

## Casimir Force Control with Optical Kerr Effect (Kawalan Daya Casimir dengan Kesan Optik Kerr)

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### ABSTRACT

*The control of the Casimir force between two parallel plates can be achieved through inducing the optical Kerr effect of a nonlinear material. By considering a two-plate system which consists of a dispersive metamaterial and a nonlinear material, we show that the Casimir force between the plates can be switched between attractive and repulsive Casimir force by varying the intensity of a laser pulse. The switching sensitivity increases as the separation between plate decreases, thus providing new possibilities of controlling Casimir force for nanoelectromechanical systems.*

*Keywords: Casimir effect; optical kerr effect (OKE)*

### ABSTRAK

*Kawalan daya Casimir antara dua plat selari boleh dicapai dengan mencetuskan kesan optik Kerr dalam suatu bahan tak linear. Dengan mempertimbangkan suatu sistem dua-plat yang terdiri daripada satu plat metamaterial dengan satu bahan tak linear, kami menunjukkan bahawa daya Casimir antara plat-plat tersebut boleh ditukar antara daya tarikan Casimir serta daya tolakan Casimir dengan mengubah keamatan laser. Tahap kesensitifan pertukaran tersebut meningkat apabila jarak pemisah antara plat-plat tersebut dikurangkan, justeru mencetus idea baru untuk mengawal kesan Casimir bagi sistem mekanikal nanoelektrik.*

*Kata kunci: Kesan Casimir; kesan optik Kerr*

### INTRODUCTION

As boundary conditions are being introduced in a quantized electromagnetic field, the vacuum energy level changes. This change is then observed as a vacuum force on boundaries. This effect was first theoretically derived by Casimir (1948). Since then, Casimir force for various geometries and boundary surfaces has been investigated (Boyer 1968; Lambrecht & Marachevsky 2008; Rahi et al. 2010). Furthermore, various corrections to the ideal cases, including temperature corrections had been investigated (Canaguier-Durand et al. 2010; Milton 2001; Weber & Gies 2010). However, in most cases the Casimir force is found to be attractive (Yang et al. 2010).

Recent development in microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS) leads to increasingly complex NEMS designs. Downscaling size of NEMS will inevitably bring up the issue of Casimir interaction between two surfaces in close proximity, such as stiction (Chan et al. 2001; De Los Santos 2003; Serry et al. 1998). This problem may be avoided if the Casimir force is repulsive (Capasso et al. 2007; Yang et al. 2010). Therefore recently the repulsive Casimir force has attracted renewed interest due to its practical significance (Kenneth et al. 2002; Levin et al. 2010; Munday et al. 2009).

Recently, metamaterials with controllable electromagnetic properties were fabricated experimentally. These metamaterials may have either negative permittivity

$\epsilon$  or permeability  $\mu$  (single-negative materials, SNG) (Pendry et al. 1996, 1999) or simultaneously negative permittivity  $\epsilon$  and permeability  $\mu$  over a band of frequency (left-handed materials, LHM) (Lezec et al. 2007; Parazzoli et al. 2003; Smith et al. 2000; Zhang et al. 2009). As metamaterials may possess noticeable magnetic properties, the Casimir force involving metamaterials may be repulsive (Yang et al. 2010). In particular, repulsive Casimir force is obtained from two parallel, perfectly conducting plates when a LHM lens is introduced between the plates (Leonhardt & Philbin 2007). Casimir force between LHM plates has also been investigated (Rosa et al. 2008; Yang et al. 2008; Yannopoulos & Vitanov 2009). Furthermore, it is also possible to control the Casimir force by adjusting the frequency dependent electromagnetic properties of materials (Yang et al. 2010).

In this article, the Casimir force between metamaterial plates driven by external high intensity laser source, which introduces optical Kerr effect (OKE) is presented. The optical Kerr effect is a nonlinear optical effect due to third order polarization in the presence of a strong electromagnetic field. Although OKE is well-known in nonlinear optics (Chen & Raymond 2011), it has not been utilized to manipulate the Casimir force. We show that with an external laser source which introduces OKE, it is possible to gain control of the sign and magnitude of Casimir force between plates. Particular interest is focused on repulsive Casimir force.

