

## **SIMULATION MODELLING FOR EVALUATING PERFORMANCE OF COMMUTER SYSTEM WITH ARENA**

(Pemodelan Simulasi untuk Menilai Prestasi Sistem Komuter dengan Arena)

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### *ABSTRACT*

Capital investment on upgrading commuter transportation system may be ineffective if demand of passengers towards commuter is not accounted for. Hence, simulation study is suggested to study the commuter system performance. The system is evaluated from the aspects of waiting time of passengers and number of cases of passengers being left out at the stations. Input data such as arrival time of commuter, arrival process of passengers, capacity of coaches, proportion of passengers to certain destinations, and dwelling time of commuter at stations were collected, modelled, validated and used as input to the simulation model developed using Arena. The developed model is used to generate data related to the performance of the system. An example on application of the model in comparing performance of two classes of commuter is discussed.

*Keywords:* simulation modelling; stochastic process; transportation; performance; Arena

### *ABSTRAK*

Pelaburan modal dalam menaik taraf sistem pengangkutan komuter mungkin tidak berkesan sekiranya permintaan penumpang terhadap komuter tidak dipertimbangkan. Oleh itu, suatu kajian simulasi telah dicadangkan untuk meninjau prestasi sistem komuter. Prestasi sistem dinilai dari segi masa penumpang menunggu komuter dan bilangan kes penumpang ditinggalkan oleh komuter di stesen-stesen yang berkaitan. Data input seperti masa ketibaan komuter, proses ketibaan penumpang, kapasiti koc, perkadaran penumpang ke destinasi tertentu, dan tempoh masa komuter berhenti di stesen dikumpulkan, dimodelkan, disahkan dan digunakan sebagai input kepada model simulasi yang dibangunkan menggunakan Arena. Model simulasi yang dibina telah digunakan untuk menjana data output yang berkaitan dengan prestasi sistem tersebut. Contoh penggunaan model dalam membandingkan prestasi dua kelas komuter telah dibincangkan.

*Kata kunci:* pemodelan simulasi; proses stokastik; pengangkutan; prestasi; Arena

## **1. Introduction**

Capital investment on upgrading commuter transportation system may lead to a loss if the demand of passengers towards commuter is not considered. Although schedule of commuter may affect the arrival pattern of passengers at each station, it is undeniable that arrival of passengers may vary from time to time. The uncertainty of arrival process will affect the sales of commuter ticket and then lead to review on strategy of commuter scheduling. Hence simulation modelling is suggested as a tool of measuring the performance of commuter transportation system to meet the market demand.

There are two main objectives of this paper, firstly to build a simulation model for commuter network system with Arena 14.5. Secondly to apply the simulation model built in comparing performance of two commuters with different capacity by measuring passengers' waiting time at stations.

The paper is organised as follow: Section 2 contains a literature review related to various study on measurement of transportation system performance, simulation modelling and application of the Arena software in simulation modelling. In Section 3 strategies used to achieve the objectives mentioned in this article are discussed. Section 4 shows the development of simulation model for the commuter transportation system. Section 5 illustrates the application of simulation model in comparing performance of two commuters with different capacity by measuring passengers' waiting time at stations and its output analysis. Conclusion is made in Section 6 followed by recommendations for future research.

## **2. Literature Review**

### **2.1. Measurement for Performance of Transportation System**

There are different types of indicators to measure the performance of a transportation system. Krueger (1999) suggested the application of Parametric Model in Canadian National Railway for improvement of track asset utilisation through measurement and monitoring of system track capacity. Martínez *et al.* (2001) built a SIMAN-Arena model of the train system in United States of America (USA) that allow analysis on track layout, operation strategies, modal coordination, on-time performance and compare schedule operation and headway operation of the system. Quan *et al.* (2004) used graphical technique to develop deadlock-free algorithm for dispatching each train to its destination with minimal travel time while considering the speed limit constraint and headways between trains in Los Angeles County, USA. Hofman and Madsen (2005) studied factors that affect the robustness in train scheduling and studied scheduling recovery methods by modelling train network in Denmark using simulation tool. Alfieri *et al.* (2006) presented solution for effective railway circulation on single line in the Netherland based on integer multi commodity flow model. Noble (2009) compared list of rail enhancements for improvement of on-time passenger service and reduce freight delays in the USA.

