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Stochastic Lead Time Demand Estimation via Monte Carlo Simulation Technique in Supply Chain Planning

(Anggaran Permintaan Masa Lopor Stokastik Melalui Teknik Simulasi Monte Carlo dalam Perancangan Rantaian Bekalan)

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ABSTRACT

This paper considers a Monte Carlo simulation based method for estimating cycle stocks (production lot-sizing stocks) in a typical batch production system, where a variety of products is scheduled for production at determined periods of time. Delivery time is defined as the maximum lead time and pre-assembly processing time of the product's raw materials in the method. The product's final assembly cycle and delivery time, which were obtained via the production schedule and supply chain simulation, respectively, were both considered to estimate the demand distribution of product based on total duration. Efficient random variates generators were applied to model the lead time of the supply chain's stages. In order to support the performance reliability of the proposed method, a real case study is conducted and numerically analyzed.

Keywords: Cycle stock; inventory; lead time demand; Monte Carlo; supply chain

ABSTRAK

Kertas ini mengambil kira kaedah simulasi Monte Carlo untuk menganggarkan kitaran stok (tempat keluaran-saiz stok) dalam sistem pengeluaran tipikal kelompok, dengan pelbagai produk dijadualkan untuk pengeluaran pada jangka masa yang ditetapkan. Dalam kaedah ini, masa penghantaran ditakrifkan sebagai masa lopor maksimum dan masa sebelum pemprosesan produk bahan mentah. Kitaran pemasangan akhir produk dan masa penghantaran masing-masing yang diperoleh melalui jadual pengeluaran dan simulasi rantaian bekalan diambil kira untuk menganggarkan pembahagian permintaan produk berdasarkan jumlah tempoh. Penjana pengubah rawak yang cekap digunakan sebagai model masa lopor peringkat rantaian bekalan. Dalam usaha untuk menyokong kebolehpercayaan prestasi kaedah penilaian yang dicadangkan, kajian kes sebenar dijalankan dan dianalisis secara berangka.

Kata kunci: Inventori; kitaran stok, Monte Carlo; permintaan masa utama; rantaian bekalan

INTRODUCTION

In any batch production system, various kinds of products are produced in determined period of time according to the scheduled master production plan. Thus each product is composed of a different bill of material (BOM), according to which there are determined supply chains, where production planning and inventory control for each product and its BOM is in place (Sarker & Parija 1996). The system design is based on customer demand, market strategies and present supply chain conditions (Zhao 2009).

There are a few challenges in such design: diversity of products and their related supply networks; variation of inventory control and warehousing policies; differences in manufacturing processes and final assembly lines and sources of uncertainty throughout the supply networks. It is quite clear such implications impose time and expenses on the whole production system.

Sources of uncertainty are both external and internal factors influencing the system design like demand quantity, lead time, transportation and logistics time as external factors and products' preparation and pre-assembly process, raw material inspection time intervals and the acceptance and rejection results of inspections as internal factors.

There are published studies aimed at solving the above mentioned challenges, some of which focus on improving the inventory level and specifying optimum order point. Some operational research techniques (Crdenas-Barrón 2009; Dogrua et al. 2008; Hoque & Goyal 2000; Timpe & Kallrath 2000), simulation modeling (Baudet et al. 1995; Fuchino et al. 1999; Wu & Olson 2008) and heuristic or Meta heuristic algorithm (Hoquea & Goyal 2006; Sarker & Khan 1999) based on various models of production and inventory planning have been proposed. For example, Sarker et al. (1995) and Sarker and Khan (1999) developed a variety of models for a typical manufacturing system under continuous supply chain and a constant rate of demand. However, the assumption of having a constant rate of customer demand is very restrictive during the growth and decline phases of the product life cycle where demand is either increasing or decreasing with time. Following this idea, some improved models were developed later based on deterministic time-varying demand process (Omar & Smith 2002), or multiple demand classes (Xua et al. 2010), linear trend in demand (Rau & OuYang 2008), compound Poisson demand (Zhao 2009), normally distributed demand (Kevork 2010) and stock dependent demand (Mohsen et al. 2010). But, they mostly worked on a limited range of specific distributions.

