	3.70259E-13	1.03695E-13
1.50	5.00155E-14	3.35953E-13	8.03801E-14
1.55	5.88696E-14	2.921E-13	9.9809E-14
1.60	5.07649E-14	3.12639E-13	8.11573E-14
1.65	4.98213E-14	3.26073E-13	7.26086E-14
1.70	6.15341E-14	3.57714E-13	1.06581E-13
2.00	7.63556E-14	3.20854E-13	9.65894E-14
2.50	5.80647E-14	3.40172E-13	9.71445E-14
3.00	6.42264E-14	2.69673E-13	1.03861E-13
3.50	8.18789E-14	2.49578E-13	1.21292E-13
4.00	7.17759E-14	1.71196E-13	1.10967E-13
4.50	7.26641E-14	1.6509E-13	1.06082E-13
5.00	8.11018E-14	1.82965E-13	1.01585E-13
5.50	9.55902E-14	2.08833E-13	1.11078E-13
6.50	9.37028E-14	1.93512E-13	1.24567E-13

At first, we choose the smallest value for sum of residuals to determine σ in filtering step for the Gaussian scale-space algorithm. Figure 5 shows the filtered (dot-dashed line) and unfiltered (line) data with $\sigma = 1.65$ for smoothing ankle joint angle. It can be observed that the spikes are eliminated and the data are very well smoothed.

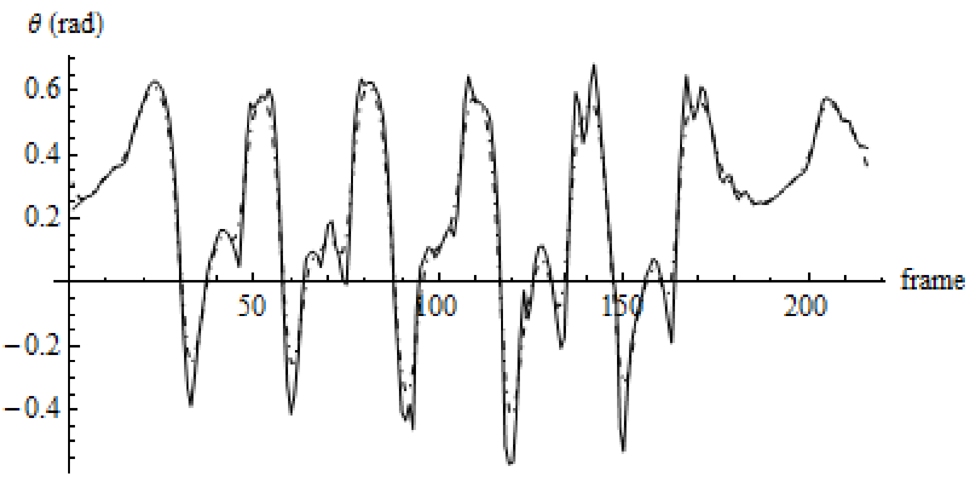


Figure 5: Filtered (dot-dashed line) and unfiltered (line) data when $\sigma = 1.65$ for smoothing ankle joint angle

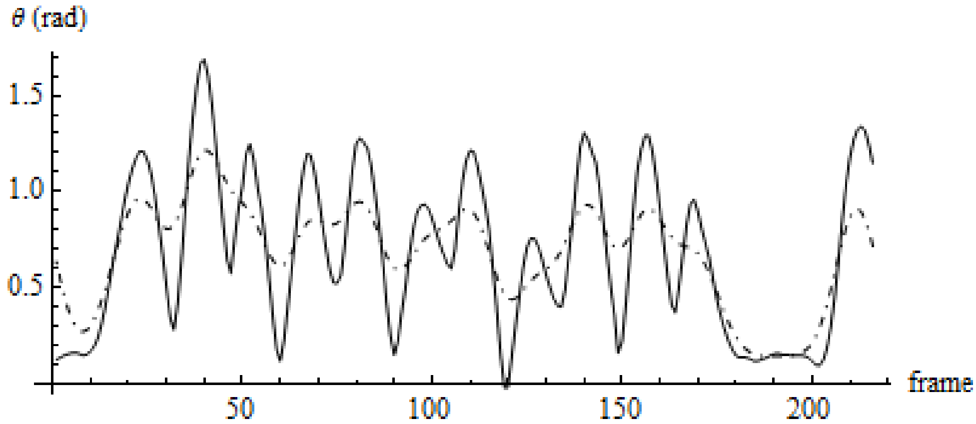


Figure 6: Filtered (dot-dashed line) and unfiltered (line) data when s = 4.50 for smoothing knee joint angle