## BASIC THEORY

The Casimir force between two parallel, infinite plates A and F, separated by a distance of  $a$  in free space is expressed by the Lifshitz formula.

$$F_c = -\frac{\hbar}{\pi} \text{Re} \int_0^\infty d\omega \int \int \frac{d^2k}{(2\pi)^2} \sqrt{\frac{\omega^2}{c^2} - k^2} \times \sum_{p=TE, TM} \frac{r_p^A(\omega, k) r_p^B(\omega, k) e^{2i\omega a} \sqrt{\omega^2/c^2 - k^2}}{1 - r_p^A(\omega, k) r_p^B(\omega, k) e^{2i\omega a} \sqrt{\omega^2/c^2 - k^2}}$$

$$= \frac{\hbar}{2\pi^2} \int_0^\infty d\xi \int_0^\infty k dk \sqrt{\frac{\xi^2}{c^2} + k^2} \times \sum_{p=TE, TM} \frac{r_p^A(\xi, k) r_p^B(\xi, k) e^{-2a\sqrt{\xi^2/c^2 + k^2}}}{1 - r_p^A(\xi, k) r_p^B(\xi, k) e^{-2a\sqrt{\xi^2/c^2 + k^2}}}, \quad (1)$$

where  $\xi$  is the (tangential) component of wave vector parallel to the plate surface,  $\xi$  is the imaginary frequency where  $\omega = i\xi$  and  $r_p^j$  is the slab's reflection coefficient for transverse electric (TE) and transverse magnetic (TM) polarized waves. The positive (negative) sign of  $F_c$  corresponds to attractive (repulsive) Casimir force.

We then assume the plates are semi-infinitely thick with respect to the transmission of virtual photons, which then reduces the plate's reflection coefficient to single interface reflection coefficients (Yang et al. 2010).

$$r_{TE}^j = \frac{\mu_j \sqrt{\xi^2/c^2 + k^2} - n_j \sqrt{\xi^2/c^2 + k^2} / n_j^2}{\mu_j \sqrt{\xi^2/c^2 + k^2} + n_j \sqrt{\xi^2/c^2 + k^2} / n_j^2}$$

$$r_{TM}^j = \frac{\epsilon_j \sqrt{\xi^2/c^2 + k^2} - n_j \sqrt{\xi^2/c^2 + k^2} / n_j^2}{\epsilon_j \sqrt{\xi^2/c^2 + k^2} + n_j \sqrt{\xi^2/c^2 + k^2} / n_j^2}, \quad (2)$$

where  $n_j = \sqrt{\epsilon_j \mu_j}$  is the index of refraction for plate  $j = A, B$ . From (1), it is noted that the Casimir force can be repulsive only if the reflectivity of both plates differs in sign. Hence, both plates must have very different electromagnetic properties across some resonances for the Casimir force to be repulsive. Yang et al. (2010) had investigated the changes of Casimir force due to changes in electromagnetic properties of respective plates. Generally, if two parallel plates have the same (different) electromagnetic properties in lower frequency region and different (same) electromagnetic properties in higher frequency region, the Casimir force between the plates would vary from repulsive to attractive (attractive to repulsive) as the distance between plates increases.

The optical control over the Casimir force may be realized by introducing a laser beam of intensity to induce a.c. Kerr effect on the plates (Raymond & Khoo 2012). The effective index of refraction is then given by,

$$n_j(\omega) = \sqrt{\epsilon_j(\omega) \mu_j(\omega) + \eta_{2,j}(\omega) I(\omega)}, \quad (3)$$

where  $\eta_{2,j}$  is the Kerr coefficient and  $I(\omega) = If(\omega)$  indicates the spectral shape  $f(\omega)$  or transient character of the laser pulse duration.

It is noted from (2) that the magnitude of refractive index may determine the sign of the reflectivity, by competing with the first term in the numerator. Hence, the Casimir force between plates may be controlled by  $I(\omega)$ .

## INTENSITY DEPENDENT CASIMIR FORCE

To understand the intensity dependence of Casimir force in OKE scheme, an idealized electric plate driven by intense laser field, with material constants  $\epsilon_A = 10$ ,  $\mu_A = 1$  and finite  $\eta_2$  was placed parallel to a perfectly conducting plate ( $\epsilon \gg 1$ ,  $r_{TE}^B = -1$ ,  $r_{TM}^B = 1$ ,  $\eta_2 = 0$ ). The Casimir force  $F_c$  between both plates was plotted as a function of intensity of impinging laser in Figure 1(a), for inter-plate distance  $a = 0.1\lambda_0$ . Here, the wavelengths in vacuum is  $\lambda_0 = 2\pi c/\omega_0$ , with  $\omega_0 = 2\pi \times 10^6 \text{ s}^{-1}$  and the Casimir forces are expressed in terms of  $K = hc/(64\pi^3 \lambda_0^4) \text{ Nm}^2$ . The Casimir force is expressed as positive (negative) if the force between plates is attractive (repulsive).