### **2.2. Simulation Modelling**

Various researchers applied simulation technique in system modelling. Yalçınkaya and Bayhan (2008) proposed solution for average passenger travel time optimisation based on discrete-event simulation and response surface methodology using Arena software besides building relationship between average travel time, rate of carriage fullness and headways using Minitab software. Marinov and Viegas (2010) studied the freight train operations in Portugal with mesoscopic simulation modelling using Simul8. Sánchez and Ortega-Mier (2011) conducted a case study based on commuter network in Spain and show that modelling trains and passenger behaviour is possible with discrete event simulation. Greenberg *et al.* (2013) built three simulation models for rails-centered hazard events in the US, which are industrial system simulation tool, line source plume model and economic simulation model. Woroniuk and Marinov (2013) applied the discrete event based simulation models in accessing the level of utilisation of the rail route in Eastern Spain. Yao *et al.* (2013) analysed the spatial and temporal characteristics of passenger flow distribution besides the effectiveness of transportation strategies with simulation model based on multi-agent approach.

### **2.3. Application of Arena in Simulation Modelling**

There are a number of researchers who used Arena software in simulation modelling. Cheng (2008) conducted a simulation study to look for strategy of optimising revenue from high speed rail system in Taiwan. Confessore *et al.* (2009) estimated the commercial capacity of railways in Italy. Öztürk (2012) analysed and evaluate the effectiveness of operations of metro line in Turkey. Motraghi and Marinov (2012) analysed the current urban freight train system, evaluate alternatives and maximise utilisation of proposed rail system in the United Kingdom (UK). Thai (2013) suggested the application of a Single Double Track – Train Scheduling Model for optimising average travel time of freight trains transportation system in Canada.

## **3. Methodology**

### **3.1. Collection and Analysis of Data**

Non-random values required in the simulation model are scheduled arrival time of commuter at stations and maximum passenger capacity of commuters, whereas random variables required in the simulation model are number of passengers arrived at stations in different interval of time, proportion of passengers to certain destinations and dwelling time of commuter at stations. Arena Input analyser can then be used in modelling input random variables as probability distributions. Changes of random variable with time can be modelled with stochastic process especially application of non-homogenous Poisson process on modelling of arrival pattern of passengers on different interval of time.

Validation of simulation model have to be carried out to check simulation modelling error. Statistical models can be validated using statistical hypothesis testing. Goodness-of-fit test is a suitable tool in testing how well the probability distribution could explain the pattern of random variable. For those who failed goodness of fit test, Mann-Whitney test can be used to compare actual data and data simulated from certain probability distribution.

Performance of commuter transportation system can be measured from the aspects of waiting time of passengers at stations other than number of cases of passengers being left out at the related stations and number of those passengers. Simulation have to be repeated to obtain a better estimate of performance. Output of simulation can be obtained from the report generated by Arena software. Comparison can be made between the outputs and reasonable benchmarks to know how well the commuter transportation system is performing.

### **3.2. Development of Model**

Discrete event simulation technique was used in modelling of commuter transportation system. Events such as arrival of commuter and arrival of passengers at a station were modelled such that the events occurred on a point in time within the simulation model. The Arena software was chosen as main analysis tool in this study due to capability of visualisation on commuter network system. According to Law (2007), this software has the built-in capability of modelling non-homogenous Poisson process.

The framework of simulation model as suggested by Öztürk (2012) is used in this study except his method of modelling arrival pattern of passengers, because Kelton *et al.* (2010) pointed out that this method of modelling on their arrival pattern may lead to misleading estimation of arrival rate if the change in arrival rate between different intervals of time is huge. Hence, schedule module is used as a better alternative to this issue as recommended by Seila *et al.* (2003) and Kelton *et al.* (2010).

Structural assumptions and data assumptions were made in this simulation model as shown in the following:

- a. Passengers
  - i. Passengers do not know the actual arrival time of commuters at stations.
  - ii. Passengers would enter the commuters after passengers who are inside the commuters step out from commuter.
  - iii. Principle of “first in, first out” was applied.
  - iv. If event of missing the commuters due to reaching of maximum passenger capacity of commuters occurred, those passengers would not leave the station for alternative transportation.
  - v. Passengers would step out from commuters at desired destination only, in other words, passengers would not travel beyond the destination.
  - vi. All the passengers waiting at the platform would be able to enter the commuters within the dwell time.
  - vii. Passengers would not arrive at stations in group.
- b. Commuters and tracks
  - i. Commuters arrive at stations punctually as scheduled.
  - ii. Train failure would not occur.
  - iii. Total number of passengers within the commuters would not exceed the maximum passenger capacity of commuters.
- c. Arrival pattern of passenger at stations
  - i. Number of arrival of passenger at station within certain interval of time could be modelled as Poisson distribution.
  - ii. Arrival rates of passengers would not be changed within certain interval of time.
  - iii. The arrival process of passengers in a day is independent of the arrival process of passenger in another day.
  - iv. The arrival rates of passengers in a day are the same as the arrival rate of passengers in another day within the same interval of time.
  - v. Inter arrival times of passengers are assumed to be an independent and identically distributed exponential random variable.
- d. Proportion of passengers to certain destinations
  - i. Proportions of passengers to certain destinations are assumed to be constant.
- e. Dwell time at stations
  - i. Dwell times are assumed to be uniformly distributed.
  - ii. Distributions of dwell time would not be varied with time and with day.