The ability of applying any possible distributed model for both product's demand and raw materials' lead time together with the assumption of their uncertainty has not been considered yet. Also pre-assembly processing time, which is spent on un-processed or semi-processed raw materials, to make them ready at the beginning of final assembly line, has not been yet considered following the lead times.

In this study, the mentioned potential developments have been conducted to make a dynamic model of demand distribution at the period of time between two successive production plans of a particular product (which it's itself dynamic too). A Monte Carlo based simulation method is designed for running the model to estimate the quantity of cycle stock (production lot-sizing stock) of the product (Fishman 1996). Monte Carlo is a technique for managing uncertainty in complex systems such as supply networks. It is very convenient for managers because by building just one scenario they can see results for many possible variants. This approach offers not just one outcome, but a distribution of possible outcomes (Stefanovic et al. 2009). There is also a literature of using this technique in supply chain simulation modeling and optimization of batch production systems (Kevork 2010; Nair & Closs 2006; Vilko & Hallikas 2011; Zhao 2009), which provides an efficient performance of current implementations. Random variates generators that are used to simulate supply chain performing time are derived in this paper.

The paper is organized as follows. The next section summarizes configurations of a typical batch production system belong with its supply chain and defines some related terms. The section after that formulates the proposed model of demand distribution at the period of delivery time and estimates cycle stock subsequently. The last section reviews a numerical case study followed by the conclusion.

System Configuration and Definition

In a batch production system, where a variety of products is scheduled for production at determined periods of time, different supply chains and manufacturing processes are expected to be designed for each product. Different sorts of inventory control and warehousing policies are subsequently required to be planned by considering each product's features. Each product's bill of material (BOM) is divided into unprocessed, semi-processed and processed stages. The first two stages and the last stage are also named as raw materials and processed industrial materials (parts), respectively. Unlike raw materials, which still need some manufacturing or complimentary pre-assembly processes to become ready for use at the final assembly process of the product; parts come to the assembly line directly from supplier (or probably from factory's inputs warehouse). For example assume X is one of the products in a batch production system and is composed of a bill of n materials.

So X_i ; $i = 1, 2, \dots, n$ will be the i^{th} product. A portion of them is imported to the factory as raw materials and is undertaken a process of pre-assembly operations (Op.), which will finally be sent to the main assembly line. The rest (parts) are collected from the market and are kept in the warehouses until the startup of the main assembly line.

A typical design of supply and production process for product X has been illustrated in Figure 1. All raw materials and parts enter the final assembly line (bold shapes in Figure 1) in order and the finished product is sent to stock warehouse after passing the quality control (QC) inspection. Demand will be satisfied gradually by shipping products over the market network including distribution centers (DC), retailers or even end-user customers. The supply chain of X_i can generally involve either external or both external and internal stages depending on being a part or raw material. Whatever the type of X_i is, the supply chain may have a number of stages, for j ; $j = 1, 2, \dots, m_i$ where m_i is the whole stages of the chain. The time that X_i spends on stage j is named by t_{ij} ; $i = 1, 2, \dots, n$ & $j = 1, 2, \dots, m_j$ and the total time that it spends from first stage to the beginning of final assembly line is called its delivery time.

Model Formulation

Given the definitions and assumptions in the prior section and considering the schematic model of the system, which is presented in Figure 1, delivery time of $X_{i\ n}$ can be calculated as below:

$$T_i^s = \sum_{j=1}^{j=m_i} t_{ij}; i = 1,2,\dots,n. \tag{1}$$

So the delivery time of X during the supply chain until the assembly process starts up, is named T_x^s and is calculated by (2).

$$T_x^s = \max_{i=1}^{n}\left\{\sum_{j=1}^{j=m_i} t_{ij}\right\} = \max_{i=1}^{n}\left\{T_i^s\right\}. \tag{2}$$