In the above case, it is noted that the first plate is mainly electric and the Casimir force is expected to be attractive at zero intensity. However, the Casimir force is noted to be attractive for all intensities. It is further noted that at intensity of  $\eta_2 I \approx 2$ , an optimum attractive force is observed. This slight increase in magnitude of the Casimir force is caused by the transverse electric reflectivity ( $r_{TE}$ ) which decreases quicker compared with the transverse magnetic reflectivity ( $r_{TM}$ ). At higher intensities, the Casimir force between plates becomes less attractive.

In Figure 1(b), a similar setup was used, with the first plate being substituted with material constant of  $\epsilon_A = 1$ ,  $\mu_A = 10$ . In this case, a similar response curve is noticed compared to Figure 1(a). The repulsive force between plates increases slightly when  $\eta_2 I \approx 2$ , corresponding to a quicker decrease in  $r_{TM}$  compared with  $r_{TE}$ . At higher intensities, the Casimir force between plates becomes less repulsive. It is noteworthy that in both cases, the magnitude of Casimir force decreases in magnitude but does not change in sign.

To demonstrate the possibility of laser-controlled sign change in Casimir force, an ideal, piecewise-dispersive material with material constant given by (4) is considered.

$$\epsilon = \begin{cases} 1.5, & \omega < \omega_r \\ 1, & \omega \geq \omega_r \end{cases} \quad \mu = \begin{cases} 0.5, & \omega < \omega_r \\ 5, & \omega \geq \omega_r \end{cases}. \quad (4)$$

This plate is noted to be mainly magnetic at high frequency, while being electric at lower frequency and it can be influenced by the Kerr effect. The Casimir force between this plate and another perfectly conducting plate ( $r_{TE}^B = -1$ ,  $r_{TM}^B = 1$ ,  $\eta_2 = 0$ ) is plotted in Figure 2 for  $\omega_r = 0.8\omega_0$ . Here we notice that the Casimir force is repulsive on small inter-plate distances, while slightly attractive at large inter-plate distances. At laser intensity of  $\eta_2 I \approx 15$ , the Casimir force between plates changes in sign for  $a = 0.17\lambda_0$ . Hence, by introducing piecewise-dispersive constant, couple with OKE, we are able to vary both magnitude and sign of Casimir force. This demonstration indicates that materials

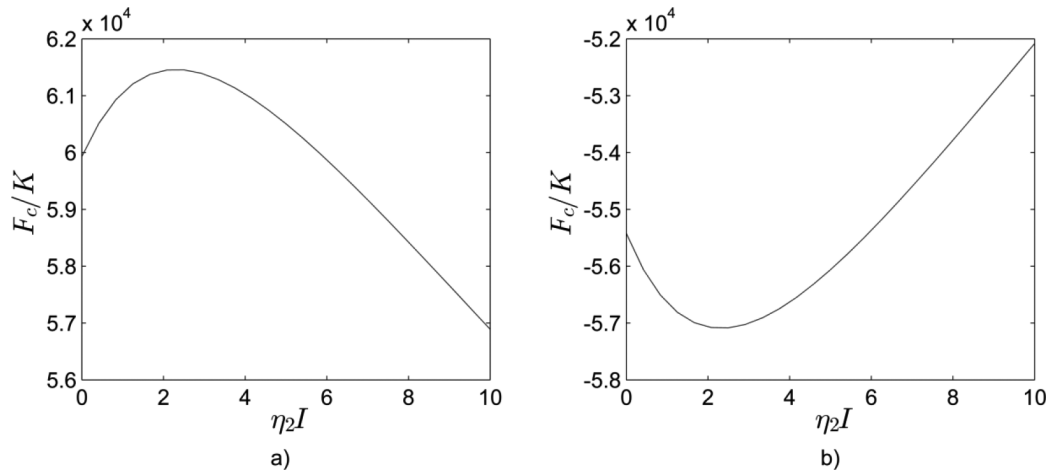


FIGURE 1. (Color online) Casimir force between a perfect conductor ( $r_{TE}^B = -1$ ,  $r_{TM}^B = 1$ ) and a plate with a)  $\epsilon_A = 10$ ,  $\mu_A = 1$  and b)  $\epsilon_A = 1$ ,  $\mu_A = 10$ , as a function of intensity of impinging laser  $\eta_2 I$ . Inter-plate separation is taken as  $a = 0.1\lambda_0$