Developed simulation model was verified from the aspects of its function and its model logic with debugging tool provided in the Arena software. Introducing trace to the simulation model allow the user to see the path of trace animation to detect possible logic errors in the

model. Testing process have been done on the output of simulation model with input whose outcome is known through alternative method such as manual calculation of value. Interactive run controller was applied in verification process by stopping the animation on anytime to check the current underlying values and random variables. The simulation model was also checked by third parties who are usually an expert in related field or those who are familiar with similar system to verify the logic of combination of various modules found in the model.

## 4. Development of Simulation Model

### 4.1. Passenger Flow

Let the number of passengers arrival at a station within the interval time of  $(0, t]$  be  $N(t), t \geq 0$ . By referring to the definition from Ross (2007), a counting process of passengers arrival at the station  $\{N(t), t \geq 0\}$  is said to be a Poisson process having arrival rate  $\lambda(t), t \geq 0$ , if

- a.  $N(0) = 0$
- b. The process has independent increments.
- c. The number of passengers arrival at a station in any interval of length  $t$  is Poisson distributed, that is for all  $s, t \geq 0$ ,

$$P(N(t+s) - N(s) = n) = e^{-[m(t+s) - m(t)]} \cdot \frac{[m(t+s) - m(t)]^n}{n!} \quad (1)$$

with  $n = 0, 1, \dots$  and  $m(t) = \int_0^t \lambda(u) du$ .

In Arena software, create module, decide module, assign modules and hold module were used in generating arrival of passengers at stations as shown in Figure 1. The create module on the left side of the figure represents the arrival of passengers at a station. The arrival rate of passengers could be scheduled as in Figure 2 with schedule module based on the definition of non-homogenous Poisson process. The decide module following the create module was assigned proportion of passengers to certain destinations from a station. After decide module, the assign module was assigned the destination of passengers. A queue at platform could be modelled by creating a hold module after assign modules.

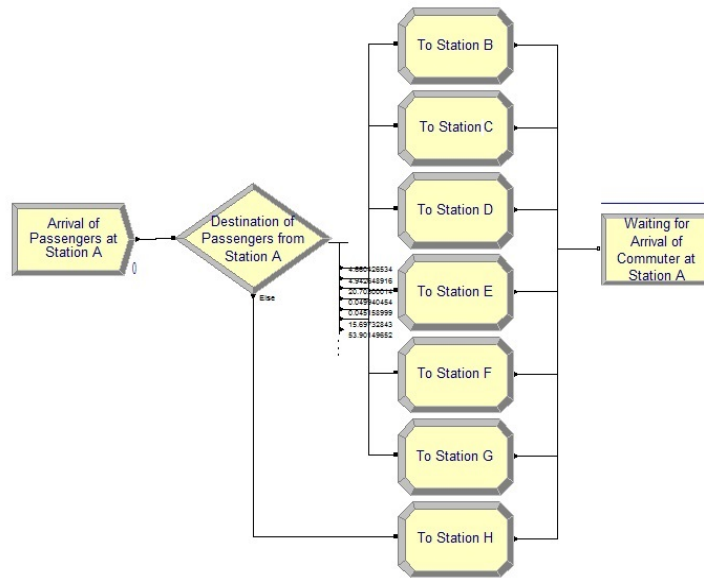


Figure 1: Example on modules used in modelling arrival of passengers at a station