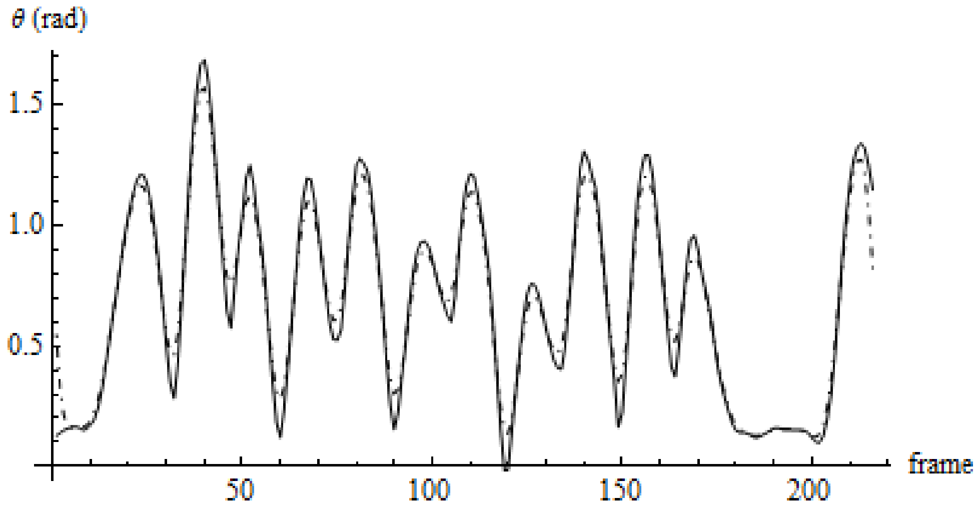


Figure 7: Filtered (dot-dashed line) and unfiltered (line) data with $\sigma = 1.65$ for smoothing knee joint angle

Figure 6 shows the filtered (dot-dashed line) and unfiltered (line) data when $\sigma = 4.50$ for smoothing knee joint angle. Based on the graph plotted, as we take a closer look at the knee data, we found that the smallest value for sum of residual could not be taken due to a high divergent between filtered and unfiltered data. As the graph depicted, we can observed that the main features of the data are loosened away. The over-smoothed data have caused the information that describes the skipping performance, especially the degree of flexion of knee in certain events, to be misleading. Hence, we choose σ value of 1.65 for the knee data where the main features of the knee joint angles are preserved while smoothing the extreme values.

Figure 7 shows the filtered (dot-dashed line) and unfiltered (line) data with $\sigma = 1.65$ for smoothing knee joint angle. It can be observed that the Gaussian scale-space method preserved the main features of the data and smooth the data well.

For smoothing hip angle data, Figure 8 shows the filtered (dot-dashed line) and unfiltered (line) data when $\sigma = 1.65$. We can observed that the main features of the signal is preserved while smoothing the data.

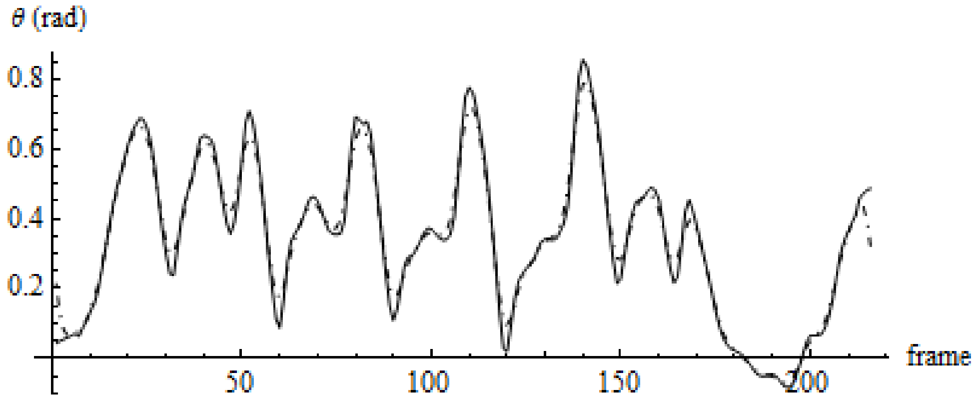


Figure 8: Filtered (dot-dashed line) and unfiltered (line) data with $\sigma = 1.65$ for smoothing hip angle

4. Discussion

Gaussian scale-space method has been widely used in computer vision to sharpen images and smooth data but none has been done on biomechanical data. Usually, different data set will use different scale to preserve the original image.

The value of σ or the chosen standard deviation for the Gaussian Scale-space depends on the individual selection, as long as the main features of the signal exist, the filtering eliminates spikes and smooths the data. This technique answers in a quantitative way which features dataset that really exists in a meaningful way, even if the emergent features appear only on a particular scale. The technique avoids the question of bandwidth selection by investigating all scales.

By using a very small value of σ, which is $\sigma = 0.5$ in this study, filtered data resembled the original data and also was not well smoothed. This means that the spikes or noise still exist in the data stream after filtering is done. On the other hand, a bigger value of σ such as $\sigma = 6.5$ produce smoothed data but the filtered data are diverged from the original data and all become positive values which did not preserve the main features.

Karim and Kong (2011) used Gaussian scale-space to filter Kuala Lumpur Composite Index (KLCI) data and found that the chosen σ value lies between four and five. The study also found that a σ value close to one will result in smoothed data which is closed to the original data. However, for the rope skipping data, values of $\sigma = 4$ or $\sigma = 5$ will cause the smoothed data to diverge from the original dataset. Thus, the values are not suitable for the biomechanical data. Instead the sum of residuals is used as the measure of a good σ value. It was found that a smaller sum of residuals will smooth the data well and eliminate spikes while preserve the main features of the data. The objective is achieved by choosing $\sigma = 1.65$ to remove the spikes, hence smoothing the data. Furthermore, using the same σ value works for the other sets of data. For that, it can be said that the Gaussian scale-space method can be applied for smoothing biomechanical data.

5. Conclusion

In this work, we apply the Gaussian scale-space method to biomechanical data, specifically data for a rope skipping activity. It is found that one empirically chosen value of σ can be applied to different sets of data without the need to do the trial and error procedure for the other data sets. Hence, it is concluded that one σ value can be used throughout different anatomical body landmark for rope skipping data.

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