Consider the assembly time of X (namely T_x^p), which can be separately estimated based on the production cycle time and order batch size and production capacity, the cycle stock, which is defined as the required amount of product to satisfy demand quantity between two successive production plan, will be obtained from the amount of demand during X's total delivery time ($T_x^s + T_x^p$). Given that the demand distribution is probabilistic on the time ($g_d(t)$) as well as supplier's lead time (t_{ij}), the cycle stock of X will be as (3):

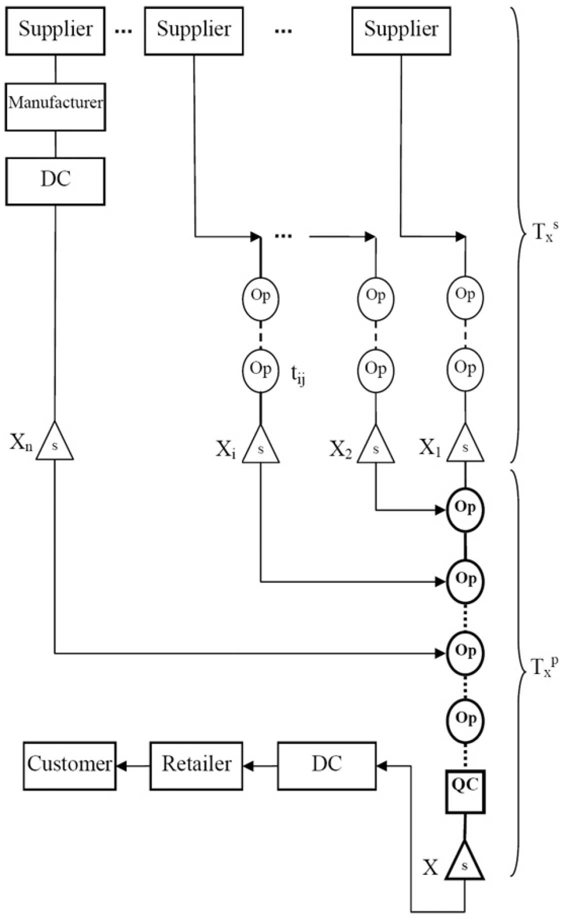

FIGURE 1. Configuration of operation process chart and supply chain design

$$I_x = D\,(T_x^s + T_x^p\,), \tag{3}$$

where D is the amount of demand per unit of time (that t_{ij} calculated based on) only if such amount could be considered deterministic and constant. While in the real world, the demand often happens under a probabilistic distribution, I_x from (3) cannot be a reliable estimation of demand at delivery time. Utilizing characteristics like mean amount of demand per time (D), will not provide the acceptable accuracy. On the other hand, t_{ij} and T_x^p are hardly constant and not deterministic in the real world. In situations where D and t_{ij} are probabilistic, calculation of I_x is not possible using (3). Monte Carlo, as a powerful and accurate technique, is capable of addressing the issue. Considering the fact that this technique utilizes random number characteristics for modeling the stochastic processes, it can simulate the demand distribution in a delivery time. If the probabilistic treatment of t_{ij} is assumed to meet a distribution function such as $f_{ij}\,(t_{ij})$; $i = 1, 2, \ldots, n, j = 1, 2, \ldots, m_i$, the cycle stock of X could be estimated by (4).

$$\begin{aligned} I_x &= g_d\left(T_x^s + T_x^p\right) = g_d\left(\max_{i=1}^{n}\left\{T_i^s\right\} + T_x^p\right) \\ &= g_d\left(\max_{i=1}^{n}\left\{\sum_{j=1}^{m_i} t_{ij}\right\} + T_x^p\right); \\ T_{ij} &\sim f_{ij}\left(t_{ij}\right);\, i = 1,2,\ldots,n,\; j = 1,2,\ldots,m_i. \end{aligned} \tag{4}$$

Such estimation for I_x using Monte Carlo technique is understood through generating a sequence of demand-in-delivery time random variates and then conducting an appropriate distribution. Each of elements in the sequence is produced by a series of random variates. The random numbers $R_1, R_2, \ldots, R_{m_i}$ are used to generate random variates T_{ij} ; $i = 1, 2, \ldots, n$ & $j = 1, 2, \ldots, m_i$ and then T_i^s ; $i = 1, 2, \ldots, n$ and T_x^s will be obtained using (1) and (2), respectively.