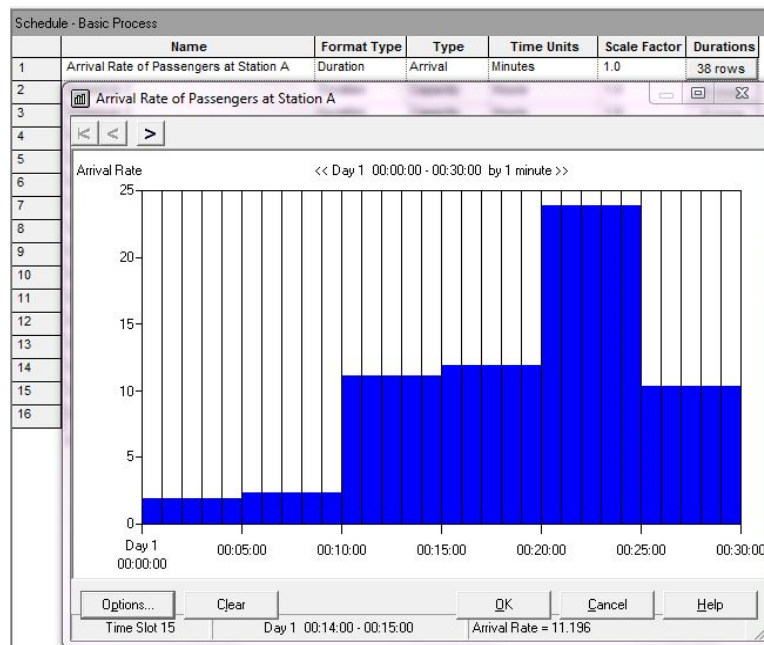


Figure 2: Example on scheduled arrival rate of passengers at a station in every five minutes

**4.2. Movement of Commuters**

Create module, process module and assign module are required at the terminal station to generate arrival of commuter as shown in Figure 3. Commuters were generated using create module. Scheduling of commuter departure which is similar to scheduling of arrival of passengers was made at the process module after the create module. Maximum passenger capacity would be assigned by the assign module, which connected the process module and the rest of the model.

Movement of commuters at a station required the combination of station module, assign modules, process module and route module as shown in Figure 4. Station module received arrival of commuter from other station. Arrival time of commuters was assigned as attribute of simulation entities such as commuters at assign module which is next to station module. Process module at the middle of the figure represents platform for picking or dropping passenger(s) at a station. Dwell time was set at this module. The assign module on the right of the process module was used to assigned departure time to another attribute of the entities. Route module was connected to this assigned module to control the movement of commuter from a station to the next station with assigned routing time and assigned next station.

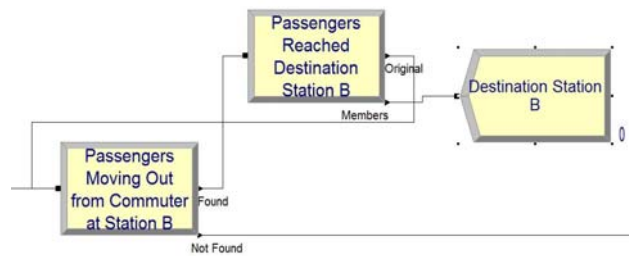


Figure 3: Example on combination of modules at terminal station

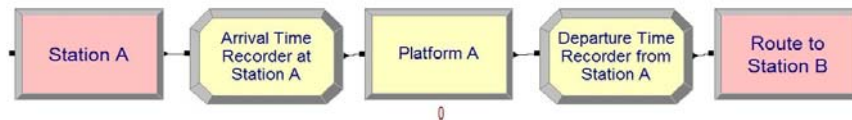


Figure 4: Example on modelling movement of commuter at a station

### 4.3. Interaction between Passengers and Commuters

Interaction between passengers and commuter involves the process of picking passengers and dropping of passengers at stations. Figure 5 shows that search module is required to determine whether station arrived by commuter is destination of passengers in the commuter. If that station is destination of a passenger, then drop module was used to release passengers from commuter. This process was repeated for every passenger in the commuter until destinations of every passenger in the commuter were checked. The passengers who reached the destination would be removed from the model by the dispose module. The remaining passengers in the commuter would be sent to the next stage.

Figure 6 shows the combination of decide module, pick modules and record modules used to represent the process of picking passenger waiting at a platform. When commuters reached decide module, two possible scenarios would happen, that are situation whereby maximum passenger capacity was not reached and otherwise. If the sum of passengers in the commuter and passengers waiting at the platform exceed the maximum passenger capacity of commuters, then decide module would send the commuter to the pick module with restricted amount of passengers picked, then the number of cases of passengers being left out at the station and its number of passengers involved will be recorded using record modules right after the pick module, else the commuter would be sent to another pick module with no restriction on the number of passengers picked.