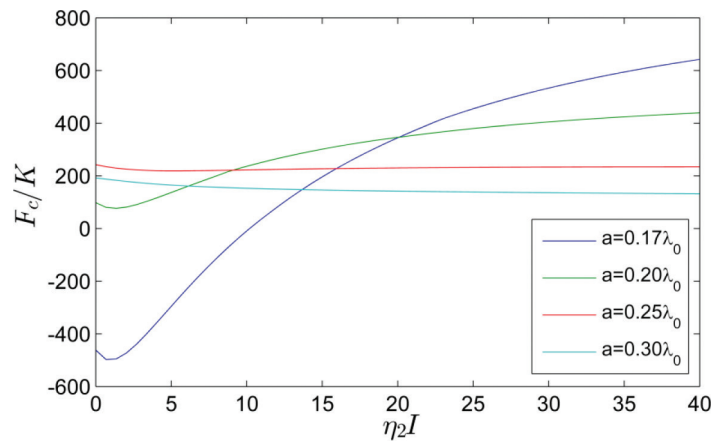


FIGURE 2. (Color online) Casimir force between a perfectly conducting plate and a piecewise-dispersive material of (4), as a function of separation between plates and laser intensity

with significant change in electric and magnetic behavior over a range of frequency may allow laser control over both magnitude and sign of Casimir force.

We now consider the Casimir force between practical materials. In essence, we let one of the plates be a Kerr material, while the other plate is a metamaterial with its dispersive constant modeled by Drude-Lorentz type of single resonance, as given by (5) below.

$$\{\epsilon, \mu\} = 1 + \frac{\omega_p^2}{\omega_r^2 - \omega^2 - i\gamma\omega}. \quad (5)$$

In (5),  $v = e, m$ , where  $v = e$  corresponds to the dielectric constant,  $\epsilon$  on the left-hand side of equation, while  $v = m$  refers to the magnetic permeability  $\mu$ . Meanwhile  $\omega_p^2, \omega_r^2, \gamma$  are the plasma frequency, the resonance frequency and the damping frequency, respectively. The metamaterial is not affected by the

Optical Kerr effect, that is  $\eta_2 = 0$ . The metamaterial's dispersion is modeled using (5) with  $\omega_p = 0.5\omega_0$ ,  $\omega_r = \omega_0$ ,  $\omega_p = 35\omega_0$ ,  $\omega_r = 0.01\omega_0$ ,  $\gamma = 10^{-2}\omega_r$ . Meanwhile, we modeled the Kerr material as a Chalcogenide glass,  $As_2S_3$ , with refractive index  $n_0 = 2.4$  and Kerr coefficient  $\eta_2 = 1.4 \times 10^{-17} m^2/W$  (Kosa et al. 1993). The Casimir force between these plates is plotted in Figure 3. Here, we notice that the Casimir force between both plates changes from repulsive to attractive at laser intensity of  $I \approx 7 \times 10^{18} W/m^2$ . The sensitivity of Casimir force towards change in intensity increases for smaller inter-plate spacing.

To further note on the feasibility of this scheme, we notice that the intensity of laser source required ( $I \approx 10^{18} W/m^2$ ) is achievable with normal diode lasers, which is far less than the current ultrahigh laser intensity  $I \approx 10^{26} W/m^2$  (Bahk et al. 2004), thus enabling practical control over the Casimir force via optical means.

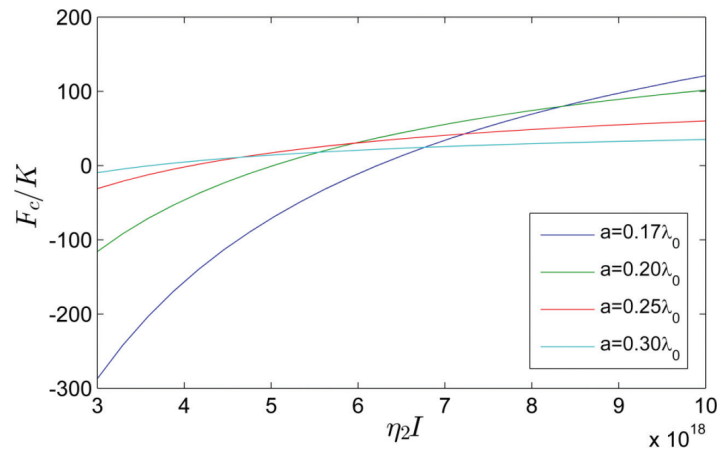


FIGURE 3. (Color online) Casimir force between the  $As_2S_3$  plate with Kerr effect, and a metamaterial with dispersive constant as given in (5), as a function of separation between plates and laser intensity

### CONCLUSION

We have analyzed the possibility of controlling the Casimir force using combinations of metamaterials and nonlinear materials exhibiting optical nonlinear Kerr effect. We have shown that the force can be significantly varied and switched between positive and negative values by changing the intensity of the laser. The ability to alter the force due to the quantum vacuum by changing nonlinear optical properties of matter with intense laser sources provides new possibilities of using laser optics to control quantum electrodynamics phenomena. The potential applications include manipulating nanoresonators and integrating optical devices into nanoelectromechanical systems. A more concise model for Chalcogenide glass's dispersion will be used in future works.

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