A set of this series of mathematical and statistical operations, which is conducted under Monte Carlo technique, will generate one I_x value. Replications for a given number of times (e.g. k) produce a sequence of I_{x_z} ; $z = 1, 2, \ldots, k$, which is a random statistical sample of demand-in-delivery time for product X and is used to calculate the production lot-sizing stock. One of the characteristics of the suggested method is its capability for modeling supply chain stages effect of a given part or raw material on each others, which subsequently affects the whole delivery time of the part or material and the final product. For example, assume that in stage j of X_i supply chain, there are two suppliers (e.g. α and β), with different performance qualities, which in addition to having a different t_{ij} value, (e.g. t_{ij}^{α} and t_{ij}^{β}), causes a different time period in one of the following stages (e.g. t_{iz} ; $z > j$) related to pre-assembly operations process of the given part or material. In other words, if the given part is provided by supplier α, the pre-assembly operation time in stage z will also be t_{iz}^{α} or otherwise, it is t_{iz}^{β} . Therefore, for the product X_i, delivery time (T_i^s) could be either $T_i^{s(\alpha)}$ or $T_i^{s(\beta)}$ depending on the supplier of stage j. The case of supplier selection may be also done through a particular probability model as well. In the real world, such cases are very common and include a great portion of challenges in supply chain management's decision making and planning.

The method proposed in this study is capable of including all the effects of uncertainty and its modeling in the supply chain design. In fact the procedure that is capable of including effects of uncertainties with high accuracy and validity is Monte Carlo technique.

A Numerical Case Study

In an industrial manufacturing company, kinds of outdoor floodlight are produced. One of these products with the trade name 'Floodlight 400 W' is composed of a quantity of parts which are collected directly from market (lamp, electro-transformer, starter, sealing washer, screws and bolts). Some raw materials (reflector sheet, aluminium bullion and metal sheet) that are provided from supplier would take a couple of pre-assembly operations to enter final assembly line.

This product (namely X) is scheduled in the production plan periodically. The problem is that there is no accurate estimation for order point of the product, which could satisfy the demands during the two production period. In other words the main concern is estimating cycle stock. In order to apply the proposed model, some preliminary studies have been conducted and required data collected. The data concerns the bill of material, quantity, stages and periods for product delivery in supply chain and pre-assembly processes for all parts and raw materials collected.

In Tables 2-15 the results of statistical analyses on collected data aimed at probabilities distribution of each stage's period in materials supply chain are shown. Time unit is day. Table 1 summarizes all estimated probability distribution or density functions (PDF) and followed by Tables 2 to 15, which describe addressed experimental distributions in Table 1.

Random variates generators for exponential and uniform densities and experimental distributions are made by inverse transformation method and normal density through direct transformation method. In such a way that for any given random variable T_{ij} there will be one generator and in modeling stage it will be utilized for generating random variates generating.

A considerable literature has been reviewed on the topic of random variates generators to apply the most efficient generators in each case (Banks & Carson 1984; Box & Muller 1958; Fishman 1978; Schmeiser 1981; Schmidt & Taylor 1970). Samples of the generators for T_{11}, T_{53}, T_{54} and T_{14} are presented in (5) to (8), respectively.

$$T_{11} = 0.5(1 + R). \tag{5}$$

$$T_{53} = \frac{-1}{0.6}\ln(1-R). \tag{6}$$

$$T_{54} = \begin{cases} 1 & ; \quad 0 < R \le 0.41 \\ 2.5 & ; \quad 0.41 < R \le 0.73. \\ 4 & ; \quad 0.73 < R < 1 \end{cases} \tag{7}$$

$$T_{14} = \begin{cases} 1.7\left[(-2\ln R_1)^{1/2}\, Sin(2\pi R_2)\right] + 2.5 \\ 1.7\left[(-2\ln R_1)^{1/2}\, Cos(2\pi R_2)\right] + 2.5 \end{cases}. \tag{8}$$