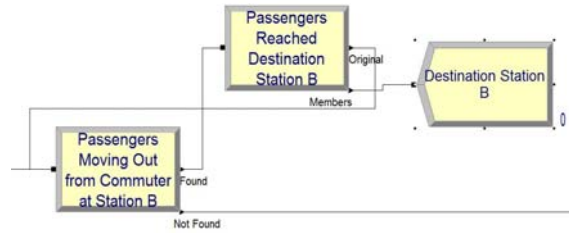


Figure 5: Example on process of dropping passenger at a station

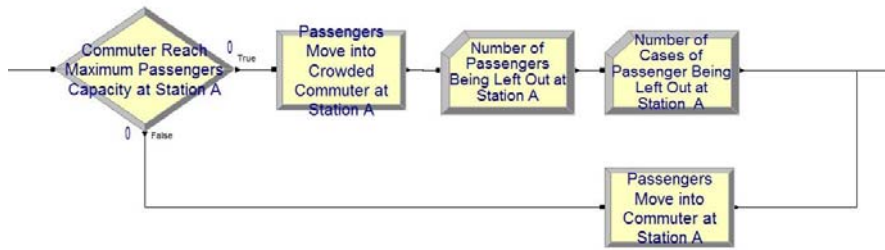


Figure 6: Example on process of picking passenger at a station

#### 4.4. Animation of Simulation Model

The combinations of various processes at a station could be arranged as shown in Figure 7. For terminal stations, the commuters were introduced into the system at the first station and were removed from the system at the last station. After the arrival of commuter at a station, the simulation model is designed such that the passengers move out from the commuter, followed by the movement of passengers who were waiting at the platform into the commuter. The whole process ended with the departure of commuter from a station.

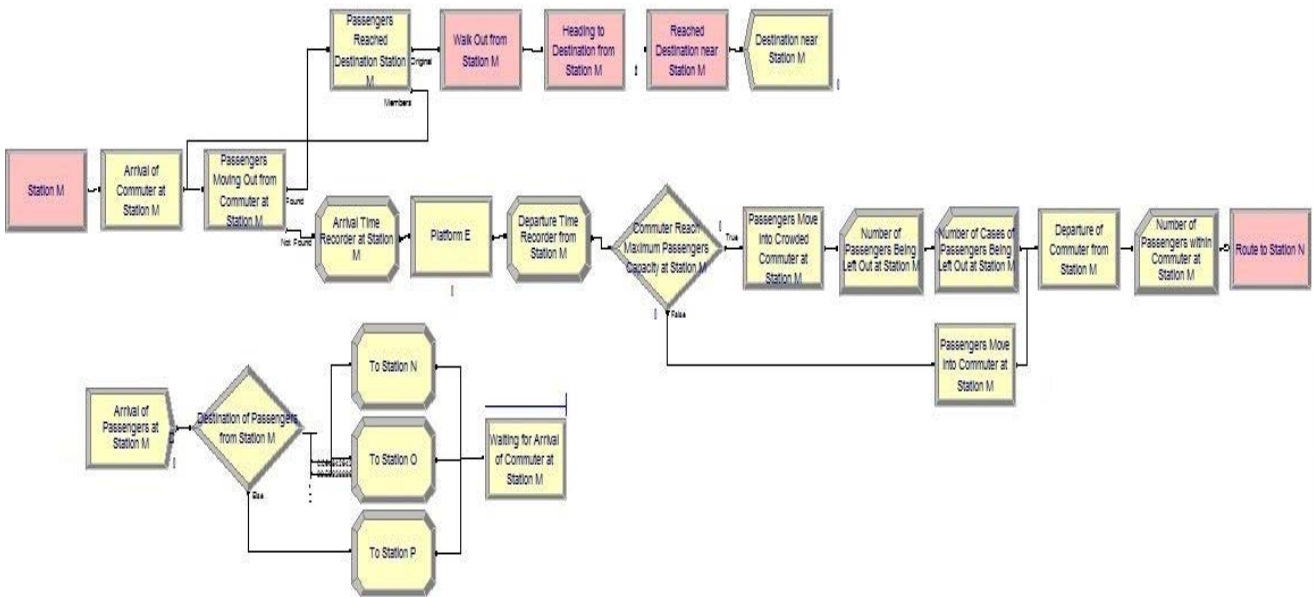


Figure 7: Example on combination of modules within a station



The whole process starting from the arrival of commuter to the departure of commuter can be placed within a submodel to represent a station in order to simplify the model presentation and to make the animation clearer. The queue at the platform, movement of commuters between stations and the movement of passengers coming out from the station can be displayed together with number of passengers in the queue, the cumulative number of passengers going to the destination station and simulation clock in the model. Animation tool such as route module, station module, store module, unstore module and others can be added to the model to make a better animation.

## 5. Application of Simulation Model

### 5.1. Background

The simulation model built was applied in comparing performance of two commuters with different capacity within commuter transportation network operated in Malaysia. The older class of three-coach commuters can be occupied up to approximately 450 passengers, whereas newer class of six-coach commuters can carried up to approximately 1118 passengers. Arrival process of passengers at eight stations were selected in the simulation study as shown in Figure 8. There are others stations beyond station P which are not shown in the figure. Only one side of direction of movement was considered to reduce complexity of simulation study.

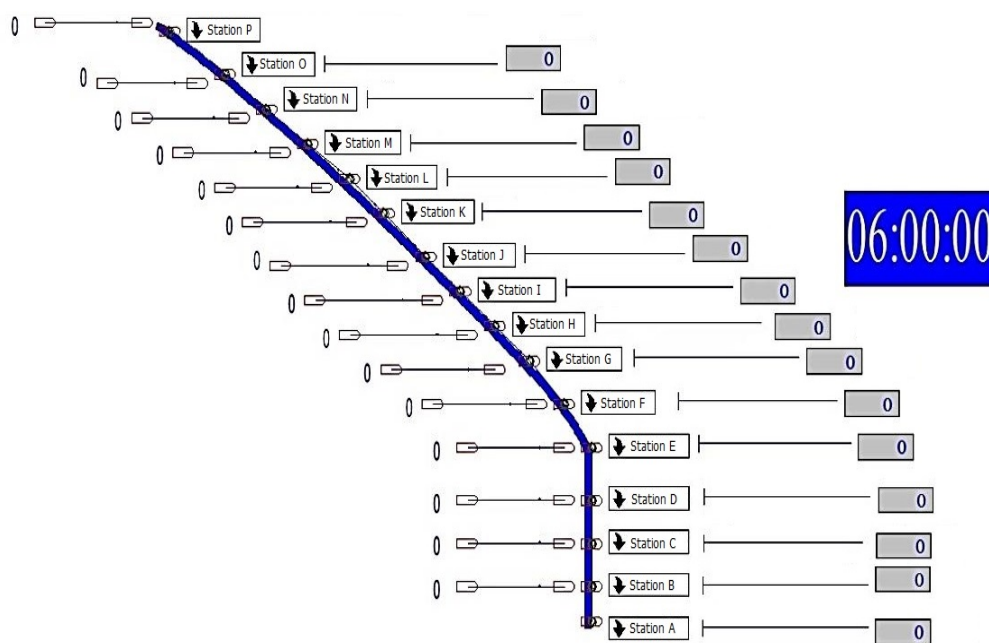


Figure 8: Example on application of model in commuter transportation system

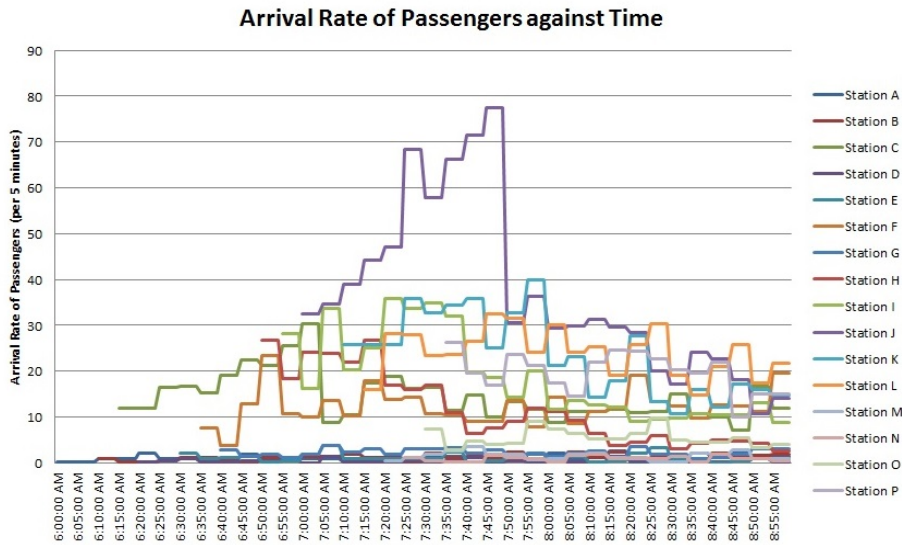


Figure 9: Arrival rate of passenger per 5 minutes against time for each station

The arrival rate of passenger against time in Figure 9 was measured such that it is the mean of number of passengers within time interval of 5 minutes. The continuous uniform distribution from 15 seconds to 30 seconds was applied for dwelling time of every station in the simulation model except for station P with continuous uniform distribution from 30 seconds to 60 seconds. Figure 10 shows the proportion of passengers from stations on the first column of the table to stations on the first row of the table. These proportions are used in the modelling arrival of passengers at a station.