Running Table 1 for every time results a set of statistical samples for all T_{ij} and provides a possibility to generate a random variate sample for T_x^s, which is shown in Table 16. Consecutive replication of the sampling by utilizing Microsoft Excel programming, leads to generating a sequence of T_x^s values. A 32-element sequence generated through the Monte Carlo method for T_x^s is shown in Table 17. The collected data from customers' daily order quantity

TABLE 1. Estimated pdf for delivery times of x 's supply chain ($f_{ij}(t_{ij})$)

T_{ij}		j = 1	2	3	4	5	6	7
i	1	Uniform [0.5,1]	Table 2	Uniform {1,2,3,4}	Normal (2.5,1.7)	Table 3		
	2	Uniform [0.5,1]	Normal (4.5,1.3)	Table 4	Constant 2			
	3	Uniform [0.5,1]	Table 5	Uniform [5,8]	Table 6			
	4	Uniform [0.5,1]	Table 7	Table 8	Table 9	Uniform {2,2.5,3}	Normal (1.5,1)	
	5	Uniform [1,2]	Constant 1	Exponential (0.6)	Table 10	Uniform {2,3}	Table 11	Table 12
	6	Uniform [1,2]	Table 13	Table 14	Constant 2.5	Exponential (0.7)		
	7	Uniform [1,2]	Normal (2.2,1.1)	Table 15	Uniform {0.5,1,1.5,2}	Constant 0.8	Table 16	

TABLE 2. Estimated pdf for T_{11}

t_{11}	0.5	1	1.5	2
$P(t_{11})$	0.21	0.36	0.38	0.05

TABLE 3. Estimated pdf for T_{15}

t_{15}	1.2	1.3	1.4	1.5	1.6
$P(t_{15})$	0.17	0.14	0.20	0.25	0.24

TABLE 4. Estimated pdf for T_{23}

t_{23}	2	3	4	5	6	7	8
$P(t_{23})$	0.08	011	0.31	0.26	0.12	0.06	0.07

TABLE 5. Estimated pdf for T_{32}

t_{32}	2	3
$P(t_{32})$	0.43	0.57

TABLE 6. Estimated pdf for T_{34}

t_{34}	4.2	4.3	4.4	4.5
$P(t_{34})$	0.55	0.19	0.18	0.08

TABLE 7. Estimated pdf for T_{42}

t_{42}	2	2.5	6	6.5
$P(t_{42})$	0.125	0.125	0.345	0.345

TABLE 8. Estimated pdf for T_{43}

t_{43}	0.1	0.2	0.3	0.4
$P(t_{43})$	0.61	0.16	0.12	0.11

TABLE 9. Estimated pdf for T_{44}

t_{44}	1	2	3.5	4
$P(t_{44})$	0.48	0.33	0.11	0.08

TABLE 10. Estimated pdf for T_{54}

t_{54}	1	2.5	4
$P(t_{54})$	0.41	0.32	0.27

TABLE 11. Estimated pdf for T_{56}

t_{56}	2	2.5	3	3.5
$P(t_{56})$	0.15	0.21	0.28	0.36

TABLE 12. Estimated pdf for T_{57}

t_{57}	2.1	2.2	2.3	2.4	2.5	2.6	2.7
$P(t_{57})$	0.12	0.13	0.19	0.17	0.15	0.14	0.09