Proportion (%)	Destination Stations															
Stations	Station B	Station C	Station D	Station E	Station F	Station G	Station H	Station I	Station J	Station K	Station L	Station M	Station N	Station O	Station P	
Station A	0.062762	0.397634	0.02954	0.053672	4.390877	0.057303	0.297016	1.524564	4.342895	4.605888	19.29243	0.046538	0.042082	14.62781	50.22899	
Station B		0.397884	0.029559	0.053706	4.393635	0.057339	0.297202	1.525521	4.345622	4.608781	19.30454	0.046567	0.042109	14.637	50.26053	
Station C			0.029677	0.053921	4.411186	0.057568	0.29839	1.531615	4.362982	4.627191	19.38166	0.046753	0.042277	14.69547	50.46131	
Station D				0.053937	4.412496	0.057585	0.298478	1.53207	4.364277	4.628565	19.38741	0.046767	0.042289	14.69983	50.47629	
Station E					4.414877	0.057616	0.298639	1.532897	4.366632	4.631063	19.39788	0.046792	0.042312	14.70776	50.50353	
Station F						0.060277	0.312433	1.603698	4.568317	4.844962	20.29382	0.048953	0.044266	15.38709	52.83618	
Station G							0.312621	1.604665	4.571073	4.847884	20.30606	0.048983	0.044293	15.39637	52.86805	
Station H								1.609698	4.585408	4.863087	20.36974	0.049137	0.044432	15.44465	53.03385	
Station I									4.660427	4.942649	20.703	0.04994	0.045159	15.69733	53.9015	
Station J										5.184257	21.71501	0.052382	0.047366	16.46465	56.53633	
Station K											22.90233	0.055246	0.049956	17.36489	59.62758	
Station L												0.071657	0.064796	22.52324	77.34031	
Station M													0.064843	22.53939	77.39577	
Station N														22.55401	77.44599	
Station O															100	

Figure 10: Proportion of passengers to certain destination stations

### 5.2. Simulation Output and Interpretation

After simulations like the animation shown in Figure 11 were replicated 100 times for six-coach commuters, the simulation output is compared with the output generated by replacing six-coach commuters with three-coach commuters. According to Table 1, none of the passengers waited for the arrival of six-coach commuter in more than the benchmark duration that was set to be 21.5 minutes. There is no case of passengers being left out by the

commuters at the stations. This is an acceptable performance because the waiting times of passengers for all the stations are 7 minutes to 9 minutes in average.

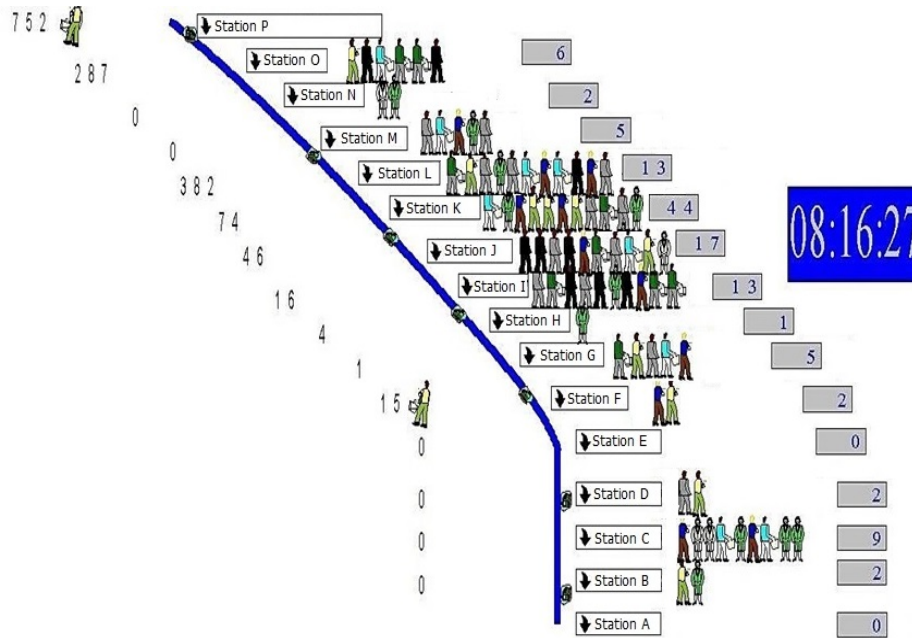


Figure 11: Screenshot on animation of commuter transportation system simulation model