TABLE 13. Estimated pdf for T_{63}

t_{63}	2	3	4	5
$P(t_{63})$	05	0.3	0.1	0.1

TABLE 14. Estimated pdf for T_{73}

t_{73}	1.8	2.1	2.6	3	4
$P(t_{73})$	0.22	0.19	0.11	0.18	0.30

TABLE 15. Estimated pdf for T_{76}

t_{76}	2	3	4	5
$P(t_{76})$	0.63	0.12	0.14	0.11

TABLE 16. Generated delivery times as random varates (t_{ij})

t_{ij}		j							T_i^s
		1	2	3	4	5	6	7	
	1	0.6	1	3	3.1	1.7			9.4
	2	0.9	2.5	7	2				12.4
	3	0.7	2	6.2	4.3				13.3
i	4	0.8	2.5	0.4	2	3	1.1		9.8
	5	1.3	1	2.2	1	2	3	2.4	12.6
	6	1.5	6	3	2.5	2.1			15.1
	7	1.1	1.5	3	2	0.7	4		12.3
T_x^s									**15.1**

for product X explains that daily demand distribution is consistent with Table 18. On the other hand, the final assembly time for product X, (which initiates upon fully delivery of all materials), depends on factors including production cycle time, daily production capacity and production order quantity. It is assumed that there is no strategy for production capacity changing in the factory. Production cycle is usually stabilized after startup period, while the demand quantity varies for different periods and depends on factors like period of time between two

TABLE 17. Generated sequence of x's delivery time (T_x^s)

15.1	9.2	10.9	18.0	13.5	18.2	11.9	13.4
12.0	17.5	14.6	12.6	17.0	12.7	15.7	12.6
13.8	11.1	13.4	15.8	16.3	10.0	16.2	13.0
16.7	12.8	16.0	14.3	14.0	12.5	17.4	14.8

TABLE 18. Estimated pdf for daily demand ($g_d(t)$)

D	260	280	800	320	340	360	380	400
P(D)	0.09	0.12	0.15	0.19	0.22	0.11	0.10	0.02

scheduled productions of *X*, warehousing conditions and sales strategies. In other way, once the final assembly process is initiated and the final product batches are imported to warehouses, the customers' order satisfying could be started. Thus, there is no need for postponing the shipment process until the completion of production plan of *X*.

Regarding the given product, production cycle time in final assembly line is 2 min or 1/240 of working day and a capacity of 240 units per working day is planned for final assembly. Taking into account the final quality control time and product warehouse input or output process, put time and preparing operations for dispatching, the final production estimated cycle for *X* is 4 min.

The mean quantity of each customer's daily order is estimated for 110 units of *X* which is consistent with the final production capacity of 120 units per day. Thus, an acceptable estimation for value $T_x^p = 1$ could be assumed. Now using the generated sample of delivery times (T_x^s) obtained from Table 17, daily demand values from Table 18 and constant value $T_x^p = 1$, it will be possible to simulate the demand distribution of the whole delivery time of product *X* through Monte Carlo technique. The simulation results are collected in Table 19 and an analyzing test on the results (Figure 2) shows that they meet Normal density function appropriately.

The estimated distribution can be used directly to inventory control and production planning of the system especially for determining the order point, order quantity and cycle stock of product *X*. Application of the proposed model for all products provides company a really well-designed production lot-size ordering system considering uncertainty of supply chain performance and demand distribution during the delivery time.

CONCLUSION

This paper considered a batch-production system, in which a variety of products is scheduled for production at determined periods of time and different supply chains and manufacturing processes are expected to be designed for each product. Different sorts of inventory control and warehousing policies are subsequently required to be planned considering each product's features in this system. The most important factor in the planning is demand distribution upon lead time, which is key element to calculate fundamental measures such as optimum order point, economic order quantity and production schedules. Diversity of products and their related bill of material and manufacturing processes along with sources of uncertainty are some inherent challenges that critically affect supply chains in batch production systems and particularly make estimating the demand distribution during lead time and processing time hard to estimate.

If pre-assembly processing time (for raw materials) adds to the lead time and named delivery time, estimating demand distribution during the delivery time of each product is a crucial problem in batch production systems.

TABLE 19. Monte carlo replication reselts for I_x

Rep.	$T_x^s + T_x^p$	I_x	Rep.	$T_x^s + T_x^p$	I_x	Rep.	$T_x^s + T_x^p$	I_x	Rep.	$T_x^s + T_x^p$	I_x
1	16.1	5240	9	11.9	3820	17	14.5	5100	25	12.9	4180
2	13	4040	10	15.6	5240	18	18.0	6000	26	16.7	5420
3	14.8	5020	11	14.4	4940	19	17.3	5880	27	17.2	5440
4	17.7	5740	12	17.0	5460	20	15.0	4860	28	18.4	6420
5	10.2	3300	13	19.0	6060	21	19.2	6300	29	14.4	4600
6	18.5	6560	14	13.6	4440	22	13.7	4720	30	13.6	4240
7	12.1	3600	15	16.8	5940	23	11.0	3460	31	14.0	4800
8	13.8	4320	16	15.3	5360	24	13.5	4560	32	15.8	5020

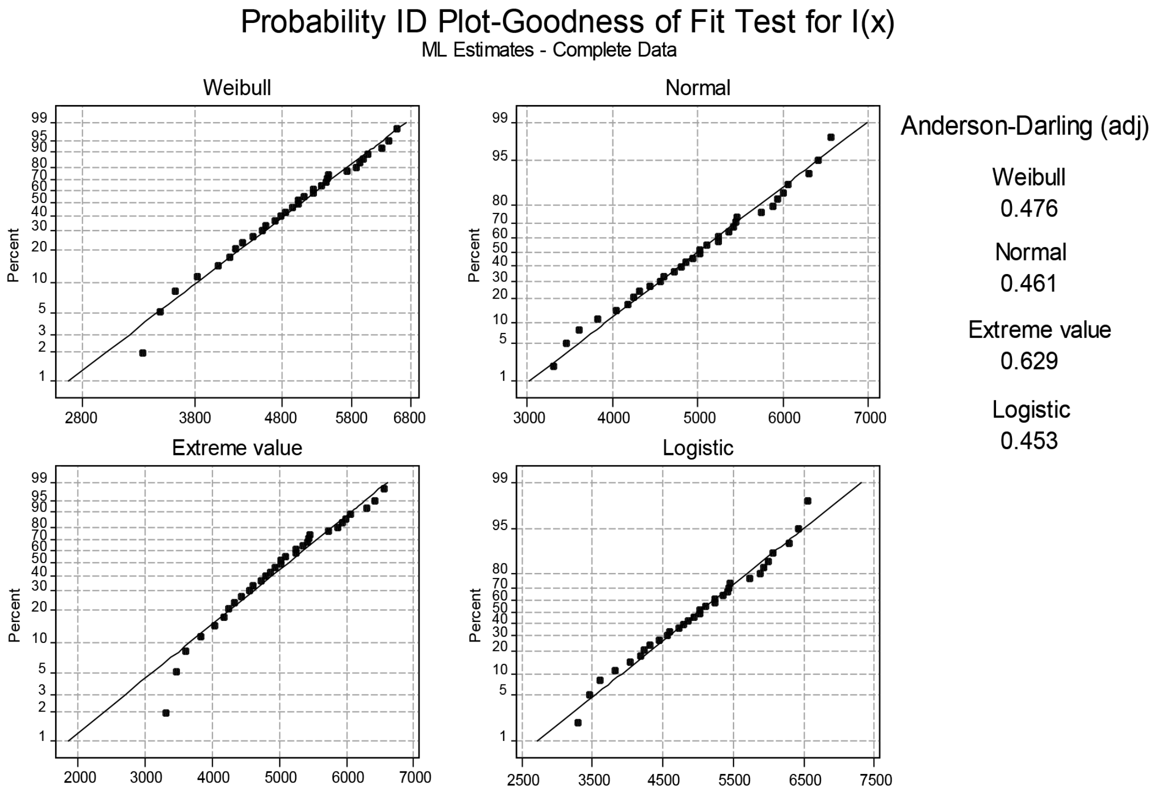

FIGURE 2. Goodness of fit test result for I_x

In this study a Monte Carlo simulation based technique is proposed for dealing with this problem via modeling demand distribution at the period of products' delivery time and final assembly process time. Order point, order quantity and cycle stock (production lot-sizing stock) of each product are estimated subsequently. Efficient random variates generators are applied to simulate timing models of supply chain stages. To support the proposed method, a real case study is conducted and numerically analyzed.

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