Table 1: Waiting time of passengers at each station in the case of six-coach commuters

Station	Waiting Time (minutes)		
	Average	Half Width of 95% Confidence Interval	Maximum Value
Station A	8.5044	0.16	21.1339
Station B	7.8384	0.17	20.5900
Station C	7.6752	0.05	21.1237
Station D	7.5981	0.28	21.1971
Station E	7.0737	0.18	21.1117
Station F	8.1200	0.05	21.1646
Station G	7.5187	0.12	21.1084
Station H	8.0630	0.07	21.1607
Station I	7.8201	0.04	21.1561
Station J	7.7656	0.03	21.1902
Station K	7.8883	0.04	21.2248
Station L	7.5996	0.04	21.2036
Station M	8.4683	0.20	21.1546
Station N	8.0288	0.22	19.9500
Station O	7.6943	0.10	21.1381

According to Table 2, some passengers took more than 21.5 minutes to wait for the arrival of three-coach commuters at station J, station K, station M and station N. The usage of three-coach commuter during the peak hour proved to be not appropriate because the waiting time of passengers is 7 minutes to 16 minutes in average. In addition to that, there are passengers who wait for arrival of commuters in duration of more than 43 minutes. In other words, it is likely that those passengers may miss the commuter twice consecutively.

There are case of reaching maximum passengers capacity of three-coach commuters at station J, station K, station L and station N. Table 3 shows that station K has the highest number of case of reaching maximum passengers capacity, that is approximately 3cases per day in average. However, Table 4 shows that station J has the highest number of passengers being left out by the commuters, that is nearly 117 passengers per case. This number may reach as high as 199 passengers per case. In short, the total average on number of case of reaching maximum passengers capacity of three-coach commuters is approximately 7 cases per day, and for every case, there are approximately 227 passengers, on average, that may involved in those cases.

Table 2: Waiting time of passengers at each station in the case of three-coach commuters

Station	Waiting Time (minutes)		
	Average	Half Width of 95% Confidence Interval	Maximum Value
Station A	8.5043	0.16	21.1339
Station B	7.8384	0.17	20.5900
Station C	7.6750	0.05	21.1237
Station D	7.5983	0.28	21.1971
Station E	7.0734	0.18	21.1117
Station F	8.1205	0.05	21.1646
Station G	7.5186	0.12	21.1084
Station H	8.0635	0.07	21.1607
Station I	7.8208	0.04	21.1561
Station J	8.9942	0.07	24.3032
Station K	15.7386	0.51	43.6654
Station L	7.7917	0.06	21.2036
Station M	11.1897	0.39	35.7031
Station N	10.2976	0.56	53.7150
Station O	7.6917	0.10	21.1381

Table 3: Number of case of reaching maximum passengers capacity of three-coach commuters

Station	Number of Cases	
	Average	Half Width of 95% Confidence Interval
Station J	1.03	0.03
Station K	3.26	0.09
Station L	0.70	0.10
Station M	0.98	0.09
Station N	1.01	0.13

Table 4: Number of passengers being left out by the three-coach commuters at the related stations

Station	Number of Passengers			
	Average	Half Width of 95% Confidence Interval	Minimum Value	Maximum Value
Station J	116.8800	4.78	1	199
Station K	88.9992	3.91	2	159
Station L	10.6350	2.10	0	46
Station M	7.5950	0.83	0	18
Station N	3.2050	0.49	0	10

## 6. Conclusion and Recommendation

This paper has shown that it is possible to build a simulation model for commuter network system with Arena 14.5. The simulation model can be used to evaluate the performance of commuter transportation system from the perspective of waiting time of passengers at stations. The output of the simulation study in Malaysia showed that newer class of commuters outperform older class of commuters.

The scope of this simulation study can be extended to wider scope. Delay on arrival of trains on station due to train failure, maximum safe speed at certain location of track, traffic light along the track and headway can also be considered in the simulation study. Instead of waiting time of passengers at station, other indicators such as total travelling time of passengers, utilisation of trains and tracks can be used as additional measures to performance of train transportation system. Efficiency of system and optimisation of system performance are possible choices of study on train transportation system.

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